Two Samples per Bit Interval Detection of NCFSK Signal in Presence of White Gaussian Noise and Interference

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Abstract – **In this paper, an analysis of non – coherent FSK signal in the presence of Gaussian noise and channel interference has been performed. In receiver, decision is made on the bases of two samples during one bit – interval. We express the conditional probability density function for signals in both detector branches. Also, bit error probability versus signal to noise ratio, for different values of signal to interference ratio, is plotted.**

Keywords – **NCFSK receiver, White Gaussian noise, Interference, Bit error probability.**

I. Introduction

Frequency shift keying (FSK) is commonly used form of digital modulation in the high-frequency radio spectrum, and has important applications in telephone circuits. Binary FSK (usually referred to simply as FSK) is a modulation scheme allowing the data transmission by shifting the frequency of a continuous carrier in a binary manner to one or the other of two discrete frequencies. One frequency is designated as the "mark" frequency and the other as the "space" frequency. The mark and space correspond to binary one and zero, respectively. By convention, mark corresponds to the higher radio frequency. FSK modulation (Frequency Shift Key) is commonly believed to perform better than ASK and PSK in the presence of interfering signals. However, it is usually more difficult and expensive to implement. [1]

FSK signal can be transmitted coherently or noncoherently. Coherency implies that the phase of each mark or space tone has a fixed phase relationship with respect to a reference. Coherent FSK is capable of superior error performance but noncoherent FSK is simpler to generate and is used for the majority of FSK transmissions. Noncoherent FSK has no special phase relationship between consecutive elements, and, in general, the phase varies randomly.

Noncoherent FSK modulation is based on the system modeled with two matched filters centered at ω_0 and ω_1 with envelope detectors summed to a decision circuit [2].

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This model suggests that non-coherent FSK modulation can be treated as two ASK signals, one at frequency ω_0 and the other at frequency ω_1.

Optimum demodulation of non-coherent FSK can be achieved by envelope detection of the signal filter outputs in a filter-type demodulator. The outputs of the mark and space filters are envelope-detected and then compared to determine which has greater magnitude.

The phase information is not required. With the "right" filter shape, performance of this type of demodulator approaches the theoretical optimum for noncoherent FSK. The "right" filter shape for a white noise interference environment is one that has the same spectral shape as the transmitted signal. For the "rectangular" modulation of FSK, the right shape is a function (sin x)/x bandpass filter centered about the desired mark or space tone.

II. The System Analysis

Gaussian noise and disturbances are inevitable in telecommunication systems. Typical interferences usually have sinusoidal form. It is interesting to analyze the influence of sinusoidal interferences and Gaussian noise on the system error.

This paper presents an analysis of the digitally modulated signal with two symbols in presence of Gaussian noise and sinusoidal interference. In Figure 1. the block diagram for noncoherent detection of FSK signal is shown.

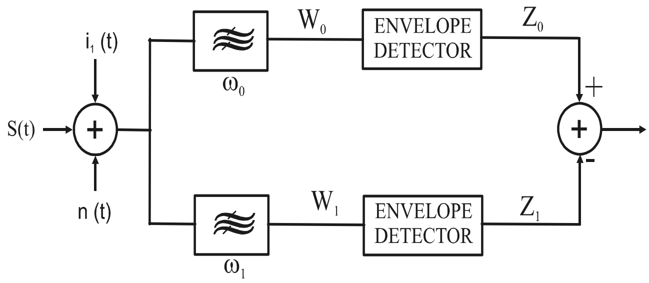

Fig. 1. Block diagram of the receiver for noncoherent detection of 2 - FSK signal

We supposed that filter in the upper branch passes just the elemental signal of ``1``, while the filter in the lower branch passes the elemental signal of ``0``. Besides, the central frequencies ω_0 and ω_1 are sufficiently spaced so the bands of these filters are not overlapping in the frequency domain.

Each branch contains an ideal envelope detector after filter. In the moment of decision making (if ``0`` or ``1`` is sent), the instantaneous difference of the two envelopes is compared to the decision threshold.

We consider a signal containing two components at frequencies ω_0 and ω_1 and presented with two pulses of duration T.

In real systems it is generally T >> T_0, T_1, so the modulated carrier signal can be analytically presented as:

H_0: ``0`` is sent:

$$\begin{aligned} S(t) &= A\cos\omega_0 t \quad 0 \le t \le T/2 \\ S(t) &= A\cos\omega_1 t \quad T/2 < t < T \end{aligned} \tag{1}$$

H_1: ``1`` is sent:

$$\begin{aligned} S(t) &= A\cos\omega_1 t \quad 0 \le t \le T/2 \\ S(t) &= A\cos\omega_0 t \quad T/2 < t < T \end{aligned} \tag{2}$$

Gaussian noise can be analytically presented as:

$$n(t) = \sum_{i=1}^{\infty} N\cos(2\pi i\Delta f + \theta_i) \tag{3}$$

A disturbance signal generally interferes with one of the components, so it can be taken as:

$$i_1(t) = A_1\cos(\omega_0 t + \theta_1) \tag{4}$$

Filtering the signal at the output of band pass filter at ω_0 results in component of the signal at frequency ω_0, narrow band noise and disturbance at frequency ω_0. Likewise, the output of band pass filter at ω_1 consists of the signal at frequency ω_1 and narrow band noise, since the disturbance is eliminated. In proposed cases, this can be written as:

H_0:

$$\begin{aligned} W_0(t) = (&A\cos\omega_0 t + x\cos\omega_0 t + y\sin\omega_0 t + \\ &+ A_1\cos(\omega_0 t + \theta_1))\,(h(t) - h(t - T/2)) \end{aligned} \tag{5a}$$

$$\begin{aligned} W_0(t) = (&x\cos\omega_0 t + y\sin\omega_0 t + A_1\cos(\omega_0 t + \theta_1) \\ &(h(t - T/2) - (t - T)) \end{aligned} \tag{5b}$$

$$W_1(t) = (x\cos\omega_1 t + y\sin\omega_1 t)(h(t) - h(t - T/2)) \tag{5c}$$

$$\begin{aligned} W_1(t) = (&A\cos\omega_1 t + x\cos\omega_1 t + y\sin\omega_1 t) \\ &(h(t - T/2) - h(t - T)) \end{aligned} \tag{5d}$$

H_1:

$$\begin{aligned} W_0(t) = (&x\cos\omega_0 t + y\sin\omega_0 t + A_1\cos(\omega_0 t + \theta_1)) \\ &(h(t) - h(t - T/2)) \end{aligned} \tag{6a}$$

$$\begin{aligned} W_0(t) = (&A\cos\omega_0 t + x\cos\omega_0 t + y\sin\omega_0 t + \\ &+ A_1\cos(\omega_0 t + \theta_1))(h(t - T/2) - h(t - T)) \end{aligned} \tag{6b}$$

$$\begin{aligned} W_1(t) = (&A\cos\omega_1 t + x\cos\omega_1 t + y\sin\omega_1 t) \\ &(h(t) - h(t - T/2)) \end{aligned} \tag{6c}$$

$$W_1(t) = (x\cos\omega_1 t + y\sin\omega_1 t)(h(t - T/2) - h(t - T)) \tag{6d}$$

In the frequency domain, the envelope detector translates the spectrum of the useful signal from the region of the high frequencies to baseband. In the time domain, it forms a sequence of pulses containing the modulating digital signal.

Digital signal envelope detectors contain also a decision block, which generates a sequence of pulses, despite of presence of noise and disturbances. The output of envelope detector in the upper branch in case of hypothesis H_0 is the symbol 1 in the first half - period and 0 in the second, and in case of hypothesis H_1 it will be 0 in the first half - period and 1 in the second. Likewise, the envelope detector output in the lower branch will be 0 in the first and 1 in the second half - period in the case of hypothesis H_0, and 1 in the first and 0 in the second half period in the case of hypothesis H_1.

III. Determination of the Probability Density Function

At the output of the envelope detector, the signals z_{ijk} are analyzed. In this case we can use the following notations:

i = {0, 1}, whether 0 or 1 is sent; (code domain)

j = {0, 1} whether upper or lower branch with bandpass filter is considered; (space domain)

k = {1, 2} whether the first or the second half - period is considered; (time domain).

Since both branches are subject to noise and one is subject to disturbance, we derive joint conditional probability density functions for different hypothesis in time and space domains.

$p(z_{001}, z_{002})$- represent combined probability density function of the signal “0” in upper branch during the whole period given by:

$$\begin{aligned} p(z_{001}, z_{002}) = &\int_{-\pi}^{\pi} d\varphi_1 \int_{-\pi}^{\pi} d\varphi_2 \int_{-\pi}^{\pi} d\theta\ p(\theta) \frac{z_{001} z_{002}}{(2\pi\sigma^2)(1-R)} \times \\ &\times \exp\left\{-\frac{(z_{001}\cos\varphi_1 - A - A_1\cos\theta)^2 + (z_{001}\sin\varphi_1 + A_1\sin\theta)^2}{2\sigma^2(1-R)^2}\right\} \times \\ &\times \exp\left\{\frac{(z_{002}\cos\varphi_1 - A_1\cos\theta)^2 + (z_{001}\sin\varphi_2 + A_1\sin\theta)^2}{2\sigma^2(1-R)^2}\right\} \times \\ &\times \exp\left\{-\frac{2R(z_{001}\cos\varphi_1 - A - A_1\cos\theta)(z_{002}\cos\varphi_2 - A_1\cos\theta)}{2\sigma^2(1-R^2)}\right\} \times \\ &\times \exp\left\{\frac{2R(z_{001}\sin\varphi_1 + A_1\sin\theta)^2 + (z_{002}\sin\varphi_2 + A_1\sin\theta)^2}{2\sigma^2(1-R^2)}\right\} \end{aligned} \tag{7}$$

R represents reference parameter of the envelope for both sampling moments. We suppose the uniform distribution of $p(\theta)$. φ_1 and φ_2 represent the refererent phasing for the z_0 and z_1 processes. They are given by the following expressions:

$$\varphi_1 = arctg\frac{y - A_1\sin\theta_1}{A + x + A_1\sin\theta_1}$$
$$\varphi_2 = arctg\frac{y - A_1\sin\theta_1}{x + A_1\sin\theta_1} \tag{8}$$

$p(z_{011}, z_{012})$- combined probability density function of the signal "0" in lower branch during the whole period and is given by:

$$\begin{aligned} p(z_{011}, z_{012}) &= \frac{z_{011}z_{012}}{(2\pi\sigma^2)(1-R^2)}\times \\ &\times\exp\left(-\frac{z_{012}^2 + z_{012}^2 + A^2}{\sigma^2(1-R^2)}\right)\times \\ &\times\int_{-\pi}^{\pi}\exp\left(-\frac{z_{011}RA}{\sigma^2(1-R^2)}\right)\cos\varphi_1 d\varphi_1\times \\ &\times\int_{-\pi}^{\pi}\exp\left(\frac{z_{012}A\cos\varphi_2}{\sigma^2(1-R^2)} + \frac{z_{011}z_{012}\cos(\varphi_1-\varphi_2)}{\sigma^2(1-R^2)}\right)d\varphi_2 \end{aligned} \tag{9}$$

$p(z_{101}, z_{102})$ - combined probability density function of the signal "1" in upper branch during the whole period. It is given by:

$$\begin{aligned} p(z_{101}, z_{102}) &= \int_{-\pi}^{\pi}d\varphi_1\int_{-\pi}^{\pi}d\varphi_2\int_{-\pi}^{\pi}d\theta\, p(\theta)\frac{z_{101}z_{102}}{(2\pi\sigma^2)(1-R^2)} \\ &\times\exp\left\{-\frac{(z_{101}\cos\varphi_1 - A_1\cos\theta)^2 + (z_{101}\sin\varphi_1 + A_1\sin\theta)^2}{2\sigma^2(1-R^2)}\right\}\times \\ &\times\exp\left\{\frac{(z_{102}\cos\varphi_1 - A - A_1\cos\theta)^2 + (z_{102}\sin\varphi_2 + A_1\sin\theta)^2}{2\sigma^2(1-R^2)}\right\}\times \\ &\times\exp\left\{-\frac{2R\{(z_{101}\cos\varphi_1 - A_1\cos\theta)(z_{102}\cos\varphi_2 - A - A_1\cos\theta)\}}{2\sigma^2(1-R^2)}\right\}\times \\ &\times\exp\left\{\frac{+(z_{101}\sin\varphi_1 + A_1\sin\theta)(z_{102}\sin\varphi_2 + A_1\sin\theta)}{2\sigma^2(1-R^2)}\right\} \end{aligned} \tag{10}$$

$p(z_{111}, z_{112})$ - Combined probability density function of the signal "1" in lower branch during the whole period and it is given by:

$$\begin{aligned} p(z_{111}, z_{112}) &= \frac{z_{111}z_{112}}{(2\pi\sigma^2)(1-R^2)}\exp\left(-\frac{z_{111}^2 + z_{112}^2 + A^2}{\sigma^2(1-R^2)}\right)\times \\ &\times\int_{-\pi}^{\pi}\exp\left(\frac{z_{111}A}{\sigma^2(1-R^2)}\cos\varphi_1\right)d\varphi_1\times \\ &\times\int_{-\pi}^{\pi}\exp\left(-\frac{z_{112}AR\cos\varphi_2}{\sigma^2(1-R^2)}\right)\exp\left(\frac{z_{111}z_{112}\cos(\varphi_1-\varphi_2)}{\sigma^2(1-R^2)}\right)d\varphi_2 \end{aligned} \tag{11}$$

The system output during the whole period in case the symbol "0" is sent, is:

$$H_0:\quad \begin{aligned} z_1 &= z_{001} - z_{011} \quad\rightarrow\quad z_{001} = z_1 + z_{011} \\ z_2 &= z_{002} - z_{012} \quad\rightarrow\quad z_{002} = z_2 + z_{012} \end{aligned} \tag{12}$$

Combined probability density function for symbol "0" is therefore:

$$\begin{aligned} p_0(z_1, z_2) &= \int_{z_{011}}\int_{z_{012}} p_1(z_1 + z_{011}, z_2 + z_{012})\times \\ &\times p_2(z_{011}, z_{012})dz_{011}dz_{012} \end{aligned} \tag{13}$$

The output during the whole period if the symbol "1" is sent will be:

$$H_1:\quad \begin{aligned} z_1 &= z_{101} - z_{111} \quad\rightarrow\quad z_{101} = z_1 + z_{111} \\ z_2 &= z_{111} - z_{112} \quad\rightarrow\quad z_{111} = z_2 + z_{112} \end{aligned} \tag{14}$$

Combined probability density function for symbol "1" is therefore:

$$\begin{aligned} p_1(z_1, z_2) &= \int_{z_{111}}\int_{z_{112}} p_1(z_1 + z_{111}, z_2 + z_{112})\times \\ &\times p_2(z_{111}, z_{112})dz_{111}dz_{112} \end{aligned} \tag{15}$$

The decision area $\Im_1$, for symbol "1", contains all the (z_1, z_2) pairs giving the likelihood ratio $\lambda(r) > \lambda_0$ [3]. On the other side, the decision area $\Im_0$, for symbol "0", contains all the (z_1, z_2) pairs giving the likelihood ratio $\lambda(r) < \lambda_0$. From these equations the error probability can be determined [4]..

IV. The Bit Error Probability

The main parameter that characterizes the system performances, that is, the quality of telecommunication service, is the bit error probability. Analytical expression for the bit error probability *Pe* is derived directly from previous equations:

$$\begin{aligned} Pe &= P(H_1)\iint_{\Im_0} p_1(z_1, z_2)dz_1dz_2 + \\ &+ P(H_0)\iint_{\Im_1} p_0(z_1, z_2)dz_1dz_2 \end{aligned} \tag{16}$$

where $p_0(z_1, z_2)$ and $p_1(z_1, z_2)$ represent probability density functions for H_0 and H_1, respectively. Integrals $\iint_{\Im_0}$ and $\iint_{\Im_1}$ represent the probabilities that ``0`` is detected when ``1`` is sent, and that ``1`` is detected when ``0`` is sent, respectively. The decision area is obtained relative to the likelihood ratio $\lambda(r) > \lambda_0$ [5].

The bit error probability dependence on SNR, for three values of signal to interference ratio, is given in Figs. 2 and 3.

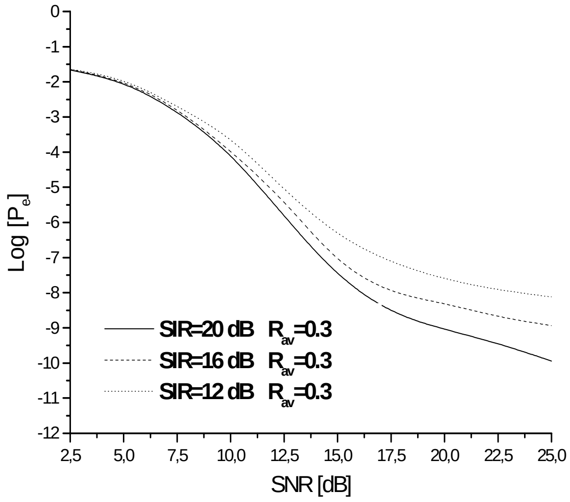

Fig. 2. The bit error probability of the system with respect to SNR, for three different values of SIR and parameter R_{av} = 0. 3.

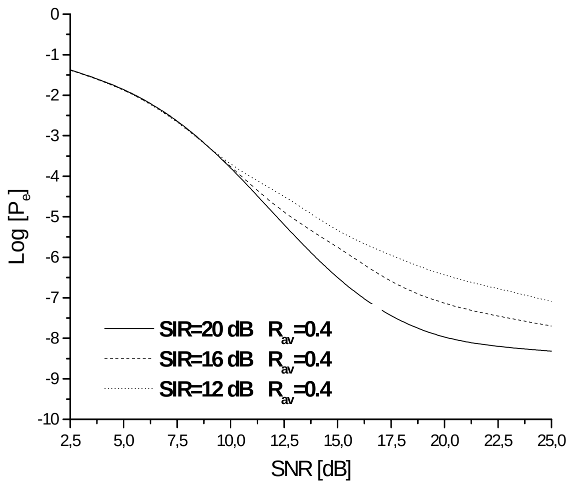

Fig. 3. The bit error probability of the system with respect to SNR, for three different values of SIR and parameter R_{av} = 0. 4.

These plots show that increasing the SNR results in decreasing the bit error probability. The best performances are gained for SIR = 20 dB

V. CONCLUSION

This paper presents expressions to evaluate the bit error probability for noncoherent frequency shift keying (NCFSK) system over digital channel with white Gaussian noise and channel interference. In receiver, decision is made on the bases of two samples per bit interval. The probability density function with respect to whether ``0`` or ``1`` is sent, is determined for both cases.

The system performances, i.e. the bit error probability is discussed in terms of different values of the SNR. As expected, the bit error probability is decreasing (the performances are better) when rising of SNR. Such system gives significantly better performances but is somewhat less cost effective.

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