Utilization of the Serial Analysis Method of the Date for Reliability Evaluation of the Electronic Devices

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***Abstract* - In the contemporary manufacture of the semiconductor devices and integral circuits, the time is decreased from the idea to the realization of the ready production. Because of that, the time, for testing and estimating the reliability indicators of the manufactured semiconductor devices, should be decreased. Nowadays, that is one of the main reasons that we should pay attention to the statistical method for serial analysis of the dates, received when the devices are examined. In the presented paper, the possibilities, for serial statistical method application by reliability evaluation of the semiconductor devices, are examined. Its advantages and disadvantages are analyzed. It is shown the specified method application. It is also paid an attention to the risk of the consumer and the risk of the manufacturer.**

***Keywords* - failure analysis, reliability of the electronic device.**

I. THEORY

When the determined statistical control limits are at least equal to or within the specification limits, a process is defined as capable and deemed incapable whenever the control limits lie outside of the specification limits. The capability index

$$C_P = \frac{USL - LSL}{6\sigma} \tag{1}$$

where *USL* and *LSL* represent the upper and lower specification limits and σ the population standard deviation, is a handy measure of Process Capability.

If $C_P<1$, the process is deemed incapable, by convention; if $C_P=1$ or $C_P>1$, the process is considered *marginally* or *definitively capable* (Ex: $C_P \geq 1.33$). Hence, for a selected *USL-LSL* specification range, any sampling decision on whether a process is *capable* is dependent upon the process standard deviations σ_0 ($C_P>1$) σ_1 ($C_P<1$) and corresponding type *I* α and type *II* β errors for hypothesis acceptance or rejection. With the exception of double sampling, most accept/reject inspection decisions assume that the observational numbers are independent of the results and require a predetermined sample size. Serial test or inspection plan sample sizes are dependent on the outcome of the observations and require three decisions: (2) Accept the test hypothesis, (3) Take another observation, (4) Reject the test hypothesis.

In 1943, Serial testing results from the theory of serial analysis that was created by Wald for war time military equipment development and inspections. Numerous examples of the theory's practical applications and sampling saving economies were later published1 by the Statistical Research Group of Columbia University.

The basis for Wald's serial test are the probability inequality ratios:

$$\frac{P_{1N}}{P_{0N}} \leq \frac{\beta}{1-\alpha} \tag{2}$$

$$\frac{\beta}{1-\alpha} < \frac{P_{1N}}{P_{0N}} < \frac{1-\beta}{\alpha} \tag{3}$$

$$\frac{P_{1N}}{P_{0N}} \geq \frac{1-\beta}{\alpha} \tag{4}$$

where P_{0N}, P_{1N} are the probability density functions corresponding to the test H_0 and alternate H_1 hypothesis for the serial set of observations $X_1...X_N$, and α, β are the selected type *I* (producer's risk) and type *II* (consumer's risk) errors. The test is continued when inequality ratio (3) applies and discontinued at the first occurrence of the ratios (2) or (4) resulting in the acceptance of hypotheses H_0 or H_1 respectively.

Consider the sequence of observations $X_1 \div X_N$, as an example of test principle usage, from a normally distributed process with a known mean μ and unknown standard deviation σ, where the test hypothesis is $\sigma \leq \sigma_0$ and the alternate $\sigma \geq \sigma_1$. For each value of *N*, the probability ratio P_{1N}/P_{ON} is calculated. Thus, the test hypothesis $\sigma \leq \sigma_0$ is accepted when (2) $P_{1N}/P_{ON} \leq \beta/(1-\alpha)$ and the alternate $\sigma \geq \sigma_1$ when (4) $P_{1N}/P_{ON} \geq (1-\beta)/\alpha$. The test is continued when (3) $\beta/(1-\alpha) < P_{1N}/P_{ON} < (1-\beta)/\alpha$. The expansion of inequality (3) results in expression (5) below:

$$\frac{2\log_e\left(\frac{\beta}{1-\alpha}\right)+N\log_e\left(\frac{\sigma_1^2}{\sigma_0^2}\right)}{\frac{1}{\sigma_0^2}-\frac{1}{\sigma_1^2}} < \sum_{i-1}^{N}(X_i-\mu)^2 < \frac{2\log_e\left(\frac{1-\beta}{\alpha}\right)+N\log_e\left(\frac{\sigma_1^2}{\sigma_0^2}\right)}{\frac{1}{\sigma_0^2}-\frac{1}{\sigma_1^2}} \tag{5}$$

and further simplification results in the straight line forms that are used to create the table (Table 2) and/or graph (Figure 1) of a Wald serial test procedure for process variability:

Accept standard deviation σ_0 at step *N* if the summed observations (In the cases where the mean μ is unknown the sum of squares is replaced by the squares of the deviations from the sample mean and the limit *N* is replaced by *N*-1. Five useful points of both the operating characteristic *Pa*(σ) and average sample number *ASN*(σ) curves can be calculated from the formulas in Table 1) $\Sigma(X_i\ \mu)^2$ satisfy In 1943, Serial testing results from the theory of serial analysis that was created by Wald for war time military equipment development and inspections. Numerous examples of the theory's practical applications and sampling saving economies were later published1 by the Statistical Research Group of Columbia University.

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$$\sum (X_i - \mu)^2 \le Y_0(N) = SN + h_0 \quad (6)$$

or

$$\sigma_1 \quad \text{if} \quad \sum (X_i - \mu)^2 \ge Y_1(N) = SN + h_1 \quad (7)$$

and continue sampling if

$$Y_0(N) < \sum (X_i - \mu)^2 < Y_1(N) \quad (8)$$

where:

$$S = \frac{\log_e\left(\frac{\sigma_1^2}{\sigma_0^2}\right)}{\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}} \qquad h_0 = \frac{2\log_e\left(\frac{\beta}{1-\alpha}\right)}{\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}} \qquad h_1 = \frac{2\log_e\left(\frac{1-\beta}{\alpha}\right)}{\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}} \quad (9)$$

TABLE 1: FIVE USEFUL POINTS

σ	0	σ_0	$\sqrt{S}$	σ_1	∞
$PA(\sigma)$	1	$1-\alpha$	$h_2/(h_1+h_2)$	β	0
$ASN(\sigma)$	$\frac{h_1}{S}$	$\frac{(1-\alpha)h_1 - \alpha h_2}{S - \sigma_0^2}$	$\frac{h_1 h_2}{2S^2}$	$\frac{(1-\beta)h_1 - \beta h_2}{\sigma_1^2 - S}$	0

The Average Sample Number $ASN(\sigma)$ estimates the average number of samples required for a decision at a specified standard deviation s since the exact number is not known beforehand.

II. SERIAL STATISTICAL METHOD APPLICATION BY RELIABILITY EVALUATION OF THE SEMICONDUCTOR DEVICES

The method choice for examining is a serious task for the specialist in the area of reliability. In the presented paper is made a reliability estimation of the semiconductor devices with the help of the serial method. It is preferred because smaller sample and smaller duration are required. The volume of the examinations is decreased 50% in comparison with the another methods.

It should be selected one procedure, which requires the less time, from all the different procedures by which the serial analysis is made. As it is known [1], procedures possess these characteristics by which the decisive rule is based on the serial criteria of probability relation. With the help of the serial analysis of the results of the examinations is obtained a complete and accurate information. Actually, if $\underline{R}$ appears to be $\underline{R} < R_{specified}$, then it means that $R < R_{specified}$, so maybe the volume is still not enough and the upper written conditions will be met with the increase of the number of the conducting examinations K. A serial procedure for control of the conducting examinations with measured values of the function of the serviceability of the semiconductor devices will be presented in this paper. We may accept the hypothesis that f has a Gausov distribution, then:

$$R = P\{p > 0\} = F^*\left\{\frac{m_f}{f_f}\right\} = F^*\{r\}. \quad (10)$$

We may accept that if $R = \overline{R}$, the semiconductor devices are capable, but if $R = \underline{R}$, then semiconductor devices are incapable. So, the serial analysis may be presented as 1. Test of the hypothesis R_0 $\{R = \overline{R}\}$ by alternative R_1 $\{R = \underline{R}\}$ and 2. The set reliability for an error of type I α (risk for the manufacturer) and of type II β (risk for the customer).

On account of the monotony of the function $F^*\{r\}$ in reference of r, the hypothesis R_0 and R_1 may be presented as R_0 $\{r = \overline{r}\}$ by alternative R_1 $\{r = \underline{r}\}$, where $\overline{r}$ and $\underline{r}$ are quintiles of the normal distribution corresponding to $\overline{R}$ and $\underline{R}$.

Let the sample, which is characterized with f_k and volume k, is defined. From the given sample $\overline{f_k}$, are defined the mathematical expectation $\hat{m}_f$ and the quadratic metan deviation $\hat{\sigma}_f$.

$$\hat{m}_f = \frac{\sum_i^k f_i}{n}; \qquad \hat{\sigma}_f = \sqrt{\frac{\sum_i^k (f_i - \hat{m}_f)^2}{k-1}}. \quad (11)$$

It's obvious that the quantity $\hat{r} = \hat{m}_f / \hat{\sigma}_f$ is also random. In accordance with the serial criteria, the relation of the probabilities of the hypothesis R_0 may be accept if:

$$\lambda = \frac{\varphi(k-1, \overline{r}\sqrt{k}, y)}{\varphi(k-1, \underline{r}\sqrt{k}, y)} \le \frac{\beta}{1-\beta}, \quad (12)$$

where $\hat{r}\sqrt{k}$ is the value found from the sample f_K.

The production is incapable if

$$\lambda = \frac{\varphi(k-1, \overline{r}\sqrt{k}, y)}{\varphi(k-1, \underline{r}\sqrt{k}, y)} \ge \frac{1-\beta}{\alpha}, \quad (13)$$

and the examinations continue if

$$\frac{\beta}{1-\alpha} < \frac{\varphi(k-1, \overline{r}\sqrt{k}, y)}{\varphi(k-1, \underline{r}\sqrt{k}, y)} \le \frac{1-\beta}{\alpha}. \quad (14)$$

We would analyze the problems connected with the serial control examinations by exponential distribution. We would examine the reliability with the help of the index - mean time to failure. For the plan development of testing, it should be given the levels for capability and incapability (corresponding T_0 and T_1) and the risks of the manufacturer (α) and the customer (β). It is evaluated after each failure whether the level of the reliability is too high in order to be accepted the elemnts for capable, or it is too low in order to be accpted for incapable. An index for the evaluation is the total mean time to failure of the elements of the sample.

The condition for capability or incapability of the elements is the following:

$$\frac{t_\Sigma}{T_0} \ge \frac{T_1}{T_0 - T_1}\left[r \ln\frac{T_0}{T_1} - \ln\frac{\beta}{1-\alpha}\right] \quad (15)$$

and

$$\frac{t_\Sigma}{T_0} \leq \frac{T_1}{T_0 - T_1}\left[r \ln \frac{T_0}{T_1} - \ln \frac{1-\beta}{\alpha} \right], \quad (16)$$

where t_Σ is the total duration of the examination.

The lines of correspondence *II* and discrepancy *I* are made from the equations (15) and (16) (fig.1).

The control has the following algorithm. From the batch of elements is made a random sample where $N>r_0$. If at the initial moment of the examination failure $r>r_0$ elements, the batch is incapable. If failures occur in the process of examination, the graphic of the serial control of reliability is a steplike line (fig. 1).

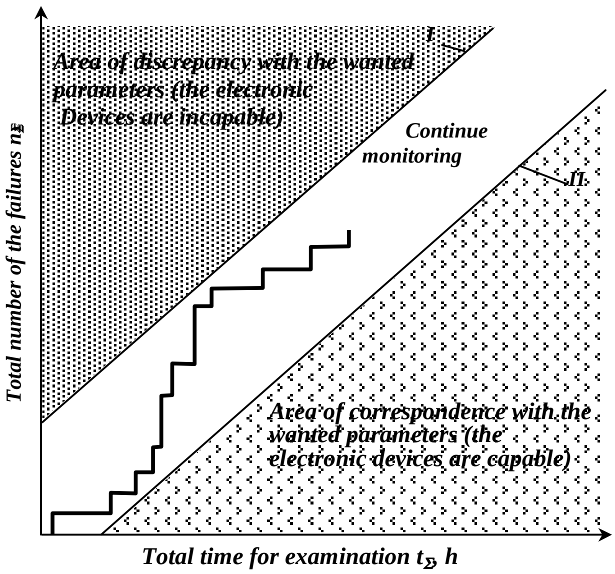

Fig.1. Graphical model of the serial method for examination

III. SAMPLE APPLICATION AND EXAMPLE RESULTS

In table 2, are registered sample data for application of the serial statistical method application by reliability evaluation of the semiconductor devices. The graphic model of the sample data, registered in table 2 is illustrated in fig. 2. The sample data corresponds to the starting data: Producer's risk 0,001, Consumer's risk 0,001, Group size 1, Cum. Samples 18, Standard deviation choices *Std1*, *Std*2 (Lowest *Std*1 12,5 and Highest Std2 24,3); starting condition: by $C_P \leq 1 \rightarrow$ incapable, $1<C_P<1.33 \rightarrow$ continuing monitoring. Calculated Process Capability Indexes are the following: $C_P \geq 1.33 \rightarrow$ capable; C_P *Std*1 is 1,33, C_P *Std*2 is 0,686.

A Chi-square test X^2 is usually applied, when a classical test of significance is applied to question of whether the sample variance s^2 differs significantly from the desired population variance σ_0. The test hypothesis H_0 specifies the population variance as H_0: $s^2 = \sigma_0^2$ and the alternate hypothesis H_1: $\sigma_1^2 > \sigma_0^2$. If H_0 is correct, then X^2/f is the distribution for s^2/σ_0^2 for f degrees of freedom. Thus, a rejection of the test hypothesis H_0 occurs at some chosen significance level a such that

$$\frac{S^2}{\sigma_0^2} > \frac{X^2_{1-\alpha}}{f} \, . \quad (17)$$

TABLE 2: SERIAL SAMPLING PROCESS CAPABILITY DETERMINATION

ampl size	Calculated Criteria Sums				)eviati on Sqr Value	Compare)eviation Sqr Sur (sum number of the failure n_Σ)
	·or·	Accept Std1	·or·	Accept Std2		
1	no	-2652	no	3216	2	2
2	no	-2369	no	3499	79	81
3	no	-2087	no	3781	106	187
4	no	-1804	no	4064	6	193
5	no	-1522	no	4346	3	196
6	no	-1239	no	4629	41	237
7	no	-957	no	4911	97	334
8	no	-674	no	5194	161	495
9	no	-392	no	5476	332	827
10	no	-109	no	5759	101	928
11	no	171	no	6041	21	949
12	no	454	no	6324	140	1089
13	no	736	no	6606	255	1344
14	no	1019	no	6889	62	1406
15	no	1301	no	7171	74	1480
16	no	1584	no	7454	199	1679
17	no	1866	no	7736	202	1881
18	**yes**	2149	no	8019	111	1992

The test's sample size can be estimated from the power of the discriminating test needed to distinguish between the population variances σ_1^2 and σ_0^2 at f degrees of freedom:

$$\frac{\sigma_1^2}{\sigma_0^2} = \frac{X^2_{1-\alpha}(f)}{X^2_{\beta}(f)} \quad (18)$$

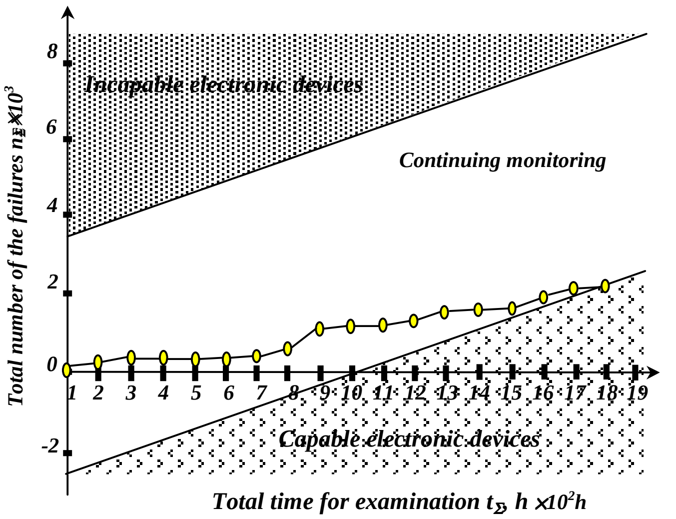

Fig.2. Graphical model of the serial statistical method application by reliability evaluation of the semiconductor devices

The substitution of the textile process parameters, σ_0=12.5, σ_1=24.3, α=0.001, β=0.001, in the discriminating test equation (18) yields $\sigma_1^2/\sigma_0^2=3.78=X^2_{.999}(f)/X^2_{.001}(f)$ at about 47 degrees of freedom, which translates into a required sample size of 48 for the classical test. The average sample number *ASN* requIred for a serial test decision at a standard deviation level of σ_0=12.5 is 23. For an unknown mean, the sample size is increased by one and the saving's percentage is reduced to 50%.

IV. CONCLUSION REMARKS

Compensations are not made for the effects of sample grouping on type *I* is a limitation of the Serial method, type *II* errors, *ASN* and *OC* curve values. When compared with single item serial plans, grouping by sample increments results in an increase in the number of items inspected and advantageously lower type *I* and type *II* errors. If the process is evaluated is very good or bad, expeditious decisions occur. However, these accept or reject decisions are dependent upon the closeness of the standard deviation parameters σ_0, σ_1 and the degree of risk α, β that the manufacturer or organization is willing to accept. Borderline processes often require indefinite sample sizes and a truncation point leading to an increased α or β risk has to be agreed upon. The method does not exempt the manufacturer or organization from continuous monitoring since processes do change in time. Also, the example discussed is based on the assumption of a centered normally distributed process.

The major advantage in the use of serial probability ratio test arises from the ease at which it can be automated, enhanced and extended to the Binomial, Poisson and Reliability Test areas. Algorithms can be devised that require only computer familiarity and data entry skills on the part of the user. AddItionally, hardware as simple as a programmable pocket calculator can be used for many of the applications.

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