Theoretical analysis of Frequency, Pulse and Transitional characteristics of Loudspeaker (Part I)

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***Abstract* - In the work, a researching pulse response of an electrodynamic loudspeaker with a direct radiating is considered. The given method for measuring the system displays: the frequency response, the pulse and the transitional characteristics using the program MathCad. These characteristics are particularly important for the sound quality of the loudspeaker.**

***Keyword* – Loudspeaker, Frequency response, Pulse response.**

I. INTRODUCTION

The Loudspeaker can be described with the lineal four-plus [1].We can describe the lineal four-plus, using linear fluxional equation to the n-th degree where x(t) is an admission of the loudspeaker and y(t) - an outlet signal or a sound level on the radiating axis of the loudspeaker at a distance of one meter.

$$b_0 y(t) + b_1 \frac{dy(t)}{dt} + b_2 \frac{d^2 y(t)}{dt^2} + \ldots + b_m \frac{d^m y(t)}{dt^m} = a_0 x(t) + a_1 \frac{dx(t)}{dt} + a_2 \frac{d^2 x(t)}{dt^2} + \ldots + a_n \frac{d^n x(t)}{dt^n} \qquad (1)$$

Equation (1) in an operative form (in the frequency area) is:

$$\left(b_0 + b_1 p + b_2 p^2 + \ldots + b_m p^m\right) y(p) = \left(a_0 + a_1 p + a_2 p^2 + \ldots + a_n p^n\right) x(p) \qquad (2)$$

Equation (2) leads to the transfer function of the loudspeaker, presented by the ratio between the created sound pressure and the fed level.

$$K(p) = \frac{y(p)}{x(p)} = \frac{a_0 + a_1 p + a_2 p^2 + \ldots + a_n p^n}{b_0 + b_1 p + b_2 p^2 + \ldots + b_m p^m} \qquad (3)$$

When we present the numerator and the denominator we get:

$$K(p) = \frac{a_n.(p - p_{a1}).(p - p_{a2})....(p - p_{an})}{b_m.(p - p_{b1}).(p - p_{b2})....(p - p_{am})} \qquad (4)$$

Where $p_{a1}, p_{a2},\ldots, p_{an}$ are roots of the numerator called zeroes of the transfer function, $p_{b1}, p_{b2},\ldots, p_{bm}$ are roots of the denominator called poles of the transfer function [3].

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II. THEORETICAL BACKGROUND

The coefficient of transition $K(j\omega)$ can be defined by the Fourier transform:

$$\varphi(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} x(\omega).e^{j\omega t} d\omega \qquad (5)$$

Spectrum thickness [4]:

$$s(\omega) = \int_{-\infty}^{+\infty} \varphi(t).e^{j\omega t} dt \qquad (6)$$

or just when we replace it with the complex variable quantity $p = \sigma + j\omega$. Then:

$$K(j\omega) = \frac{y(j\omega)}{x(j\omega)} = \frac{a_0 + ja_1\omega - a_2\omega^2 - ja_3\omega^3 \ldots + a_n(j\omega)^n}{b_0 + jb_1\omega - b_2\omega^2 - jb_3\omega^3 \ldots + b_m(j\omega)^m} \qquad (7)$$

$$K(j\omega) = |K(j\omega)|.\exp[j\varphi(\omega)] = \mathrm{Re}(\omega) + j\,\mathrm{Im}(\omega)$$

The module of the transfer function is expressed by:

$$|K(j\omega)| = \sqrt{\mathrm{Re}(\omega)^2 + \mathrm{Im}(\omega)^2} \qquad (8)$$

and the phase is expressed by:

$$\varphi(\omega) = arctg\left[\frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)}\right] \qquad (9)$$

The amplitude – frequency responses (fig.1-3):

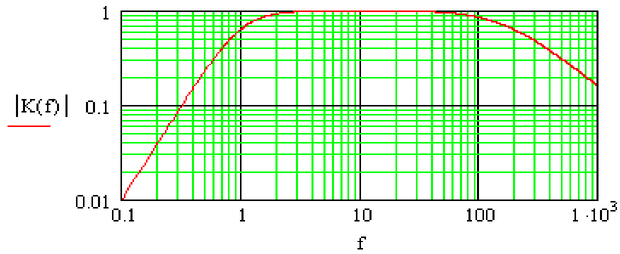

Fig.1 Normalized amplitude – frequency response of the sound pressure created by the loudspeaker

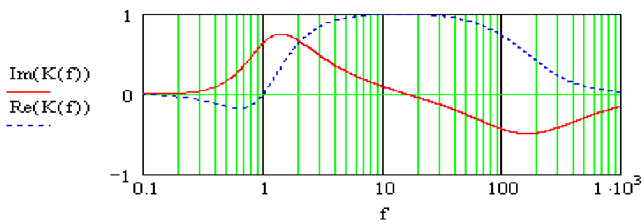

Fig.2 Amplitude – frequency response of the real and the imaginary part

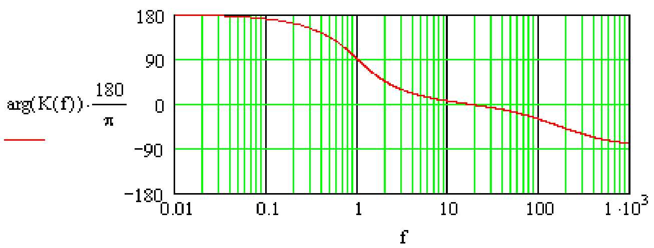

Fig.3 The phase of Amplitude – frequency response

At an analysis of the temporary area, grounded on the impulse response of the loudspeaker g(t) (fig.4) [2] determines the following:

$$g(t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\dot{K}(j\omega)e^{j\omega t}d\omega \qquad (10)$$

When we replace the math's expression (7) into (10) we get:

$$g(t)=\frac{1}{\pi}\int_{0}^{+\infty}|K(\varpi)|e^{j(\omega t+\varphi(\varpi))}d\omega$$
$$=\frac{1}{\pi}\int_{0}^{+\infty}\mathrm{Re}(\varpi)\cos\varpi t d\omega-\frac{1}{\pi}\int_{0}^{+\infty}\mathrm{Im}(\varpi)\sin\varpi t d\omega \qquad (11)$$

The simple part is a connection between the real and the imaginary part of the coefficient of transfer:

$$\frac{1}{\pi}\int_{0}^{+\infty}\mathrm{Re}(\varpi)\cos\varpi t d\omega=-\frac{1}{\pi}\int_{0}^{+\infty}\mathrm{Im}(\varpi)\sin\varpi t d\omega \qquad (12)$$

The impulse response has a real and an imaginary part g(t):

$$\mathrm{Re}\,g(t)=\frac{2}{\pi}\int_{0}^{\infty}\frac{\mathrm{Re}(\omega)}{\omega}.\sin(\omega t).d\omega$$
$$\mathrm{Im}\,g(t)=K(0)+\frac{2}{\pi}\int_{0}^{\omega}\frac{\mathrm{Im}(\omega)}{\omega}.\cos(\omega t).d\omega \qquad (13)$$

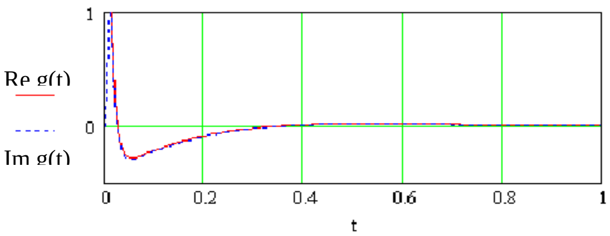

Fig.4 An impulse response

At the computer design of the impulse response it is appropriate to limit the upper border of the integrals to a real cost (fig. 2) at which the graphic images of Re g(t) and Im g(t) for t > 0 coincide. The normalized step response with MathCad coincides with the theoretical analysis [1, pp.809, fig.6] and the real results, presented by the producers of loudspeakers.

Since the impulse response g(t) is a reaction of x(t), the transitional response h(t) is designed by the integral of g(t) [2]:

$$h(t)=\int_{0}^{t}g(t)dt\,, \qquad (14)$$

or it is expressed by the transfer function:

$$h(t)=\frac{1}{\pi}\int_{0}^{\infty}K(j\omega).e^{j\omega t}d\omega \qquad (15)$$

If we use the real and the imaginary part, we can present the transitional response with the expressions (fig.5):

$$h(t)=\frac{2}{\pi}\int_{0}^{\infty}\mathrm{Re}(\omega).\cos(\omega t).d\omega$$
$$h(t)=\frac{2}{\pi}\int_{0}^{\infty}\mathrm{Im}(\omega).\sin(\omega t).d\omega \qquad (16)$$

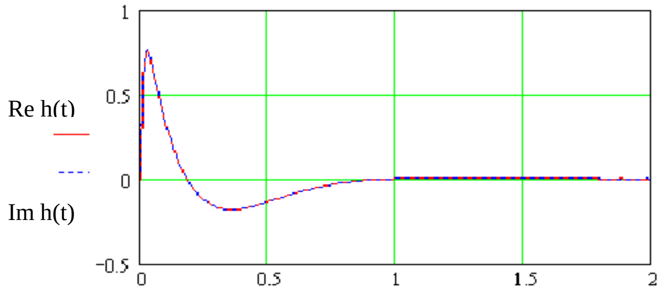

Fig.5 A transitional response

At the computer design of the impulse and the transitional responses, the precision of the graphic design at costs of t at the beginning of the transitional process t→0, are defined from the chosen step and the computing possibilities of the computer and the program which is used.

Conclusions

The presented method gives wide opportunities for an investigation of the transitional and the impulse characteristics of the loudspeakers, as well as for a valuation of their qualities and abilities compared with the catalogue data.

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