A novel hybrid ARQ scheme using randomly interleaved product codes

Nikolay T. Kostov[1] and Slava M. Yorfdanova[2]

Abstract - **A novel hybrid automatic repeat request (HARQ) scheme using randomly interleaved two-dimensional product codes is proposed. Throughput and bit error rate (BER) performance of the proposed scheme are studied through simulations and compared to the conventional forward error correction scheme.**

Keyword – **Automatic repeat request, product codes, simulation.**

I. INTRODUCTION

The possible implementations of automatic repeat request (ARQ) protocols fall into different categories [1], all of which include automatic requests for retransmission of data that are deemed unreliable by the receiver. A hybrid ARQ (HARQ) scheme [2] uses a forward error correction (FEC) code in conjunction with a retransmission scheme. Typically a cyclic redundancy check (CRC) code is used for frames error detection and this is an example of a so-called two-code approach since two different error control schemes are used for HARQ purpose [3]. Another HARQ method is the so-called one-code approach since only one error control code is used to identify some sort of reliability information within the decoding process that can be used to determine whether a retransmission is needed or not [4].

In this paper a novel one-code approach HARQ scheme based on randomly interleaved product codes (RIPC) is proposed. The performance (throughput and bit error rate) of the proposed HARQ scheme on additive white Gaussian noise channel (AWGN) is studied through simulations and compared to the conventional forward error correction scheme.

II. SYSTEM MODEL

Let us first consider the encoding/decoding processes of the conventional randomly interleaved product code (RIPC) [5]. The data frame to be transmitted (a square block of $(n-1)\times(n-1)$ bits) is first encoded by the component single parity check codes of a single two-dimensional product code (2D-PC), then it is randomly interleaved end re-encoded by the same 2D-PC. The overall RIPC is composed from the original data bits and all parity bits from the single parity check codes. The code rate R_c of the considered RIPC is

$$R_c = \frac{(n-1)^2}{2n-(n-1)^2}, \tag{1}$$

[1]Nikolay T. Kostov is with the Technical University of Varna, Department of Radio engineering, Studentska Street 1, Varna 9010, Bulgaria, E-mail: n_kostov@mail.bg

[2]Slava M. Yordanova is with the Technical University of Varna, Department of Computer Science and Technologies, Studentska Street 1, Varna 9010, Bulgaria, E-mail: slava_yordanova@mail.bg

where n is the length of the component single parity check codes and $(n-1)^2$ is the number of information bits in the encoded frame. Now, consider binary phase shift keying (BPSK) transmission via an AWGN channel. The decoding process is as follows: the original (non interleaved) noisy frame is first decoded using a soft-input/soft-output (SISO) decoding method [6] for a single decoding cycle, then the interleaved data frame is SISO decoded also for a single decoding cycle and so on. The two constituent decoders exchange the so-called extrinsic information (the error correction term gained from the decoding) at each full iteration and the decoding iterations are executed until a predetermined stopping criteria is satisfied. The final soft decision Λ_i of the *i*th data bit is given by

$$\Lambda_i = L_i^{ch} + L_i^1 + L_i^2, \tag{2}$$

where L_i^{ch} is the noisy channel observation for the *i*th data bit and L_i^1, L_i^2 are the extrinsic information terms from the first and second decoder, respectively. The corresponding hard decision a_i of the *i*th data bit is

$$a_i = \begin{cases} 1, \text{ if } \Lambda_i > 0 \\ 0, \text{ if } \Lambda_i < 0. \end{cases} \tag{3}$$

Let us now consider the proposed HARQ based on the described RIPC. The decoding algorithm is as follows:

1. Apply SISO iterative decoding (with a predetermined maximum number of iterations) on the received RIPC frame, checking all parity equations of the component single parity check codes after each full iteration. If all parity equations are satisfied (e.g., the stopping criterion is fulfilled) go to 3. Else go to 2.
2. Request a retransmission of the decoded frame by sending a negative acknowledgement to the transmitter via the feedback channel. If the maximum number of retransmissions is reached, go to 3.
3. Output hard quantized data.

It can be observed that with this decoding algorithm the frame error detection (and consequently the retransmission request) in the proposed HARQ is based on the single parity check equations satisfaction of the overall randomly interleaved product code. Because of the random interleaving, a significant improvement in terms of undetected erroneous frames rate can be expected for the considered error control scheme as compared to the conventional two-dimensional product codes. Further, using the proposed HARQ scheme an overall performance improvement in terms of post-decoded bit error rate (BER) and frame error rate (FER) can be obtained as compared to the RIPC alone.

III. Performance Results

A MATLAB-based Monte Carlo simulation was executed in order to estimate the performance of the considered HARQ scheme. The simulation of the considered error control scheme assumes BPSK signaling over an additive white Gaussian noise (AWGN) channel with at most two retransmissions allowed end error-free feedback channel. Two HARQ schemes were studied: HARQ1 with parent code rate $R_c \approx 3/4$ and HARQ2 with parent code rate $R_c \approx 4/5$. The normalized effective throughput R_{eff} of the HARQ scheme is defined in this paper as

$$R_{eff} = \frac{R_h}{R_c}, \qquad (4)$$

where R_h is the HARQ scheme code rate at a given signal-to-noise ratio (SNR) and R_c is the parent scheme (e.g., the randomly interleaved 2D-PC) code rate. Notice that $R_h \le R_c$ and $R_{eff} = 1$ only in case of no retransmissions. The retransmission efficiency R_{ret} (at a given SNR) of the proposed HARQ scheme can be defined as

$$R_{ret} = \frac{N_s}{N_s + N_u} \times 100 \quad [\%], \qquad (5)$$

where N_s is the total number of successful retransmissions and N_u is the total number of accepted erroneous frames including undetected bad frames. The retransmission efficiency versus the SNR of the considered HARQ schemes is shown in Figs. 1.

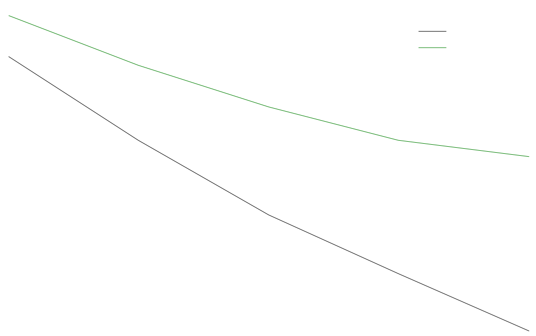

Fig.1. Retransmission efficiency versus the SNR of the HARQ1 end HARQ2 schemes.

According to Fig.1 the retransmission efficiency of the HARQ2 scheme is significantly better than those of the HARQ1 scheme over all range of SNRs of interest. This is due to the better post-decoding frame error rate of the HARQ2 scheme as a result of the larger frame size and interleaving used. It should be mentioned that there is a threshold SNR value of about 4.0 dB over which both HARQ schemes start to outperform the corresponding parent codes. The required SNRs for the HARQ schemes to achieve typical bit error rates (BERs) are shown in Table I. In Table I the BER performance of the parent codes is also given.

TABLE I

ESTIMATED HARQ AND RIPC SCHEMES PERFORMANCE

	HARQ1	RIPC1 $R_c \approx 3/4$	HARQ2	RIPC2 $R_c \approx 4/5$
SNR(dB) for $BER = 10^{-5}$	4.75	5.25	4.25	4.85
SNR(dB) for $BER = 10^{-6}$	5.3	5.8	4.85	5.5

As can be observed from Table I, a coding gain of at least 0.5 dB is obtained with the considered HARQ schemes over the conventional RIPC schemes. Another coding gain of approximately 0.5 dB can be obtained by using higher rate HARQ2 scheme instead of HARQ1 scheme. At $BER \le 10^{-5}$ the normalized effective throughput R_{eff} of both HARQ schemes is close to one (e.g., $R_h \approx R_c$) because of the small overall number of retransmissions.

IV. Conclusion

In this paper a novel one-code approach HARQ scheme is proposed. A coding gain of 0.5 dB or higher is obtained over the corresponding parent FEC schemes with practically no bandwidth expansion for moderate-to-high SNR values. The BER performance of the considered HARQ schemes is dominated by the undetectable erroneous frames and no significant performance improvement can be expected with more than two retransmissions allowed.

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