Method and examples of calculating the sampling frequency when the maximum rate of change and amplitude of the signal are known

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Abstract: **The proposed method of calculating the sampling factor N and the sampling frequency Fd when the maximum amplitude error Emax and the maximum rate of change (the slew rate) Vamax are given is based on the following basic steps:**

1. **We are calculating the maximum frequency of interest Fsmax = Vamax /(π*2*Am), [Hz]**
2. **We are calculating the sampling factor N=180/(90-arcsin(1-Emax)), [-]**
3. **We are calculating the sampling frequency Fd=N*Fsmax.**

Keywords: **sampling factor, slew rate, amplitude error**

I. Introduction

From the practical point of view it is important to calculate in a simple, clear and verifiable way and with acceptable amplitude error Emax, the sampling factor N=Fd/Fs, the sampling frequency Fd and the number of the converter's bits n when a signal with "arbitrary" form (a "real" signal from a "real" sensor) is converted into digital codes.

Sometimes we know or we could find easily:

1. The maximum allowed rate of change (the slew rate) Vamax into the analog channel ending with analog to digital converter (ADC) or starting with digital to analog converter or (DAC). Vamax could be easily calculated, measured or is known from the technical parameters of the equipment;
2. The amplitude Am or the amplitude from peak to peak App of the analog signal to be sampled. We could use a spectrum analyzer, oscilloscope, AC voltmeter plus band pass filters, technical data sheets, etc. to evaluate Am or App of the frequency component of interest;
3. The maximum permissible amplitude error during the conversion Emax. Emax is the maximum difference between the amplitude value (the maximum) of the analog signal Am and the maximal digital code when an ideal converter with infinity number of bits is used.

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4. The reference voltage Vref and the full scale voltage Vfs of the converter.

In most of the cases with n-bit converter (ADC or DAC) we have the following equations:

n>=3 bit (1)

LSB=Vref/2exp(n) (2)

LSB<<App<=Vfs (3)

LSB<<Am<=Vfs/2 (4)

where

- LSB is the least significant bit of the converter (ADC or DAC),
- n is the number of bits of the converter,
- Vref is the reference voltage the converter,
- Vfs is the full scale voltage of the converter.

II. The method

In [1] a parameter called "sampling (encoding) factor" N was introduced by the formulas

N=Fd/Fs (5)

and

N=Fd/Fmax (6)

where

- Fd is the sampling (discretization, encoding) frequency,
- Fs is the signal frequency when a sinusoidal signal (SS) or co-sinusoidal signal (CS) has to be converted,
- Fmax is the maximum signal frequency of interest if a band wide or multi-tone signal has to be converted into digital codes.

If the frequency component is an SS it is given by the equation:

A=Am*sin(2*π*F*t+φ) +Bdc (7)

We could find the maximum slew rate of a SS by differentiation:

Vamax = dA/dt=Am*d(sin(2*π*F*t+φ))/dt (8)

or

Vamax = dA/dt = 2*π*F*Am = π*F*App, [V/s] (9)

In [2] a theorem of C.E. Shannon (1948) was given and could de used as general guidance for calculating the sampling frequency Fd. Here we will try more practical approach. We could calculate the sampling factor N and the sampling frequency Fd in the following steps:

1. We know the maximum slew rate Vamax and the amplitude of the signal Am (or the amplitude from peak to peak, for example App=2*Am). Consequently, we can calculate the frequency of the signal Fmax and we could admit that Fmax is the maximum frequency

in the spectrum of the signal that has to be converted into digital codes with amplitude error not greater than a given value Emax. Because that is only one signal (frequency) component we could use the next formula for a SS and CS:

$$Fsmax = Vamax / (\pi * 2 * Am), [Hz] \qquad (10)$$

2. The maximum amplitude error Emax (0<=Emax<=1) is given and we can calculate the sampling factor N according to [1]:

$$N = 180/(90 - \arcsin(1 - Emax)) > 0, [-] \qquad (11)$$

3. Now, we can calculate the sampling frequency Fd with the next formula

$$Fd = N * Fsmax. \qquad (12)$$

Normally, the maximum rate of change of the signal Vamax (slew rate) is measured in V/s or V/us. The following equations are applicable:

$$Vamax[V/us] = Vamax[V/s]/1000\,000 \qquad (13)$$

And

$$Vamax[V/s] = Vamax[V/us] * 1000\,000 \qquad (14)$$

III. The method of calculating the sampling frequency Fd in function of the maximum slew rate Vamax in examples

Example 1:

Let us calculate the minimum frequency of sampling Fd for an analog signal with unknown form, for which we know that the slew rate could not be higher than Vamax = 500 000V/s=0.5V/us and the amplitude from peak to peak could not be higher than App=2*Am=1V. The amplitude error Emax during the analog to digital conversion due to the sampling factor N should be not greater than –3dB. Also we know that the equation 1*LSB<<App<=Vfs is satisfied. (The given Vamax could be the maximum slew rate of the analog channel between the source and the converter and could be equal to the maximum slew rate of the used amplifiers, sample and holds, multiplexers and filters. For example, Vatyp for operational amplifiers 741 and 1458 is approximately 0.5V/us.)

Solution:

We are admitting that Vamax=0.5V/us is for the frequency component with the maximum frequency Fmax, which could not have amplitude from peak to peak higher than 1V. This could be verified in practice by testing the channel with real signal. Moreover, we could admit that only one frequency component has slew rate Vamax=0.5V/us and it has sinusoidal or cosinusoidal form because it is only one frequency component.

Now, it is possible to calculate the frequency of the signal Fsmax with Vamax and the amplitude Am. For a SS we have the equation (10) and we are able to draw the conclusions that:

1. If we increase the frequency of the signal Fs that will increase proportionally its maximum slew rate Vamax;
2. If we increase the amplitude of the signal Am (or the amplitude from peak to peak App) that will increase its maximum slew rate Vamax.

Obviously, the amplitude of the signal from peak to peak is limited from the converters full scale voltage Vfs.

We are using the equation (10) in order to calculate the maximum frequency Fsmax in the spectrum of the signal to be sampled

$$Fsmax = Vamax[V/s] / (\pi * App) =$$
$$= 500\,000 V/s/(\pi * 1V) = 159160\ Hz = 160KHz$$

As it was published in [1] the frequency component Fsmax with enough amplitude Am (LSB<<Am<Vfs/2) will be converted into digital codes with amplitude error less than –3dB, if we select the sampling frequency Fd according to the equation

$$Fd >= 4 * Fs \qquad (15)$$

In this example we have:

$$Fd >= 4 * Fs >= 4 * 160KHz >= 642KHz$$

The equation above could be drawn from the basic equation (11) for calculating the sampling factor N in function of the maximum amplitude error Emax for a SS.

Example 2:

In the analog channel before the ADC are used blocks (filters, amplifiers, etc.), which are limiting the slew rate of the signal (for example from a temperature sensor) at the ADC input to Vamax=120 V/s and the amplitude from peak to peak App=2*Am=2.55Vpp.

a/ We have to calculate the sampling frequency Fd with a guarantee that the maximum difference between the maximal digital code and the amplitude value of the analog signal Am is less than 10% (or to any other applicable value between 0% and 100%) when we are using an ideal ADC with error close to zero;

b/ We have to choose a real ADC with number of bits n under the same conditions (amplitude error is less than 10%).

Solution:

a/ We are calculating the signal component with the maximum frequency Fsmax which has to be converted into digital codes. We will use the formula (10) and we have

$$Fsmax = 120V/s / (3.1416 * 2.55V) = 15\ Hz$$

We have the formula giving the relation between the sampling factor N and the maximum amplitude error Emax (11). If we apply this formula we will have:

$$N = 180/(90 - \arcsin(1 - 0.1)) = 180/(90 - 64.16) = 7, [-]$$

We have found Fsmax and N and we are calculating Fd

$$Fd = 7 * 15Hz = 105Hz$$

In this case if we are sampling with frequency Fd=105Hz or higher, with infinity fast ADC with infinity high number of bits n ($n \rightarrow \infty$) and with infinity small conversion error Eadc we could guarantee that the maximum difference between the maximal digital code and the amplitude of the signal (or the amplitude error) is less than -10% or approximately –1dB.

b/ We will try to choose the real ADC with number of bits n less than the infinity and to maintain the total conversion error Econv according to the equation

$$Econv=Eadc+Emax<=10\% \quad (16)$$

In order to calculate the minimum number of bits of the ADC in function of the sampling factor N (and in function of the maximum amplitude error Emax) the author is proposing the simplified formula:

$$n>= lg(N) + (2\ bit), [bit] \quad (17)$$

where lg is the logarithm in base 2 and 2=<N<=48.

In this case

$$n>=lg(7)+(2\ bit) >= 2.73\ bit + (2\ bit) >= 4.73\ bit$$

We are choosing the effective number of converter bits n = 6 bit. It is known that the maximum conversion error Eadc introduced by an 6-bit ADC is

$$Eadc = +-0.5*100\%/(2exp(n))=$$
$$=+-0.5*100\%/(2exp(6)) = +-0.5*100\%/64 = +-0.78125\%$$

The root mean square error (Eadcrms) introduced by the 6-bit ADC is given by the widely used formula:

$$Eadcrms = 1*LSB\%/sqr(12) \quad (18)$$

We could calculate

$$Eadcrms=(100\%/64)/sqr(12) = 1.5625\%/3.464 =0.451\%.$$

If we apply the model of the maximum error (in this model we add all errors in order to obtain the maximum possible error) we will have the following error distribution in order to have a maximum conversion error less than 10%.

- Eadc = 0.78125% (error from the ADC);
- Emax=10%-0.78125% = 9.22% (error from the sampling factor N=Fd/Fs).

Now we have Emax and we could calculate the sampling factor N with the formula (10)

$$N= 180/(90-arcsin(1-0.0922))=180/(90-65.20)= 7.26\ [-]$$

We have calculated the sampling factor N and we could calculate the sampling frequency with (12)

$$Fd = N*Fs = 7.26*15=109Hz.$$

The solution of the Example 1 was found and we can draw the conclusions that:

1. The decreasing of the effective number of converter's bits n could be compensated only partially (in small degree) by the increasing of the sampling factor N;
2. The increasing of the sampling factor N from 4 to infinity will reduce the maximum amplitude error Emax from -29.3% (or -3dB) to 0% (when $n\rightarrow\infty$).

IV. How can we found the maximum slew rate Vamax?

There are many ways to know the maximum slew rate of the analog channel between the sensor (the source of information) and the ADC:

1. From the data sheet of the sensor.
2. From the data sheet of the operational amplifier(s) and filter(s) used to amplify and filter the signal from the sensor.
3. From the data sheets of the filters between the sensor and the ADC.
4. With an "almost ideal" square wave signal and oscilloscope we could measure the rise and the fail times between the levels of 10% and 90% of the maximum levels on the output of the analog channel. (Usually Trise=Tfall=Tr). Consequently, it is possible to calculate two slew rates (the rate of rise of the signal Vr and the rate of the fall of the signal Vf), to choose the greater from them (Vamax=max(Vr, Vf)) and to use it as a maximum slew rate of the channel. In most of the cases between the rise time Tr and the bandwidth of the analog channel BW(-3dB) we could apply the following equation [3]:

$$BW(-3dB)= 0.35/Tr \quad (19)$$

Table 1 is giving an idea about the maximum slew rate Vamax of some SS and CS with frequency Fs, Am=1V and App=2*Am=2V.

Table 2 is giving some basic values for the sampling factor N=Fd/Fs and the corresponding possible amplitude error during the SS and CS conversion with $n\rightarrow\infty$ if the formulas (11) is used for the calculation of Emax and the minimum number of the converter bits n, according to the equation (17). There is an obvious amplitude modulation, depending on the phase of the signal (SS or CS) to be converted into digital codes.

Table 1
The maximum slew rate of some SS and CS with frequency Fs, Am=1V and App=2*Am=2V.

Fs	Vamax		Fs	Vamax
1Hz	6.283V/s		2Hz	12.567V/s
1kHz	6283V/s		2kHz	12567V/s
1MHz	6.283V/us		2MHz	12.567V/us
1GHz	6283V/us		2GHz	12567V/us
1THz	6.283V/ns		2THz	12.567V/ns

Table 2
Some basic values of the sampling factor N=Fd/Fs and the possible amplitude error during the SS and CS conversion

N	Possible amplitude error ($n\rightarrow\infty$)	Variation of the phase	n, [bit] (nadc, ndac)	Eadc, +-LSB/2
2	0 to -100%	0 to 90 deg.	>3	6.2%
4	0 to –29.3%	0 to 45 deg.	>4	3.1%
7	0 to -10%	0 to 25.7 deg.	>4.8	1.8%
22.2	0 to -1%	0 to 8.1 deg.	>6.5	0.55%

V. Calculating the number of bits of the converter n

Once we have calculated the sampling factor N and the sampling frequency Fd we have to calculate the minimum acceptable number of bits for the converter (e.g. the ADC).

As a first step we are calculating the (minimal) number of converters bits n (in order to consider the converter as an “ideal” one) from the sampling factor N with simplified formula (17):

$$n1 >= lg(N) + (2\ bit),\ [bit] \quad (20)$$

As a second step the well known formula giving the Signal To Noise Ratio (SNR) in function of the effective number of converter’s bits n could be applied [4]:

$$n2 = (SNR[dB]-1.76dB)/6.02 \quad (21)$$

Finally, we could choose the minimal acceptable number of the converter bits n, according to the formula

$$n = max(n1,n2) \quad (22)$$

If the signal amplitude from peak to peak App is less than Vfs/2 we are able to calculate the lost of the converter bits L using the following formula

$$L = lg(Vfs/App),\ [bit], \quad (23)$$

Where lg is a logarithm in base 2.

As we mention before the lost of converters bits L could be compensated only partially with higher than calculated sampling factor N (or sampling frequency Fd). This conclusion could be drawn from the following formula:

$$Emax = 1-sin(90-180/N)) \quad (24)$$

VI. Conclusion

The paper is giving a simplified and useful approach for calculating:

1. The sampling factor N=Fd/Fs;
2. The sampling frequency Fd;
3. The minimal number of converters bits n;

when the maximal slew rate Vamax, the maximum amplitude Am of the signal and the maximum amplitude error during the conversion Emax are known.

The method discussed in the present paper could be easily tested with a direct current (DC) signal, SS or CS with a DC component and with a sum of SS and DC signals with or without a DC component.

The method proposed in the paper is very useful because every real signal has a finite slew rate Vamax and finite amplitude Am or App which could be measured or calculate easily and accurately. It should be noted that only real signals with finite slew rate could be represented accurately with Fourier series. The artificial signals with infinity high slew rate ($Vmax \rightarrow \infty$) like rectangular or saw-tooth signal (one or both of the edges of the signal are infinitely short or a part of the signal is infinitely short) cannot be represented fully with Fourier series.

VII. References

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