Physical and Mathematical Modeling of an Optical Medium – Quartz Fiber

Ivan S. Kolev and Ivelina S. Stoeva

Abstract – **Nowadays the most often used optical medium is the quartz fiber. Fibers are divided into one-mode and multimode fibers. The multimode fibers on their part are divided into step and gradient fibers. The target of the present paper are the quartz multimode fibers with a step profile of the index of refraction.**

- **Keywords – Optical fibers, Mode dispersion; Wave (spectral) dispersion, Quartz Fiber.**

I. OPERATION MODES

The following media are used as optical media in communications:

- Air and vacuum;
- Optical fibers (Quartz or polymer);

There are three main factors which influence the change of the output pulse parameters towards the input pulse:

- Mode dispersion;
- Wave (spectral) dispersion;
- Delay of the fastest mode of the output signal towards the input.

All these three factors have been mathematically described by means of links with pure delay and two aperiodic links.

A physical model of the optical fiber has been developed on this basis. A mathematical expression of the slowest mode towards the fastest has been worked out. This expression contains parameters accessible for the company - introducer.

The time of delay has been optimized towards the indices of refraction of the core and the fiber Digital expressions of the parameters have been assigned and the values received have coincided with a certain allowance with these cited by leading companies. An expression of the maximum operating frequency of the optical fiber directly related to the signal expansion has been derived.

The expressions derived have been checked for polymer multimode fibers with a step profile of the index of refraction.

The transfer function, the pulse transition function and the amplitude-frequency characteristic of the quartz fiber have been derived.

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As a result of the expressions worked out, it can be recommended to change the known refractive indices of the fiber and the core in order to widen the frequency band of the fiber.

The expressions derived allow for a computer simulation of the optical fiber operation. This is a subject our team is working on at present.

$$P(L) = P(0).10^{\frac{-aL}{10}}, W \qquad (1)$$

Where:
P(0) – optical power in W set in the beginning of the fiber;
L – length in km;
A – coefficient, measure of attenuation in dB per unit of length in km.

$$A = a.L.10\lg\frac{P(0)}{P(L)}, dB \qquad (2)$$

Where A is an optical attenuation
hence, the coefficient of optical attenuation A is

$$a = \frac{A}{L} = \frac{10}{L}\lg\frac{P(0)}{P(L)}, dB/km$$

The pulse dispersion (expansion) on the output will be:

$$\tau(L) = \sqrt{t_{изх}^2 - t_{вх}^2} \qquad (3)$$

$t_{изх}$ - pulse duration on the output
$t_{вх}$ - pulse duration on the input

II. THE FREQUENCY BAND FOR MULTIMODE FIBERS

The frequency band for multimode fibers can be defined by the following expression:

$$f_{\max} = \frac{0,44}{\tau} \qquad (4)$$

$$\tau = \sqrt{t_{изх}^2 - t_{вх}^2} = M(\lambda).\Delta\lambda_{FRMS}.L, ps \qquad (5)$$

M(λ) – dispersion at maximum spectral radiation, ps/(nm.km)

$$\Delta\lambda_{FRMS} = \frac{1}{\sqrt{\ln 4}} = \Delta\lambda.0,85 \qquad (6)$$

Conclusion: Dispersion influences the frequency band. (4)

III. REFRACTIVE INDEX OF THE CORE AND REFRACTIVE INDEX OF THE FIBER COATING

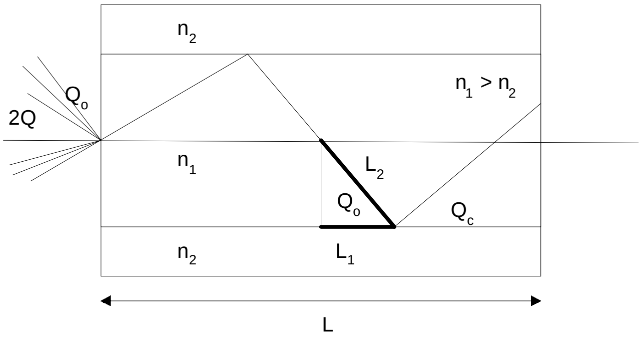

Fig.1

2Q – angle of the beam, entering the core (entrance angle)

$$\sin Q_o = \sqrt{n_1^2 - n_2^2} \qquad (7)$$

n_1 - refractive index of the core

n_2 - refractive index of the fiber coating

$$Q_c = ar\cos\frac{n_2}{n_1}$$ - critical angle

$$\frac{L_1}{L_2} = \cos Q_c$$

the shortest way (mode L' arrives fastest)

L'=ΣL$_1$

the slowest mode (mode L˝ arrives most slowly)

L˝=ΣL$_2$

If the fastest mode covers the distance L, the slowest mode will cover the distance L+ΔL

The speed of each mode (each beam) is equal at n_1=const,

$$\frac{L}{\cos Q_c} = \frac{L}{(n_2/n_1)} = \frac{L.n_1}{n_2} \approx L + \Delta L$$

$$\Delta L = \frac{L.n_1}{n_2} - L = L\frac{(n_1 - n_2)}{n_2} \qquad (8)$$

The speed of the mode $V = \dfrac{C}{n_1}$, where C – velocity of light =300 000km/s

The time lag of the slowest mode towards the fastest mode is:

$$\Delta t = \frac{\Delta L}{V} = \frac{\dfrac{L(n_1 - n_2)}{n_2}}{C/n_1} = \frac{Ln_1(n_1 - n_2)}{C.n_2} = \frac{L}{C}\frac{n_1}{n_2}(n_1 - n_2)$$

Example: L=1km; n_1=1,48; n_2=1,46; C= 300 000km/s

The signal expansion is:

$$\Delta t = \frac{1km}{300000km/s}.\frac{1,48}{1,46}(1,48 - 1,46) = 67,5ns \qquad (9)$$

(multimode quartz fiber)

The delay of the fastest mode per 1км of length is:

$$t_d = \frac{L}{C/n_1} = \frac{1km}{300000/1,48} \approx 4,93\,\mu s \qquad (10)$$

The delay of the slowest mode is: t_d+Δt, i. e. 4,93μs+67,5ns=4,9975μs.

The form of the input and the output signal is shown in Fig.2

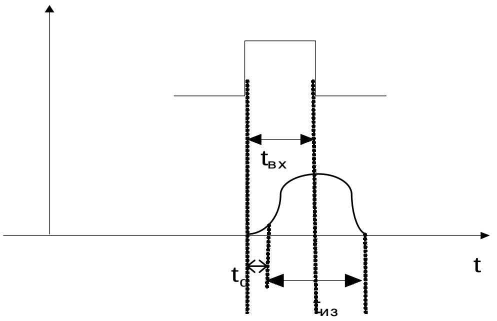

Fig.2

An example of plastic multimode step fibers:

L=1km; n_1=1,492; n_2=1,417; C= 300 000km/s

$$\Delta t = \frac{1km}{300000km/s}.\frac{1,492}{1,417}(1,492 - 1,417) = 263ns$$

Conclusion: The frequency band is

$$f_{\max} = \frac{0,44}{\sqrt{(t_{вх} + \Delta t)^2 + t_{вх}^{\ 2}}} \qquad (11)$$

We have taken into account only the mode dispersion so far. The other factor for pulse expansion is the dispersion of λ (wave dispersion).

Conclusions:

The output signal differs from the input signal as follows:

1. Pure delay of the signal (t_d) from the mode transition time
2. Dispersion of the output signal towards the input, determined by two factors:

- Mode dispersion
- Wave (spectral) dispersion.

On this basis, the electron model of the optic mean (quartz fiber) is built. It consists of three components (Fig. 3):

IV. DELAY COMPONENT

$W_1(P) = e^{-pT_1}$ (It reads the pure delay of the signal- Fig. 4)

Aperiodic component (gives the mode dispersion – Fig. 5)

$$W_2(P)=\frac{k}{1+p.T_2}$$ - Transfer function

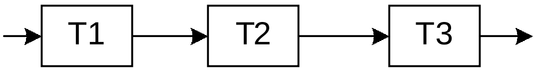

Fig.3

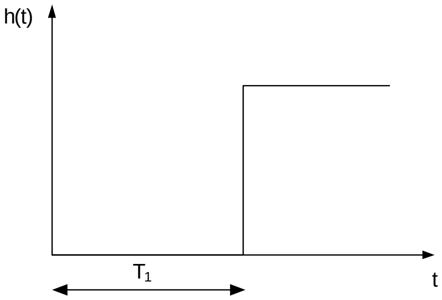

Fig.4

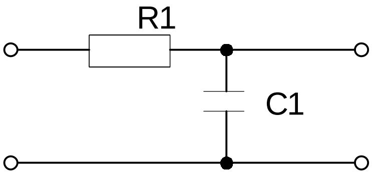

Fig.5

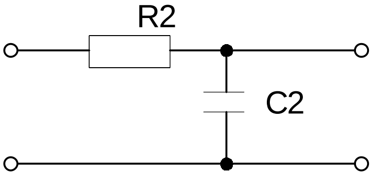

Fig.6

$$h(t)=k(1-e^{\frac{-t}{T_2}}).1(t)$$

Pulse transition function

$$L(w)=20\lg\frac{k}{\sqrt{1+w^2T_2^2}}$$

Logarithmic amplitude-frequency characteristic

$W_2(P)=\dfrac{k}{1+p.T_2}$ Se $W_2(P)=\dfrac{k}{1+p.T_2}$ - Second (aperiodic) component reads the wave dispersion

$W_3(P)=\dfrac{1}{1+p.T_3}$ - (Third aperiodic) component – Fig. 6

$$W(P)=\frac{k.e^{-p.T_1}}{(1+p.T_2).(1+p.T_3)}$$ Common transfer function

Therefore:

$$T_1=\frac{L}{C/n_1}=\frac{L.n_1}{C}$$

$$T_2=\frac{L}{C}.\frac{n_1}{n_2}(n_1-n_2)$$

$$T_3=M(\lambda).\Delta h.0,85.L$$

VI. CONCLUSION

The results of this paper will be used for lectures and seminars with students in the programmes of “Optoelectronics”, “Optoelectronic systems” and “Optoelectronics and Optical Communications”.

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