On the effect of cochannel interference on average symbol error probability of MQAM in Nakagami fading channels

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Abstract –In this paper the analytic expression is derived for the average symbol error probability of M-ary quadrature amplitude modulation (MQAM) in Nakagami fading channels in the presence of additive white Gaussian noise (AWGN). Using this expression numerical results are obtained and are shown in graphical, which represent the effect of cochannel interference on average symbol error probability for three constellations: 4-QAM, 16-QAM i 64-QAM.

Keywords - MQAM, cochannel interference, average symbol error probability, Nakagami fading channels.

I INTRODUCTION

During the transmission of digital modulated signals through communication channels, multipath propagation causes the amplitude fluctuation of received signal in short term of time. This effect is known as a fading. Development of wireless mobile communications requires transmission of data with speed in range from few tenth to hundred Mb/s. One of several problems which appears as following effect is intersymbol interference which is caused by multipath fading. This problem increase equally with increasing of transmission speed. In different modulation schemes the transmission speed of information could be effectively increased if we accept two elementary signals, thereby the multilevel modulation schemes became standards in a many of telecommunication systems [1].

In reference [1] is shown a process for determining the average symbol error probability for M-ary quadrature amplitude modulation (MQAM) scheme. This is the modulation scheme often used in the terrestrial microwave and satellite communication systems. In this paper we directly use the known symbol error probability results for MQAM [3] valid for additive white Gaussian noise (AWGN) as the conditional error probability before averaging over the probability density function (pdf) of the (SIR) signal-to-interference ratio for Nakagami fading [2].

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II SYSTEM MODEL

In this paper the effects of cochannel interference and multipath fading on the symbol error probability are considered for Nakagami fading channels. Nakagami channel model is more general than the Rayleigh channel model and is commonly used to characterize the urban and digital mobile radio environment. Considerations of system performances, which are limited by the interference in Nakagami fading environment, are based on the calculation of probability of error [2]. It is of interest, therefore, to consider the effect of cochannel interference on the average symbol error probability in Nakagami fading channels.

The desired signal power in a Nakagami fading channel is a gamma distributed random variable with probability density function (pdf) [2] given by Eq.(1)

$$p_{s}(x) = \left(\frac{m_{s}}{\Omega_{s}}\right)^{m_{s}} \frac{x^{m_{s}-1}}{\Gamma(m_{s})} \exp(-\frac{m_{s}}{\Omega_{s}}x)$$
(1)

where are:

 Ω_s - the average signal power

 m_s - Nakagami fading parameter

 $\Gamma(a) = \int_{0}^{\infty} e^{t} t^{a} dt$ is gamma function

We assume that L mutually independent Nakagami faded interference signals are also present at the receiver. When the interference are at approximately the same distance from the mobile station, they can also be assumed to have the same average power. The probability density function of the total interference power [2] is given by Eq.(2)

$$p_I(x) = \left(\frac{m_I}{\Omega_I}\right)^{m_I L} \frac{x^{m_I L - 1}}{\Gamma(m_I L)} \exp(-\frac{m_I}{\Omega_I} x)$$
(2)

where are:

 Ω_I - the average interference power

 m_1 - Nakagami fading parameter for the interference signals

In a typical microcellular environment in which the desired signal as well as the interfering signals undergo Nakagami fading, it is reasonable to assume that the cochannel interferers experience much deeper fading than the desired signal. Therefore is $m_s \ge m_I \ge 1/2$. In an interference-limited system the effect of noise may be ignored. In this case it can

be shown that the signal-to-interference ratio $\gamma = S/I$, has the pdf [2] given by Eq.(3)

$$p_{\gamma}(\gamma) = \frac{\Gamma(m_{s} + m_{I}L)}{\Gamma(m_{s})\Gamma(m_{I}L)} \left(\frac{m_{s}}{q_{SIR}m_{I}}\right)^{m_{s}} \frac{\gamma^{m_{s}-1}}{\left(\frac{m_{s}}{q_{SIR}m_{I}}\gamma+1\right)^{m_{s}+m_{I}L}} (3)$$

where the ratio $q_{SIR} = \frac{\Omega_s}{\Omega_I}$ is the average-signal to average interference ratio (SIR), which is useful in determining the

average symbol error probability.

III AVERAGE SYMBOL ERROR PROBABILITY

In an interference limited environment, the average symbol error probability of M-ary quadrature amplitude modulation (MQAM) in Nakagami fading channels [3] can be shown in Eq.(4)

$$P_e = \int_{0}^{\infty} P_e(\gamma) p_{\gamma}(\gamma) d\gamma \tag{4}$$

 $P_e(\gamma)$ is the conditional probability of error for a given SIR, $\gamma(\gamma = \frac{S}{I})$ for additive white Gaussian noise (AWGN) channels [3]. This is given by Eq.(5)

$$P_e(\gamma) = \frac{q}{2} \operatorname{erfc}\left(\sqrt{\frac{\gamma}{p}}\right) \cdot \frac{q^2}{16} \operatorname{erfc}^2\left(\sqrt{\frac{\gamma}{p}}\right)$$
(5)

where are:

$$q = 4 \left(1 - \frac{1}{\sqrt{M}} \right)$$
$$p = 2 \frac{(M-1)}{3 \log_2 M}$$

erfc(x) is the complimentary error function.

For rectangular MQAM in which $M = 2^k$, where k is even, the signal constellation is equivalent to two pulse amplitude modulation (PAM) on quadrature carriers, each having $\sqrt{M} = 2^{\frac{k}{2}}$ signal points.

Eq.(6) is derived by using substitution of Eqs.(3) and (5) in Eq.(4)

$$P_{e}(\gamma) = \frac{\Gamma(m_{s} + m_{I}L)}{\Gamma(m_{s})\Gamma(m_{I}L)} (\beta)^{m_{s}} \frac{q}{2} \int_{0}^{\infty} erfc \left(\sqrt{\frac{\gamma}{p}}\right) \frac{\gamma^{m_{s}-1}}{(\beta\gamma+1)^{m_{s}+m_{I}L}} d\gamma$$
$$-\frac{\Gamma(m_{s} + m_{I}L)}{\Gamma(m_{s})\Gamma(m_{I}L)} (\beta)^{m_{s}} \frac{q}{16}^{2} \int_{0}^{\infty} erfc^{2} \left(\sqrt{\frac{\gamma}{p}}\right) \frac{\gamma^{m_{s}-1}}{(\beta\gamma+1)^{m_{s}+m_{I}L}} d\gamma (6)$$
where is $\beta = \frac{m_{s}}{q_{SIR}m_{I}}$.



Fig.1. P_e of 4-QAM in Nakagami fading channels in the presence of AWGN for L=2, L=8, L=32, L=64 ($m_s = 0.8$ i $m_I = 0.6$)



Fig.2. P_e of 16-QAM in Nakagami fading channels in the presence of AWGN for L=2, L=8, L=32, L=64 (m_s =0.8 i =0.6)

IV NUMERICAL RESULTS

Using Eq.(6) and accepting the value of Nakagami fading parameters m_s i m_I , m_s =0.8 i m_I =0.6 we obtain numerical results, which can be illustrate on graphics. These results are represented in Fig.1., Fig.2., Fig.3. The exact average symbol error probability (P_e) curves of MQAM in Nakagami fading for various L, mutually independent Nakagami faded interference signals presented at the receiver, and various M, computed from Eq.(6), as a function of SIR, in dB, (SIR = $10\log q_{SIR}$) are shown in Fig.1., Fig.2., Fig.3.



Fig.3. P_e of 64-QAM in Nakagami fading channels in the presence of AWGN for L=2, L=8, L=32, L=64 ($m_s = 0.8$ i $m_I = 0.6$)

The values of k in Fig.1., Fig.2., Fig.3. are 2, 4, 6 respectively and in this cases M are 4, 16, 64. We noted that the results given by Eq.(6) are exact for $M = 2^k$ when k is even. On the other hand, when k is odd, there is no equivalent \sqrt{M} -ary PAM system. From the figures we observe that for a given average symbol error probability P_e , increasing the L, increases the required SIR. Rapidly decreasing of value P_e with increasing of *SIR* can be noticed for $SIR \ge 10db$.

V CONCLUSION

In this paper the analytic expression is derived for the average symbol error probability of M-ary quadrature amplitude modulation (MQAM) in Nakagami fading channels in the presence of additive white Gaussian noise (AWGN). The numerical results are shown in graphical form. The results represent influence of cochannel interference on average symbol error probability in following cases 4-QAM, 16-QAM i 64-QAM. For constant values of Nakagami parameters, m_s =0.8 and m_I =0.6 and for M=4, 16, 64 respectively, the values of average symbol error probability, P_e are computed from Eq.(6). It is shown that increasing the SIR, decreases the required P_e for a given SIR.

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