Fuzzy Logic Controller for the Inverted Pendulum

Vesna Ranković[1], Ilija Nikolić[2]

Abstract - **In this paper we propose a Takagi-Sugeno (TS) type Fuzzy Logic Controller (FLC) to stabilize a inverted pendulum. Moreover there is a need for developing efficient methods to tune membership functions i.e. to obtain optimal shapes, ranges and number of member function. In this work is used ANFIS (Adaptive-Network Based Fuzzy Inference System).**

Keywords - **inverted pendulum, fuzzy, control**.

I. INTRODUCTION

The classical problem of the inverted pendulum controlled using linear quadratic regulator has been extensively studied and implemented. The linear quadratic regulator design is based on a linear model of the system and a quadratic performance index. The result is a linear feedback of the state variable of the system multiplied by a set of gains.

The nonlinear Ljapunov-based controller is proposed [1] to stabilize the cart-pole system. The novelty is in the use of two-loop cascade controller.

Fuzzy logic, neural network and neuro-fuzzy systems have been applied to identification of nonlinear dynamics and to stabilize a inverted pendulum. Justification for using neural networks and neuro-fuzzy systems lie in their capability in learning and generalizing any complicated mapping from training examples.

In [2] is used a control scheme that combines neural control and linear quadratic regulator together. The output from neural network is used to compensate the output from a linear quadratic regulator to stabilize the pendulum when it is near the orign.

In [3] is presented a Takagi-Sugeno type fuzzy logic controller for a 4 dimensional inverted pendulum system. The rule antecedents of this controller is extracted from a cell state space based optimal control table.

In [4] is examined the problem of learning an efficient fuzzy logic rule set for the control of the inverted pendulum using an evolutionary algorithm.

[5] presents a hybrid system controller, incorporating a neural and an linear quadratic regulator. The neural controller has been optimized by genetic algorithms directly on the inverted pendulum system.

In this work is used the Takagi-Sugeno controller implemented within the framework of the adaptive network to stabilize a inverted pendulum. Takagi-Sugeno fuzzy controllers are today one of the most promising technique to describe input-output relations of nonlinear systems using fuzzy rules. A number of algorithms have been developed that address this problem of learning fuzzy rules and tuning membership function in a neural network architecture. In this work is used ANFIS (Adaptive-Network Based Fuzzy Inference System) ([6]). The learning rule proposed for this method is basically a hybrid of the gradient-descent method and the least square technique. ANFIS and all ANFIS-like systems extract fuzzy if-then rules such that the premises are connected by a t-norm. The t-norm being used is the product, because it is continuous and all over differentiable.

In the second section of the paper is presented the inverted pendulum system. In the third section are presented simulation results. The fourth section contains the concluding remarks.

II. INVERTED PENDULUM SYSTEM

The system of an inverted pendulum on a cart is depicted in Fig. 1. The force delivered to the cart is always in horizontal direction. Assume the mass of the pole is evenly distributed along the pole. The positive direction of force F, cart displacement x, and pole angle θ are defined in Fig. 1.

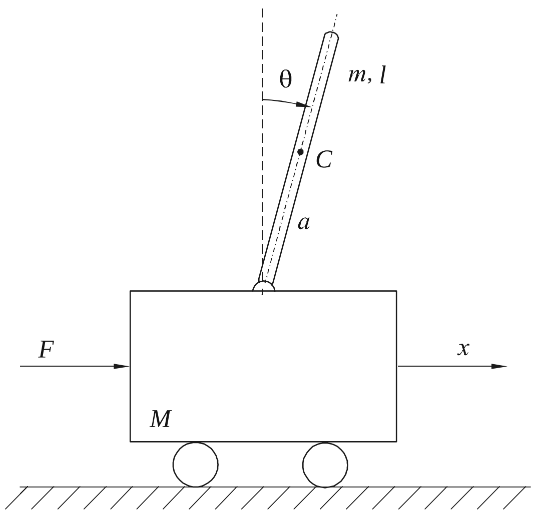

Fig. 1. Cart-pole inverted pendulum system

From the Langrangian, the general equations of motion are:

$$(M+m)\ddot{x} + ma\ddot{\theta}\cos\theta - ma\dot{\theta}\sin\theta = F \qquad (1)$$

$$\left(ma^2 + \frac{1}{12}ml^2\right)\ddot{\theta} + ma\ddot{x}\cos\theta - mga\sin\theta = 0 \qquad (2)$$

Where m and M are the masses of the pole and cart, respectively, g is the gravitational acceleration and a is

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distance from the pivot to the centre of mass of the pendulum.

The control objective of the inverted pendulum is as follows: given initial conditions $\boldsymbol{x}(\boldsymbol{0}) = \boldsymbol{x_0}$ where $\boldsymbol{x_0}$ is the state variables $\theta, \dot{\theta}, x, \dot{x}$ at time $t = 0$, find the control F to bring the system to point $\boldsymbol{x_e}$ where $\theta = 0$ and $x = x_d$.

We propose a Takagi-Sugeno type fuzzy logic controller to attain the control aim.

III. APPLICATION OF TAKAGI-SUGENO FUZZY CONTROLLER TO STABILIZE INVERTED PENDULUM SYSTEM

Simulation were performed with the program package MATLAB, by using the fuzzy toolbox. Fig. 2 are shown block diagram of the control system used in simulation. In Fig. 3 are shown SIMULINK model of inverted pendulum. The input and the output variables of the Takagi-Sugeno fuzzy controller to stabilize inverted pendulum system are shown in Fig. 4.

The inputs variables of the fuzzy logic controller:

x - cart displacement

$\dot{x}$ - cart velocity

θ - pole angle

$\dot{\theta}$ - pole angular velocity

e -cart position error ($e = x_d - x$, x_d -desired position)

and $\int_0^t edt$.

The fuzzy partitioning of the input variables of the fuzzy logic controller is realized by selection of 2 primary fuzzy sets. Selected are the Gaussian membership functions, since the best results are achieved with them.

For the system with 6 inputs and one output, set of linguistic rules is defined in the form:

R_1 : if x_1 is A_{11} and x_2 is A_{12} and x_3 and is A_{13} and x_4 is A_{14} and x_5 is A_{15} and x_6 is A_{16} then

$f_1 = p_{11}x_1 + p_{12}x_2 + p_{13}x_3 + p_{14}x_4 + p_{15}x_5 + p_{16}x_6 + c_1$

R_2 : if x_1 is A_{11} and x_2 is A_{12} and x_3 and is A_{13} and x_4 is A_{14} and x_5 is A_{15} x_6 is A_{26} then

$f_2 = p_{21}x_1 + p_{22}x_2 + p_{23}x_3 + p_{24}x_4 + p_{25}x_5 + p_{26}x_6 + c_2$

...

R_{64} : if x_1 is A_{21} and x_2 is A_{22} and x_3 and is A_{23} and x_4 is A_{24} and x_5 is A_{25} and x_6 is A_{26} then

$f_{64} = p_{641}x_1 + p_{642}x_2 + p_{643}x_3 + p_{644}x_4 + p_{645}x_5 + p_{646}x_6 + c_{64}$

Linguistic variables x_i are A_{1i}, A_{2i}, $i = 1, 6$.

The number of liguistic rules is: $p = 2^6$.

The total number of parameters, which are being adapted is 484, out of which 36 are premise parameters and 448 are consequences parameters.

The output from Takagi-Sugeno controller is:

$$y = \frac{1}{\sum_{i=1}^{64} u_i} \sum_{i=1}^{64} u_i \left(\sum_{j=1}^{6} p_{ij} x_j + c_i \right) \tag{3}$$

where:

$u_1 = \mu_{11}(x_1) * \mu_{12}(x_2) * \mu_{13}(x_3) * \mu_{14}(x_4) * \mu_{15}(x_5) * \mu_{16}(x_6)$

$u_2 = \mu_{11}(x_1) * \mu_{12}(x_2) * \mu_{13}(x_3) * \mu_{14}(x_4) * \mu_{15}(x_5) * \mu_{26}(x_6)$

...

$u_{64} = \mu_{21}(x_1) * \mu_{22}(x_2) * \mu_{23}(x_3) * \mu_{24}(x_4) * \mu_{25}(x_5) * \mu_{26}(x_6)$

* denotes certain *T*-norm.

If the membership function are taken in the Gaussian then:

$$\mu_{ij} = \frac{1}{1 + \left[\left(\frac{x_j - a_{ij}}{c_{ij}} \right)^2 \right]^{b_{ij}}}, \; i = 1, 2, \; j = 1, 6 \tag{4}$$

Characteristic values of the inverted pendulum: $M = 0.5\ kg$, $m = 0.2\ kg$, $l = 0.6\ m$, $a = 0.3\ m$. In Fig. 5, Fig. 6, Fig. 7, Fig. 8 , Fig. 9, Fig. 10 are shown the membership functions of primary fuzzy sets of variables x, $\dot{x}$, θ, $\dot{\theta}$, e, $\int_0^t edt$, respectively, after learning. In Fig. 11 is presented variation of the pole angle, and in Fig. 12 is given the force variation.

IV. CONCLUSION

Design of a fuzzy logic controller is accompanied certain problems regarding design of membership functions and choosing appropriate fuzzy rules. A number of algorithms have been developed that address this problem of learning fuzzy rules and tuning membership function in a neural network architecture. In this work is used ANFIS.

Results of simulation, presented in this paper, show that the application of the neuro-fuzzy (Takagi-Sugeno) system to stabilize inverted pendulum gives satisfactory results.

The traditional approach to building system controllers requires a priori model of the system. The quality of the model, that is, loss of precision from linearization and uncertainties in system's parameters negatively influence the quality of the resulting control.

At the same time, methods of soft computing such as neural network, fuzzy logic or neuro-fuzzy systems possess non-linear mapping capabilities, do not require an analytical model and can deal with uncertainties in the system's parameters.

Structure of the Takagi-Sugeno controller implemented within the framework of the adaptive network is similar to the structure of the RBF neural network

The controller drive the cart and the pole to its desired position (Fig. 11, Fig. 12).

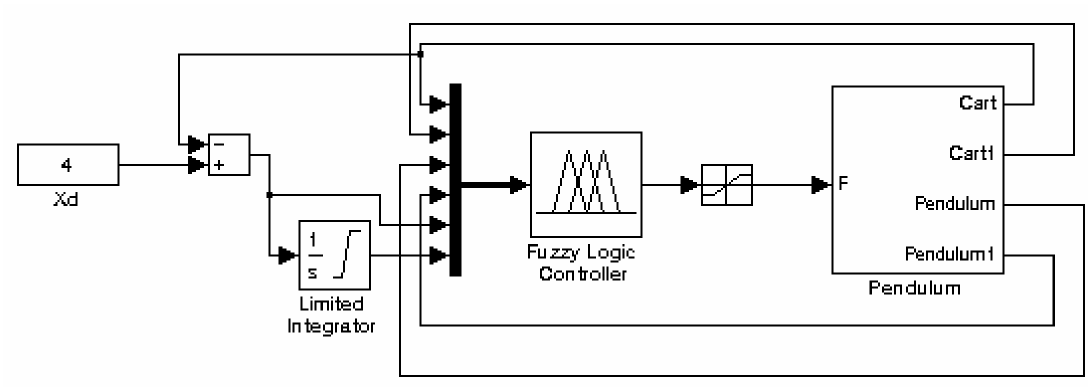

Fig. 2. Block diagram of the control system used in simulation

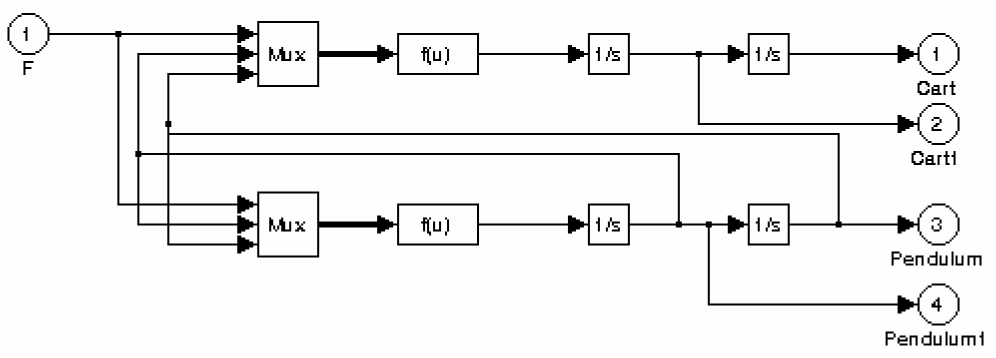

Fig. 3. SIMULUNK model inverted pendulum system

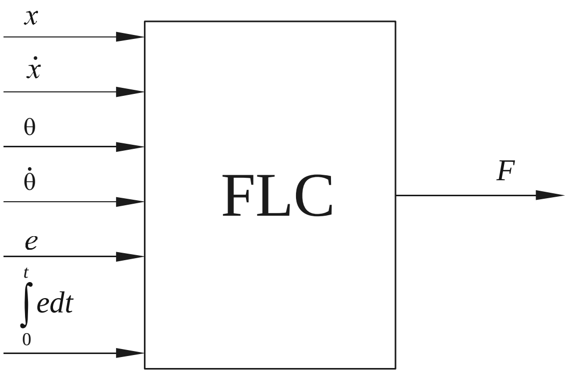

Fig. 4. Illustration of the input and output vatiables of Takagi-Sugeno fuzzy controller

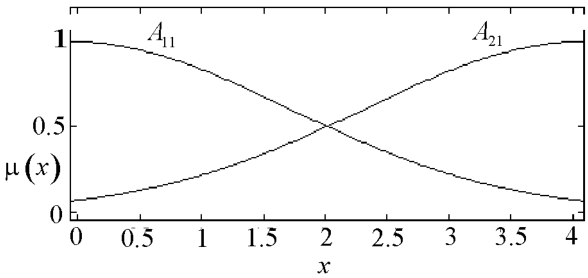

Fig. 5. Membership function of variables x after learning

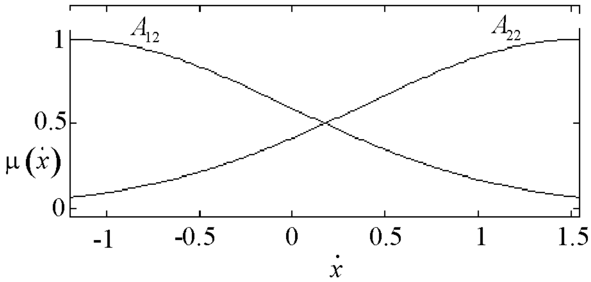

Fig. 6. Membership function of variables $\dot{x}$ after learning

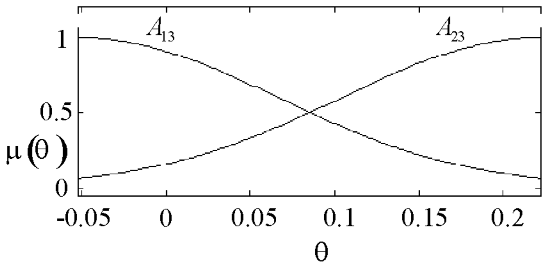

Fig. 7. Membership function of variables θ after learning

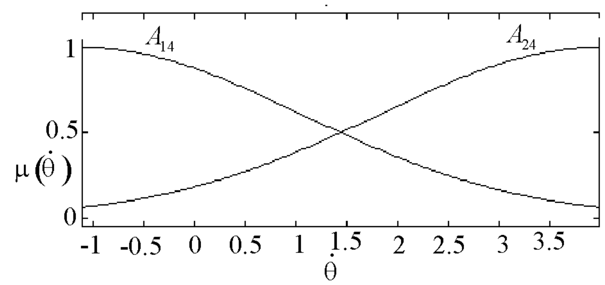

Fig. 8. Membership function of variables $\dot{\theta}$ after learning

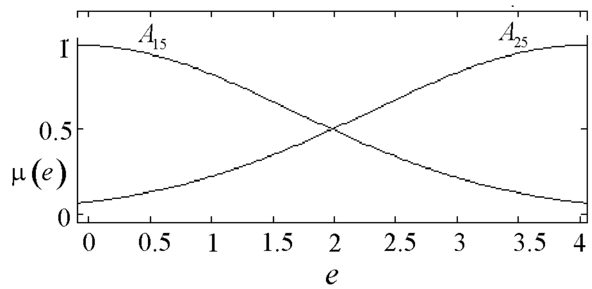

Fig. 9. Membership function of variables e after learning

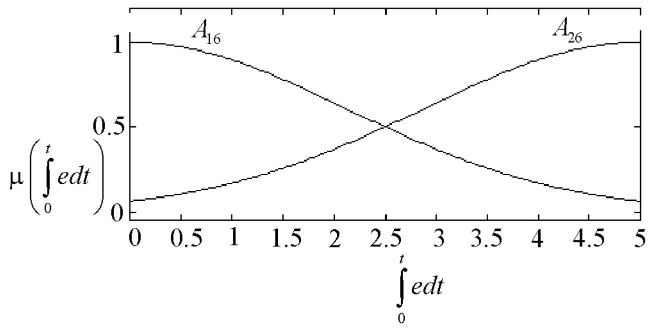

Fig. 10. Membership function of variables $\int_0^t edt$ after learning

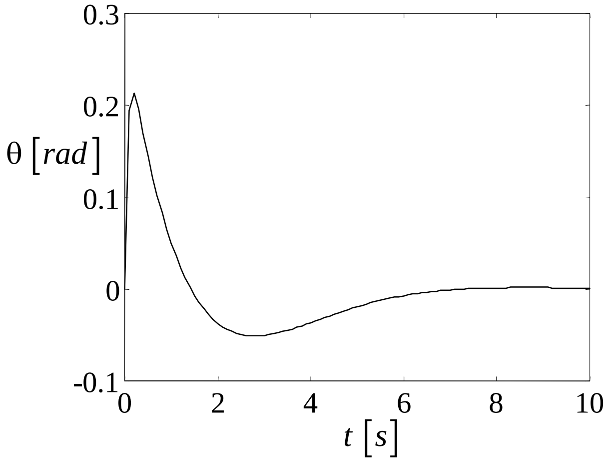

Fig. 11. Variation of the pole angle

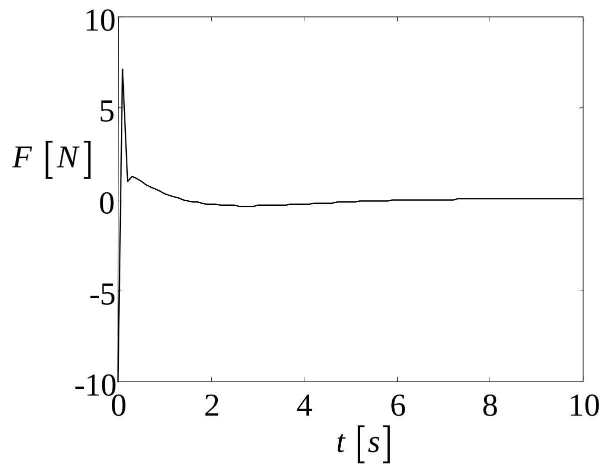

Fig. 12. Variation of the force

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REFERENCES

[1] A. Benaskeur, A. Desbiens "Application of adaptive backstepping to the stabilization of the inverted pendulum" Proceedings of IEEE Canadian Conference on Electrical and Computer Engineering, pp. 113-116, Waterloo, Canada, 1998.

[2] S. Omatu, T. Ide "Stabilization of Inverted Pendulum by Neuro-Control" Proceedings of IEEE International Conference on Neural Networks, pp. 2367-2372, Orlando, FL, 1994.

[3] F. Song, S. Smith "A Takagi-Sugeno Type Fuzzy Logic Controller with only 3 Rules for a 4 Dimensional Inverted Pendulum System" Proceedings of IEEE International Conference on System, Man and Cybernetics, pp. 3800-3805, Nashville, Tennessee, 2000.

[4] R. J. Stonier, A. J. Stacey, C. Messom "Learning fuzzy control laws for the inverted pendulum" Proceedings of ISCA 7th International Conference on Intelligent Systems, pp. 64-67, Melun, Paris, July, 1998.

[5] E. S. Sazonov, P. Klinkhachorn, R. Klein "Hybrid LQG-Neural Controller for Inverted Pendulum System" Proceedings of 35th Southeastem Symposium on System Theory, pp. 144-148, Morgantown, WV, 1993.

[5] E. S. Sazonov, P. Klinkhachorn, R. Klein "Hybrid LQG-Neural Controller for Inverted Pendulum System" Proceedings of 35th Southeastem Symposium on System Theory, pp. 144-148, Morgantown, WV, 1993.

[6] J.-S. R. Jang "ANFIS: Adaptive-Network-Based Fuzzy Inference Systems", *IEEE Transactions on Systems, Man, and Cybernetics*, Vol. 23, No. 3, pp. 665-685, 1993.