Importance of 3D Transformations for Displaying of Medical Images

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Abstract - **In this paper the types of 3D transformations in displaying of real medical images are given.**

Keywords - **Medical images, Education on treatment medical signals and images, Distance education**

I. Introduction

The creation of real 3D medical images is the combination from two fundamental processes – modeling (defining geometry of medical objects) and representing (displaying) these mathematical models. Transformations can be used to rotate, translate or scale an image to obtain an understanding of its 3D shape. This is particularly important in medicine in which the medium for displaying pictures is the two-dimensional display screen on which depth information may not be obvious. Techniques for expressing 3D transformations are represented by extending the 2D techniques. Right-handled coordinate systems are used. A point in 3D space (x, y, z) is represented by a four-dimensional position vector (x, y, z, w). This point may then be transformed by the following matrix operation:

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \qquad (1)$$

II. Types Of 3D Transformations

In order to obtain the 3D coordinates from the transformed homogeneous point, the x-, y-, z- components are divided by w- component:

$$x''=x'/w'; \quad y''=y'/w'; \quad z''=z'/w'; \qquad (2)$$

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Translation - the matrix to perform 3D translation of a medical image is:

$$\begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 1 & 0 & h \\ 0 & 0 & 1 & l \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

Matrix element **d** is the displacement added to x- coordinate, **h** – is the displacement added to y- coordinate, and **l** is added to the z- coordinates.

Scaling – 3D scaling is performed by the element on main diagonal of the matrix:

$$\begin{bmatrix} a & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & k & 0 \\ 0 & 0 & 0 & p \end{bmatrix} \qquad (4)$$

The x-, y-, and z- scale factors are given by **a**, **f**, and **k** respectively. The element **p** provides overall scaling by a factor of 1/p. The scaling is used to change the size of images.

Rotation – the terms in the upper-left 3x3 component matrix control 3D rotation of the medical images:

$$\begin{bmatrix} a & b & c & 0 \\ e & f & g & 0 \\ i & j & k & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (5)$$

The basic 3D rotations are: rotation about the x- axis, rotation about y- axis and rotation about z- axis.

A rotation about z- axis is equivalent to 2D rotation about the origin. Hence the x and y terms of the matrix may be write down straight away. Since the rotation is about z- axis, the z-coordinates should not be changed, and so the z- row and column should both be [0 0 0 1] (as in the identity matrix). Similarly for rotation about x and y , the row and column corresponding the axis of rotation are taken from the identity matrix. The cosine and sine terms are then used to fill the remaining elements of the 3x3 component matrix. The matrices for rotation about the three axes are:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ Rotation about the x-axis; (6)

$$\begin{bmatrix} \cos\alpha & 0 & \sin\alpha & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\alpha & 0 & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ Rotation about the y-axis; (7)

$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ Rotation about the z-axis; (8)

Shearing – The off – diagonal elements in the upper – left 3x3 component matrix produce 3D shearing effects:

$$\begin{bmatrix} 1 & b & c & 0 \\ e & 1 & g & 0 \\ i & j & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (9)$$

In 3D shear in x may be obtained as a function of the y- and z- coordinates. This is controlled by matrix elements **b** and **c** respectively. Similarly elements **e** and **g** control shearing in y as a function of x and z elements, **i** and **j** control shearing in z as a function of x and y.

Combination of basic 3D transformations of the medical images. A common requirement is to rotate an object about an arbitrary axis rather than one of coordinate axes. A 3D rotation about an arbitrary axis involves transforming the object and axis of rotation so that the axis coincides with one of the coordinate axes, followed by a rotation about the coordinate axis and finishing with a transformation which is the inverse of the first. The individual steps are as follows:

- Translate so that axis of rotation passes through the origin;
- Rotate medical object so that axis of rotation coincides with one of the coordinate axes;
- Perform the specified rotation about appropriate coordinate axis;
- Apply inverse rotation to bring axis of rotation back to original orientation;
- Apply inverse translation to bring rotation axis back to original position;

III. Perspective Transformations Of 3D Medical Images

All of the 3D transformations matrices examined so far have been of the form

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ 0 & 0 & 0 & p \end{bmatrix} \qquad (10)$$

i.e. elements **m**, **n**, and **o** are equal to zero. This section looks at the the effect achieved when one or more of these values is non-zero. Consider the following transformation

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \gamma & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} =$$

$$= \begin{bmatrix} x \\ y \\ z \\ \gamma.z+1 \end{bmatrix} \qquad (11)$$

After homogeneous division the real 3D coordinates of the transformed point of the medical image are (x", y", z") where

$$\begin{aligned} x'' &= x / (\gamma.z+1) \\ y'' &= y / (\gamma.z+1) \qquad (12) \\ z'' &= z / (\gamma.z+1) \end{aligned}$$

As homogeneous z–coordinates tends to infinity

$$x''\to 0;\ y''\to 0;\ z''\to 1/\gamma$$

Hence, after transformation of medical image lines originally parallel to the z- axis will appear to pass through the point (0, 0, 1/γ), known as the **vanishing point**. This kind of transformation is known as a **perspective** transformation of medical objects.

A perspective transformation has a distorting effect which gives the transformed medical object a natural appearance, similar to that which would be seen by eye from the point (0, 0, -1/γ). This is known as a **centre of projection**.

Different types of perspective transformation of medical objects are obtained if the other two elements of the bottom row are set. For example, the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \alpha & 0 & 0 & 1 \end{bmatrix} \qquad (13)$$

would create perspective transformation with a vanishing point for lines originally parallel to the x- axis at (1/α, 0, 0) and centre of projection at (-1/α, 0, 0). Similarly the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \beta & 0 & 1 \end{bmatrix} \quad (14)$$

would create a perspective transformation with a vanishing point for lines originally parallel to the y- axis at (0, 1/β, 0) and centre of projection at (0, -1/β, 0). Perspective transformations with only one vanishing point are known as one-point perspective transformation. If two or three of the matrix elements are non-zero together, a two or three point perspective transformations are obtained.

IV. Points Behind The Eye Point

In this section is shown a perspective transformation applied to point (x, y, z) with the eye point at (0, 0, c):

$$\begin{bmatrix} x' \\ y' \\ z' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/c & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} =$$

$$= \begin{bmatrix} x \\ y \\ z \\ (c-z)/c \end{bmatrix} \quad (15)$$

The value of w' varies depending on the value of the original z- coordinate: $z < c \rightarrow w'>0$; $z = c \rightarrow w' = 0$; $z > c \rightarrow w' < 0$;

This is illustrated in Fig.1.

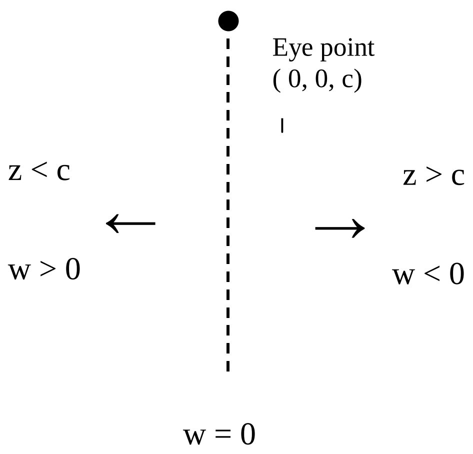

Fig. 1 - Variation of w' with original z- coordinates in a perspective transformation

Consider a line which joins the points $\mathbf{p_1}$ and $\mathbf{p_2}$ which are positioned on either side of the eye point ($\mathbf{p_1}$ has z<c and $\mathbf{p_2}$ has z>c). After transformations $\mathbf{p_1}$ will have a positive w value and $\mathbf{p_2}$ will have a negative w value. Fig. 2 shows the transformed line in homogeneous coordinate space. Homogeneous division then project the transformed points $\mathbf{p_1'}$ and $\mathbf{p_2'}$ onto the w =1 plane. The resulting line however is not the line joining the projected points $\mathbf{p_1''}$ and $\mathbf{p_2''}$. In Fig. 3 other points along the line $\mathbf{p_1'}$ $\mathbf{p_2'}$ are projected onto the w = 1 plane. It can be seen from this that the projected line is actually in two parts: $\mathbf{p_1''}$ to positive infinity and negative infinity to $\mathbf{p_2''}$. These are known as *external line segments* and is the correct interpretation of the application of a perspective transformation to a line joining points on either side of the eye point.

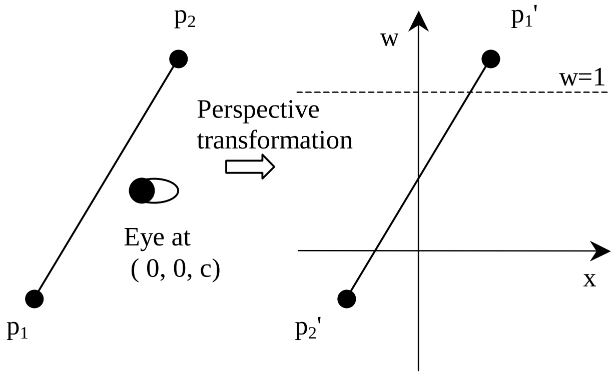

Fig.2 – Perspective transformation of the line p_1p_2

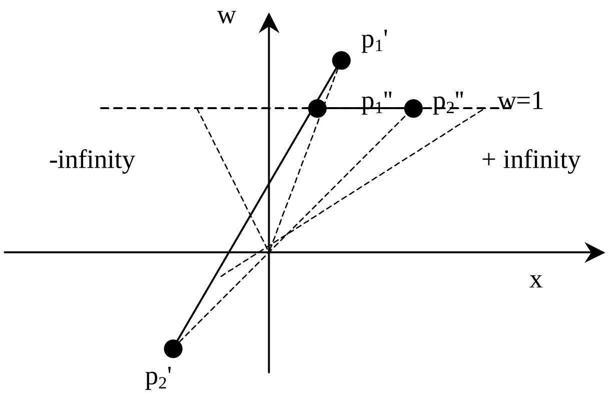

Fig.3 – Projecting line $\mathbf{p_1'}$ $\mathbf{p_2'}$ onto the w = 1 plane

Clipping – In general the results of a perspective transformations simulate what the eye would actually see when medical objects are displaying. Since the eye can only see objects in front of it, items behind the eye should be clipped out. This can be done in one of two ways:

• Clip before the perspective transformation by removing all those parts with $z \geq c$.

• Clip after perspective transformation but before homogeneous division, in this case remove parts with $w \leq 0$.

After the homogeneous division it is impossible to distinguish between points which were originally behind in front of the eye.

V. CONCLUSION

The main methods for transformation of points of 3D medical images are translation, scale, rotation and shear.

The 3D transformations of the medical objects are useful as a tool for creating and subsequently altering an image. They can also help to visualize the three-dimensional shape of the resulting medical image.

For objectives of education these methods can be used in simulations of real processes at training and examining.

REFERENCES

[1] Hearn D. and Baker M. P. , Computer Graphics, Prentice – Hall, 1986.

[2] Blinn J. F. and Newell M. E. , Clipping Using Homogeneous Coordinates, Proceedings of SIGGRAPH , pp. 245 – 251.

[3] Foley J. D. , van Dam A. , Feiner S. K. and Hughes J. F. , Computer Graphics : Principles and Practice (2nd Edition), Addison Wesley, 1990.