Some Important Parameters of the SCP Technology

Veselin Demirev[1], Alexander Efremov[2]

Abstract – **Some important parameters of the SCP technology are defined and investigated in details. The first is the Shannon capacity of a SCP system, working in thermal noise, interference and combined environment. The second is the system Spatial Cross Correlation Function and its analytical presentation, leading to formulas for predicting the side lobe level, beam-width etc.**

Keywords – **Spatial correlation processing, Shannon capacity, SCCF**

I. Introduction

Description of a new radiocommunication technology, based on random phased antenna arrays approach and Spatial Correlation Processing (SCP) is given in [1]. Matrix presentations of the signals and the basic signal processing procedures, as well as computer simulations of the system spatial resolution properties are given in [2]. Some of the basic SCP system properties were investigated by means of the probability theory [3]. The goal of this report is to determine some new and important from communication point of view SCP parameters, as follows:

II. The Shannon capacity of the SCP systems

The SCP-GSO system analysis, given in [2], uses the standard thermal noise equations, typical for coherent demodulating schemes. The SCP pilot signal is random phase spread and as it was shown in [3], it has Gaussian probability distribution. In the ideal case of pure correlation with infinite period of time average, the product between the both independent Gaussian random processes (the thermal noise and the phase spread pilot) should vanish. In the real communication systems, because of the limited time average (in order of one bit period), this product can be assumed as equal to that of the coherent demodulation schemes. The detailed investigation of the SCP thermal noise properties, based on the probability theory of the cross-power density spectrum, will be done in a future report without any expected significant quantitative changes.

The standard formula for the Shannon capacity, expressed in bps/Hz is [4]:

$$C = \log_2\left(1 + \frac{P}{N_0 B_n}\right) \tag{1}$$

where P is the power of the received signal, N_0 is the noise spectral density and B_n is the receiver equivalent noise bandwidth at baseband.

For the case of SCP system in thermal noise limited environment, the equation (1) will have the form:

$$C = \log_2\left[1 + \frac{BBO_c}{G.F.K.T_o.B_n}\right] \tag{2}$$

where BBO_c is the cooperative output signal at baseband, G is the total receiver gain, F is the receiver noise figure, K is the Boltzman,s constant, T_0 is the standard temperature (290 deg. Kelvin).

For the case of SCP system in interference limited environment, the Eq. (1) will have the form:

$$C = \log_2\left[1 + \frac{BBO_c}{G.Tr(I.p^H)}\right] \tag{3}$$

where $Tr(I.p^H)$ is the power of the interference sources at the receiver input.

For the case of SCP system with only one interference source, the equation (1) will have the form:

$$C = \log_2\left[1 + \frac{1}{SCCF_{lin}}\right] \tag{4}$$

where $SCCF_{lin}$ is the Spatial Cross Correlation Function [1] of the SCP system, represented in linear form.

For the case of SCP system in mixed thermal noise and interference limited environment (which is the real case), the Eq. (1) will have the form:

$$C = \log_2\left[1 + \frac{BBO_c}{G.F.K.T_o.B_n + G.Tr(I.p^H)}\right] \tag{5}$$

III. Analytical presentation of the SCP SCCF pattern

The SCCF was introduced for the spatial interference analysis with one interference, as follows [1]:

$$SCCF(\phi,\theta)(dB) = 10\,\mathbf{lg}\left[\mathbf{BBO}_{inter.}(\varphi,\theta)/\mathbf{BBO}_c\right] \tag{6}$$

where the interference power at baseband is given with:

$$\mathbf{BBO}_{interference} = \mathbf{timeaver}\; G.(\mathbf{I.p}) = G.\mathbf{Tr}(\mathbf{I.p}^{\mathbf{H}}) \tag{7}$$

[1] Veselin Demirev is with the Faculty of Communications and Communication technologies, TU – Sofia, "Kl. Ohridsky" blvd. N8, 1756 – Sofia, Bulgaria, E-mail: demirev@homelan.bg

[2] Alexander Efremov is with the University of Chemical Technology and Metalurgy, "Kl. Ohridsky" blvd. N8, 1756 – Sofia, Bulgaria, E-mail: efremov_a@hotmail.com

In fact SCCF is the virtual antenna pattern of the SCP system at baseband. In [2] it was computer simulated for different tilt angles and real antenna diameters in Ku frequency band. It is important to derive an analytical presentation of the SCCF in order to predict some of the most important system parameters as beam-width, side-lobe level, etc.

The interference output signal, product of the multiplication process, is:

$$G(\mathrm{i}\,.\mathrm{p}) = = G\begin{vmatrix} i_1.p_1 & i_2.p_1\ldots & i_n.p_1\ldots & i_N.p_1 \\ i_1.p_2 & i_2.p_2\ldots & i_n.p_2\ldots & i_N.p_2 \\ \ldots & \ldots & \ldots & \ldots \\ i_1.p_n & i_2.p_n\ldots & i_n.p_n\ldots & i_N.p_n \\ \ldots & \ldots & \ldots & \ldots \\ i_1.p_N & i_2.p_N\ldots & i_n.p_n\ldots & i_N.p_N \end{vmatrix} \quad (8)$$

where i_n, p_n are the interference and the pilot signal at the output of the n^{-th} random antenna array element, G is the total receiver gain and the sum of the off-diagonal terms is zero. The real part of the n^{-th} diagonal term is:

$$\begin{aligned} &\mathrm{Re}(i_n.p_n) = \\ &= \pm 0{,}5.i_n.\cos\left[k.r_n.\sin\theta_c.\cos(\phi_c-\phi_n) - \right. \\ &\left. - k.r_n.\sin\theta_m.\cos(\phi_m-\phi_n)\right] \pm \\ &\pm 0{,}5.i_n(2\omega_{II}t + \ldots..) \end{aligned} \quad (9)$$

After time-average procedure the total interference (Eq.7) is:

$$\begin{aligned} &BBO_{\mathrm{int}\,erference} = \\ &= \pm 0{,}5G.i\sum_{n=1}^{N}\cos\, k.r_n\left[\sin\theta_c.\cos(\phi_c-\phi_n) - \right. \\ &\left. - \sin\theta_m.\cos(\phi_m-\phi_n)\right] \end{aligned} \quad (10)$$

The $SCCF_{linear}$ for this particular case is:

$$\begin{aligned} &SCCF_{linear} = \frac{BBO_{\mathrm{int}\,erference}}{BBO_c} = \\ &= \frac{1}{N}\sum_{n=1}^{N}\cos k.r_n\left[\sin\theta_c.\cos(\phi_c-\phi_n) - \right. \\ &\left. \sin\theta_m.\cos(\phi_m-\phi_n)\right] \end{aligned} \quad (11)$$

Eq. (11) is equal to that of an antenna directivity pattern of a circular antenna array with uniform amplitude distribution, in-phase collimated in the direction, given with azimuth ϕ_c and zenith θ_c (the angular coordinates of the cooperative satellite).

It is well known that the analysis of such kind antenna aperture could be done in two ways, depending on the antenna diameter to wavelength ratio and the distance between the elements, as follows:

- Relatively small antenna electrical dimensions and inter-element spacing greater than the used wavelength. In this particular case, bearing in mind the random spacing, statistical approach can be used (introduced by Lo in [5]).
- High values of the electrical dimensions and small inter-element spacing (between half and one wavelength). This particular case, corresponding to the SCP technology, can be analysed with the well developed theory of the phased uniform excited circular apertures [6], as follows:

The directivity antenna pattern of a circular uniform phased aperture $F(\theta)$ is given by:

$$F(\theta) = \frac{2J_1(2\pi a.\sin\theta/\lambda)}{2\pi a.\sin\theta/\lambda} \quad (12)$$

where J_1 is the Bessel function of first order and a is antenna array radius. For this particular case the first side-lobe level is -17.6 db, the half power beam-width $2\Delta\theta_{0,5}$ and the maximum antenna gain G_o are given by:

$$2\Delta\theta_{0,5}(rad) = 1{,}02\,\lambda/2a \quad (13)$$

$$G_0 = \frac{4\pi^2 a^2}{\lambda^2} \quad (14)$$

The main principles of the proposed SCP technology are valid for all kind of random phase antenna arrays, but the Radial Line Slot Antenna (RLSA) array [1] was shown to be the most suitable for DVB-S applications. The RLSA feature is the lack of radiation slots in the middle of the aperture. It leads to non-uniformity in the aperture distribution, which influence over SCCF can be predicted by means of the Fourier transformation summing theorem [7], as follows:

$$\begin{aligned} &G_{linRLSA}(\theta) = \\ &= \frac{4\pi^2 a^2}{\lambda^2}F^a(\theta) - \frac{4\pi^2 b^2}{\lambda^2}F^b(\theta) = \\ &= \frac{4\pi^2}{\lambda^2}\left[a^2\frac{2J_1(2\pi a.\sin\theta/\lambda)}{2\pi a.\sin\theta/\lambda} - \right. \\ &\left. - b^2\frac{2J_1(2\pi b.\sin\theta/\lambda)}{2\pi b.\sin\theta/\lambda}\right] \end{aligned} \quad (15)$$

where $G_{linRLSA}(\theta)$ is the spatial gain pattern of the RLSA, a is its outer radius, b is the inner radius.

The spatial power gain patterns (in dB) of the outer and the inner circular apertures are given by:

$$G^a(\theta, dB) = 10\,\lg\left[\frac{2J_1(2\pi a.\sin\theta/\lambda)}{2\pi a.\sin\theta/\lambda}\right]^2 \quad (16)$$

$$G^b(\theta, dB) = 10\,\lg\left[\frac{2J_1(2\pi b.\sin\theta/\lambda)}{2\pi b.\sin\theta/\lambda}\right]^2 \quad (17)$$

The maximum gain of the RLSA for $\theta = 0$ will be:

$$G_{0\,RLSA} = \frac{4\pi^2(a^2-b^2)}{\lambda^2} \quad (18)$$

In the end the SCCF of the RLSA will have the form:

$$SCCF\ (dB) = 10\lg\left[\left[2J_1(2\pi a\sin\theta/\lambda)/2\pi a\sin\theta/\lambda)\ \right]^2 - \frac{b^2}{a^2}\left[2J_1(2\pi b\sin\theta/\lambda)/2\pi b\sin\theta/\lambda\right]^2\right]\left[1-\frac{b^2}{a^2}\right]^{-1} \quad (19)$$

In Fig.1 the computed by Eqs.(16) and (17) spatial gain patterns of the outer and inner circular apertures are shown for comparison $(a = 28{,}5cm, b = 5cm, \lambda = 2{,}5cm,$ $\theta = -70^0\ to + 70^0, step\ 0{,}1^0)$

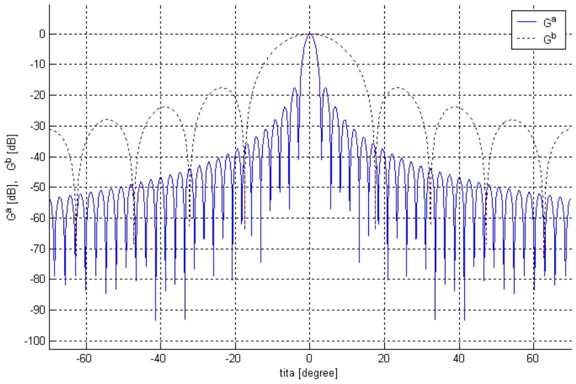

Fig.1. Spatial gain patterns of the outer and inner apertures

In Fig. 2 the computed with Eq.19 SCCF is shown. $(a = 28{,}5cm, b = 5cm, \lambda = 2{,}5cm,$ $\theta = -70^0\ to + 70^0, step\ 0{,}1^0)$:

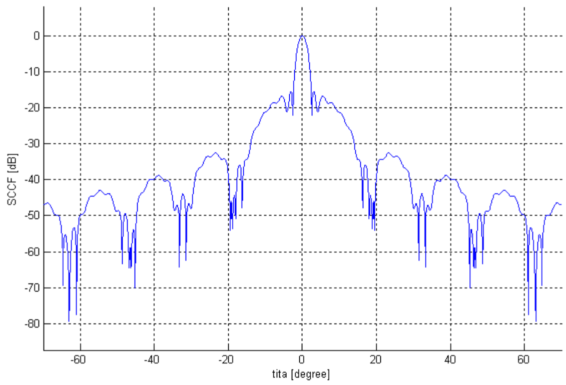

Fig.2. The computed SCCF

In Fig. 3 the computer simulated [2] SCCF is shown for comparison $(a = 28{,}5cm, b = 5cm, \lambda = 2{,}5cm,$ $\theta = -70^0\ to + 70^0, step\ 0{,}1^0)$.

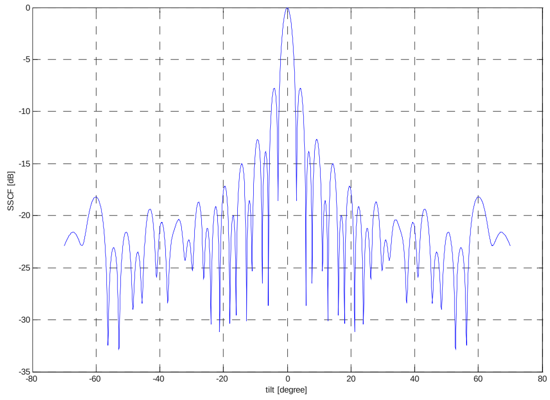

Fig.3. The computer simulated SCCF

IV. Conclusion

In this paper two important parameters of the SCP technology are defined and investigated in details. The first is the Shannon capacity of a SCP system, working in thermal noise, interference and combined environment. Equations for capacity calculations are derived. The second is the system Spatial Cross Correlation Function and its analytical presentation, leading to formulas for predicting the side lobe level, beam-width etc. Computer simulations, based on these formulas, are given too.

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