Educational Web Site "Research of the Behavior of Functions with Pseudo asymptotes and Asymptotes"

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Abstract – **The goal of the paper is to present a Web site for the research of the behavior of functions with pseudo asymptotes and asymptotes by using a system for computing algebra MATHEMATICA 4.0. A criterion for existence of pseudo asymptotes and asymptotes of differentiable functions is announced.**

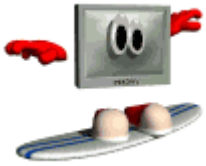
Keywords – **Criterion for the existence of the pseudo asymptotes and asymptotes of differentiable functions, systems for computing algebra**

I. WHY WAS THIS SITE CREATED?

The educational site was created to teach of students and cadets a practical exercise titled "Research of the behavior of functions with pseudo asymptotes and asymptotes". It was written in HTML by using the editor Macromedia Dream weaver MX.

II. SUMMARY OF THE SITE

At figure 1 the site's navigation toolbar is presented.



Home page | Introduction | Definitions | Criterion for pseudo asymptotes and asymptotes | Class of functions with pseudo asymptotes and asymptotes | Plan for a practical exercise | Graphics of functions with pseudo asymptotes | Graphics of functions with asymptotes | Conclusions | Literature

Fig. 1. Navigation toolbar of the site

The description of the Web pages is given below:

Page **Introduction** presents an overview of the Web site.

Page **Definition** gives a definition of the concept pseudo asymptote of functions of one independent variable [2], [3], that is not presented in the classic courses in mathematical analysis [1]. We point out the important fact that the logarithmic function is an example for such a function (its pseudo asymptote is the x-axis).

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Thereby the use of logarithmic tables is explained in a different way: big numbers are avoided.

Page **Criteria for pseudo asymptotes and asymptotes** announces a criterion for the existence of pseudo asymptotes and asymptotes of differentiable functions. This criterion separates the twice-continuous differentiable functions into three groups: functions without asymptotes, functions with pseudo asymptotes and functions with asymptotes. The proof of this criterion in details is made in [3]. Because of the importance of this criterion we describe it here:

Criterion (enough condition) for the existence of pseudo asymptotes and asymptotes of differentiable functions:

Theorem: $f : R \to R$ *is twice continuously differentiable function. Then:*

1) If $f''(x) \geq \frac{1}{1+x}$ *,*$x \geq 0$*,function f(x) has no asymptotes for* $x \to \infty$ *;*

2) *If* $\frac{1}{1+x^{1+l}} \geq f''(x) \geq \frac{1}{1+x^2}, x \geq 1,$ $0<l<1$ *f(x) has pseudo asymptote, but it has not asymptote;*

3) If $0 < f''(x) \leq \frac{1}{1+x^{2+l}}, l > 0, x \geq 0$ *f(x) has asymptote for* $x \to \infty$ *.*

Remark: By analogy we can formulate and prove a theorem for criteria (enough condition) for the existence of pseudo asymptotes and asymptotes of differentiable functions when $x \to -\infty$

Page **Class of functions with pseudo asymptotes and asymptotes** presents formulae for classes of special functions with pseudo asymptotes and asymptotes by using of the system for computing algebra MATHEMATICA 4.0 after integrating twice.

Page **Plan for a practical exercise** includes:

1. Theoretical part: the students have preliminarily attended lecture and have been acquainted with the theme and proof of the criterion. The professor accents the basic ideas and points out the existence of an extra wide class of partial and special functions with pseudo asymptotes He emphasizes the complexity of the calculations and the practical advantage of the systems for computing algebra for finding the formulas.
2. The professor suggests that the students themselves find formulae for functions with pseudo asymptotes and asymptotes.
3. The students draw graphics for the functions received in step 2.

4. Comparisons are made between the functions' graphics and the differences in their behavior toward their pseudo asymptotes and asymptotes are explained.
5. If several systems for computing algebra are installed, comparisons are made between the results received by identical tasks.
6. Conclusions: The possibility is debated of making errors and inaccuracies when working with the systems for computing algebra, finding and eliminating the errors. The advantage of work with the systems for computing algebra is emphasized for receiving and illustration of serious mathematical results.

Page **Graphics of functions with pseudo asymptotes** enables student by using of drop-down menu to choose between two functions with pseudo asymptotes, to consider their formulas and to explain their graphical behavior toward their pseudo asymptotes in intervals of different length of change of the independent variable x. In Figure 2 and Figure 3 two such graphics are visualized for the case of $f''(x) = \frac{1}{1 + x^{1,25}}$, i.e. for l = ¼. The function has a pseudo asymptote, but has not an asymptote.

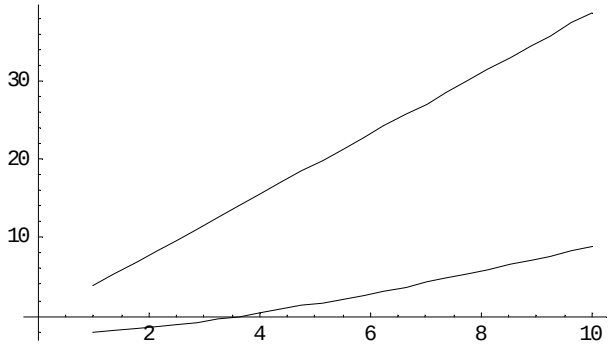

Fig. 2. When x is changed in the interval [0, 10], it is clearly visible that the function graphic has moved away from such graphic of its pseudo asymptote when x grows up.

It can be proved that the difference between the values of function and its pseudo asymptote for the same unlimited large values of x tends to $-\infty$.

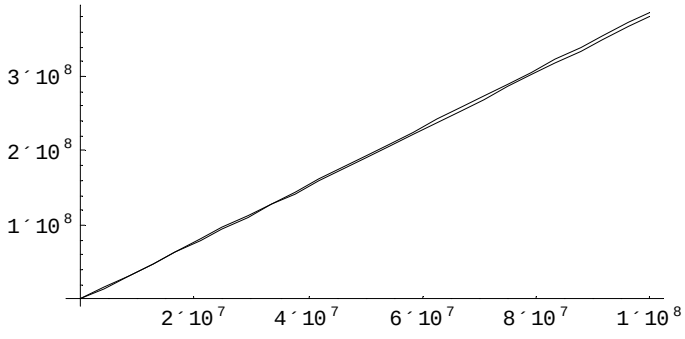

Fig. 3. For very large x the two graphics: of the function and its pseudo asymptote are almost indiscernible.

Page **Graphics of functions with asymptotes** enables student by drop-down menu to choose between two functions with asymptotes, to consider their formulas and to explain their graphical behavior toward their asymptotes in intervals of different length of change of the independent variable x. We consider the case: $f''(x) = \frac{1}{1+x^4}$, i.e. when l = 2. The function is with asymptote. At figure 4 the graphic of the function and its asymptote is shown. As expected for comparatively little x, the difference between the two graphics is slight.

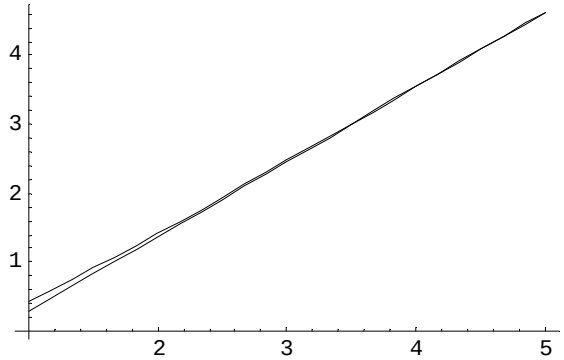

Fig. 4. For the x-values in the interval [0, 5] the difference between two graphics is slight

Page **Conclusions** presents the conclusions made when we compare the functions with pseudoasymptotes and asymptotes described in the previous pages. As expected only for large values of the argument, the graphic of a function with pseudo asymptote and such graphic of its asymptote are indiscernible, while for the functions with asymptote this is observed for considerably smaller x-values.

III. CONCLUSION

The proof of the criteria for existence of functions with pseudo asymptotes and asymptotes is made analytically. Drawing the graphics and finding the described functions is made by the system for computing algebra MATHEMATICA 4.0 The application of the systems for computing algebra realizes still more the receiving of serious mathematical results, as well its use of illustration visual aids and in this sense it must be demonstrated to the students at all levels by the professors.

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