Identification of Dynamic Systems using Spline Neural Networks

B. Danković[1], M. Milojković[2], Z. Jovanović[3]

Abstract- **This paper demonstrates how spline neural networks can be used for identification of dynamic systems. A neuron is utilized to build spline networks with locally distributed dynamics to identify input/output models of dynamic processes. For static neural network design, cubic splines are used; for dynamic part, orthogonal Malmquist rational functions are used.**

Keywords- **Identification, Spline, Neural Network**

I. INTRODUCTION

Artificial neural networks represent nonlinear parametric models which process signals without requiring a specified model structure. They are employed to a wide spectrum of problems as different as pattern recognition, communication, artificial vision and system control. They have great importance for system analysis and automatic control problems such as control tasks, fault diagnosis, real-time simulations and system identification.

A subclass of artificial neural networks, so-called mapping neural networks, perform mathematically a mapping action from a domain of its input space to the output space. The mapping task is labelled static (or spatial) if there is no time-dependency involved within the mapping action. In general, neural networks belonging to this category are capable of approximating a mathematical function to any desired degree of accuracy based upon training data pairs and thus can be applied to identify static nonlinearities. As regards the identification of static (memory-less) systems, the multilayer perceptron (MLP) an radial basis function (RBF) networks are the most commonly applied types [3], [6]. Both networks belong to the subcategory of spatial mapping neural networks, and are proved to be universal approximators of static nonlinearities [3], [6]. Therefore, the networks can fit a function to the measurements of a memory-less system to any degree of accuracy.

In this paper, spline neural networks are used instead of RBF networks. Approximating functions with splines gives smoothness during approximation, i.e. sustain continuance of function and her derivates (up to three for cubic splines).

[1]Prof. dr Bratislav Dankovic is with the Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia and Montenegro, E-mail: dankovic@elfak.ni.ac.yu

[2]Marko Milojkovic is with the Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia and Montenegro, E-mail: milojkovic@elfak.ni.ac.yu

[3]Mr Zoran Jovanovic is with the Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia and Montenegro, E-mail: zoki@elfak.ni.ac.yu

Several approaches were proposed to introduce dynamics to artificial neural networks to identify dynamic systems. The networks are essentially subdivided into structures with lumped dynamics and distributed dynamics. Structures with lumped dynamics suffer from extensive memory menagement problems since the input space dimension and training times increase with the used number of lagged measurements. Figure 1 shows the process of identification using neural networks.

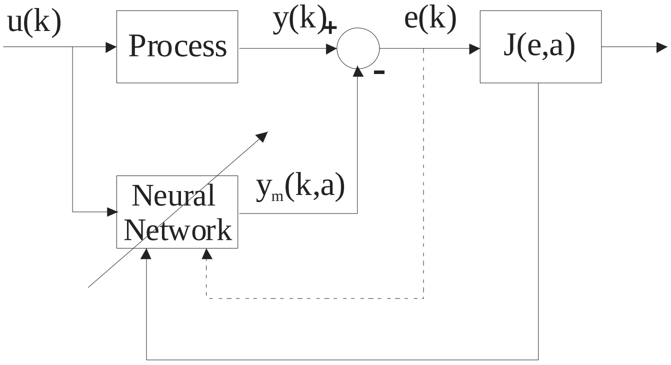

Fig 1 Process identification using neural networks

Contrary to these approaches with lumped dynamics, a novel class of neural networks arose with locally possess dynamic elements embodied within the neurons [6], [7], [8]. This class is labeled networks with distributed dynamics. In this paper, splines are used for static part of neural network; orthogonal Malmquist filter is used for dynamical part of the network instead of usual ARMAX filter [6].

II. DYNAMIC SPLINE UNITS

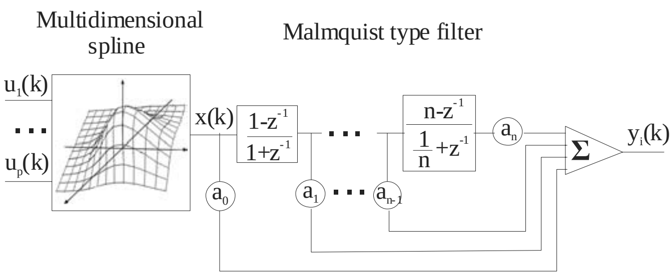

Fig. 2 Dynamic processing unit with spline function in state space representation with *p* inputs and one output

Figure 2 shows the modified structure of the dynamic neuron where spline cubic fuctions are used instead of RBFs. Instead of ARMAX filter, orthogonal Malmquist filter is used [9]. The filter input x(k) is calculated as a function of the *p* neuron inputs $u_p(k)$ by a multidimensional spline basis function [1], [2], with different biases with respect to each input. The mapping function of the spline function is given as:

$$M_i(x)=\begin{cases}0, x>2\\ \frac{1}{6}(2-x)^3, 1\le x\le 2\\ \frac{1}{6}\left[1+3(1-x)+3(1-x^2)-3(1-x)^3\right], 0\le x\le 1\end{cases} \quad (1)$$

Let $M_{ij}(x)=M_i(x-x_j)$, where x_j represents j-th bias

Function approximation using splines has a form [4]:

$$S_{iN}(x)=\sum_{l=0}^{N} C_l M_{ij}(x) \quad (2)$$

where C_l is unknown coefficient which can be determined by solving equation:

$$\mathbf{AC}=\mathbf{F} \quad (3)$$

where:

$$\mathbf{A}=\begin{bmatrix}\frac{2}{3} & \frac{1}{6} & \dots & \\ \frac{1}{6} & \frac{2}{3} & \frac{1}{6} & \dots \\ \dots & \frac{1}{6} & \frac{2}{3} & \dots \\ & \dots & \dots & \end{bmatrix}$$

$$\mathbf{F}=\begin{bmatrix}x_0 & x_1 & \dots & x_n\end{bmatrix}^T$$

$$\mathbf{C}=\begin{bmatrix}c_0 & c_1 & \dots & c_n\end{bmatrix}^T$$

for p inputs, we obtain:

$$x_q(k)=\prod_{i=1}^{p} M_{ij}(u_i) \quad (4)$$

where $q=1,\dots,M$.

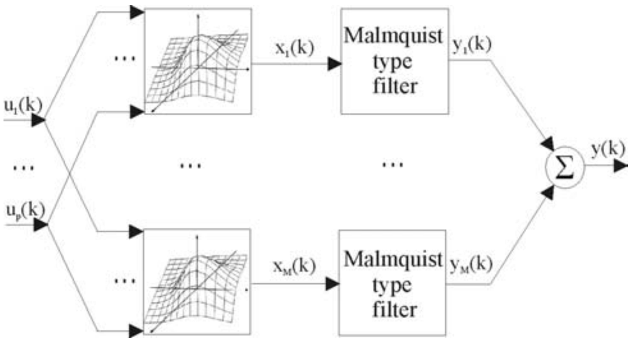

Fig. 3 Spline network with P inputs and one output

Linear part of the network is given by Malmquist orthogonal filter:

$$y(k)=a_0+\sum_{m=1}^{n} a_m \prod_{q=1}^{m} \frac{q-z^{-1}}{\frac{1}{q}+z^{-1}} \quad (5)$$

where N is order of the linear part.

Fig. 3 shows spline network with P inputs and one output comprising M dynamical spline units.

III. Parameters Optimization

Aim of algorithm for parameters optimization is to determine optimal parameter set which minimizes a quadratic performance index.

Model output has a form:

$$y_m=\sum_{i=1}^{N} y_i \quad (6)$$

where N is a number of spline units.

Output from i-th spline unit:

$$y_i=\sum_{j=1}^{Q} a_j M_j(\mathbf{U}) \quad (7)$$

where Q represents approximation order.

Mean square error:

$$J(\mathbf{a})=\frac{1}{2}\sum_{l=1}^{L}\left(y(k)-y_m(k,\mathbf{a})\right)^2=\min \quad (8)$$

where $\boldsymbol{a}$ is unknown parameter set which should be determined.

$$\mathbf{a}=(a_1,a_2,\dots,a_n)$$

Optimal parameters can be obtained in following way:

$$\frac{\partial J(a_i)}{\partial a_i}=0, \quad \text{(i=1,...,n)} \quad (9)$$

From (9) we obtain:

$$M^T M\mathbf{a}=M^T y_m \quad (10)$$

Where:

$$M=\begin{bmatrix}M_1(u_1) & M_2(u_1) & \dots & M_Q(u_1)\\ M_1(u_2) & M_2(u_2) & \dots & M_Q(u_2)\\ \dots & \dots & \dots & \dots\\ M_1(u_p) & M_2(u_p) & \dots & M_Q(u_p)\end{bmatrix}$$

From (10) we obtain [5], [6]:

$$\mathbf{a}=[M^T M]^{-1} M^T y_m \quad (11)$$

IV. Conclusion

Further studies of the proposed neural structure with respect to the identification properties have shown that the network is capable of approximating an accurate input/output model of physical processes where the nonlinearity is a function of the process inputs. For a first time, multidimensional splines for static part and Malmquist orthogonal filters for dynamic part of neural network are introduced. The final results are: smoother approximation and more accurate identification.

References

[1] Ahlberg J.H., *The Theory of Splines and their Applications*, Ac. Press, New York, 1987.

[2] Schumaker L.L., *Approximation by Spline Functions*, Ac. Press, New York, 1991.

[3] Gupta M.M., Jin L., Homma N., *Static in Dynamic Neural Networks*, John Willey and Sons, 2003.

[4] Kocic Lj., Dankovic B., "Process Identification using B-splines", *Numerical Methods and Approximation*, Nis, 1987.

[5] Dankovic B., Antic D., Jovanovic Z., *Process Control-Process Identification*, Faculty of Electronic Engineering, Nis, 1996.

[6] Ayoubi M., Isermann R., "Radial Basis Function Networks with Distributed Dynamics for Nonlinear Dynamic Systems Identification", *EUFIT*, Aachen, Germany, 1995.

[7] Ayoubi M., "Nonlinear Dynamic Systems Identification with Dynamic Neural Networks for Fault Diagnosis in Technical Processes", *IEEE Int Conf on Systems, Man and Cybernetics*, USA, 1994.

[8] Chassiakos A., Kosmatopoulos E. and Christodoulou M., "Identification of Robot Dynamics by Neural Networks with Dynamic Neurons", *Workshop on NN in Robotics*, 1991.

[9] B. Dankovic, G.V.Milovanovic, S. Rancic, "Malmquist and Muntz Orthogonal systems and applications, in Th. M. Rassias (Ed.), *Inner Product spaces and Applications*, Addison-Wesley, Harlow, pp. 22-41,1997.