(5, 2) - Formal Languages

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Abstract – **The aim of this paper is to define a (5,2)-semigroup automata on free (5,2)-semigroup, with a special attention on (5,2)-formal languages recognizable by them.**

Keywords - **(5,2)-semigroup, (5,2)-semigroup automaton, (5,2)-language**

I. INTRODUCTION

Our goal in writing this talk is to examine a (5,2)-formal language and to proof some properties about them. In that means, we are given an example.

II. (5,2)-SEMIGROUPS AND (5,2)-SEMIGROUP AUTOMATA

Here we recall the necessary definitions and known results. From now on, let B be a nonemty set and let $(B,\cdot)$ be a semigroup, where $\cdot$ is a binary operation.

A semigroup automaton is a triple $(S,(B,\cdot),f)$, where S is a set, $(B,\cdot)$ is a semigroup, and $f:S\times B\to S$ is a map satisfying

$$f(f(s,x),y)=f(s,x\cdot y), \quad (1)$$

for every $s\in S$, $x,y\in B$.

The set S is called the set of **states** of $(S,(B,\cdot),f)$ and f is called the **transition function** of $(S,(B,\cdot),f)$.

A nonempty set B with the (5,2)-operation $\{\ \}:B^5\to B^2$ is called **a (5,2)-semigroup** iff the following equality

$$\{\{x_1^5\}x_6^8\}=\{x_1\{x_2^6\}x_7^8\}=\{x_1^2\{x_3^7\}x_8\}=\{x_1^3\{x_4^8\}\} \quad (2)$$

is an identity for every $x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8\in B$. It is denoted with the pair $(B,\{\ \})$.

Example 1: Let $B=\{a,b\}$. Then the (5,2)-semigroup $(B,\{\ \})$ is given by Table 1.

This example of (5,2)-semigroup is generated by an appropriate computer program.

A (5,2)-semigroup automaton is a triple $(S,(B,\{\ \}),f)$ where S is a set, $(B,\{\ \})$ is a (5,2)-semigroup, and $f:S\times B^4\to S\times B$ is a map satisfying

$$f(f(s,x_1^4),y_1^3)=f(s,\{x_1^4y_1\},y_2^3)=$$

$$=f(s,x_1,\{x_2^4y_1^2\},y_3)=f(s,x_1^2,\{x_3^4y_1^3\}), \quad (3)$$

for every $s\in S$, $x_1,x_2,x_3,x_4,y_1,y_2,y_3\in B$.

TABLE 1
(5,2)-SEMIGROUP

{ }	
a a a a a	*(a,a)*
a a a a b	*(a,a)*
a a a b a	*(a,a)*
a a a b b	*(a,a)*
a a b a a	*(a,a)*
a a b a b	*(a,a)*
a a b b a	*(a,a)*
a a b b b	*(a,a)*
a b a a a	*(a,b)*
a b a a b	*(a,b)*
a b a b a	*(a,b)*
a b a b b	*(a,b)*
a b b a a	*(a,b)*
a b b a b	*(a,b)*
a b b b a	*(a,b)*
a b b b b	*(a,b)*
b a a a a	*(b,a)*
b a a a b	*(b,a)*
b a a b a	*(b,a)*
b a a b b	*(b,a)*
b a b a a	*(b,a)*
b a b a b	*(b,a)*
b a b b a	*(b,a)*
b a b b b	*(b,a)*
b b a a a	*(b,b)*
b b a a b	*(b,b)*
b b a b a	*(b,b)*
b b a b b	*(b,b)*
b b b a a	*(b,b)*
b b b a b	*(b,b)*
b b b b a	*(b,b)*
b b b b b	*(b,b)*

The set S is called the set of **states** of $(S,(B,\{\ \}),f)$ and f is called the **transition function** of $(S,(B,\{\ \}),f)$.

2.1⁰ Let $(S,(B,\cdot),\varphi)$ be a semigroup automaton. Then $(S,(B,\{\ \}),f)$ is a (5,2)-semigroup automaton with (5,2)-operation $\{\ \}:B^5\to B^2$ defined by

$$\{x_1^5\}=(x_1\cdot x_2\cdot x_3\cdot x_4,x_5)$$

and the transition function $f:S\times B^4\to S\times B$ defined by

$$f(s,x_1^4)=(\varphi(s,x_1\cdot x_2\cdot x_3\},x_4).$$ ■

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2.2⁰. If $(S,(B,\{\ \}),f)$ is a (5,2)-semigroup automaton, then for every $c \in B$:

i) $(B^2, *_c)$ is a semigroup, where the operation $*_c$ is defined by $(x,y) *_c (u,v) = \{xycuv\}$ for every $(x,y),(u,v) \in B^2$;

(ii) $(S,(B^2,*_c),\psi)$ is a semigroup automaton, where the transition function $\psi : S \times B \times B^2 \to S \times B$ is defined by $\psi((s,a),(x,y)) = f(s,a,x,y,c)$. ■

Example 2: Let $(B,\{\ \})$ be a (5,2)-semigroup given by Table 1 from Example 1 and $S = \{s_0, s_1, s_2\}$. A (5,2)-semigroup automaton $(S,(B,\{\ \}),f)$ is given by Table 2 and the graph in Fig. 1 .

This example of (5,2)-semigroup automaton is generated by computer.

TABLE 2
(5,2)-SEMIGROUP AUTOMATON

f	s_0	s_1	s_2
a a a a	(s_1,a)	(s_1,a)	(s_2,a)
a a a b	(s_1,a)	(s_1,a)	(s_2,a)
a a b a	(s_1,a)	(s_1,a)	(s_2,a)
a a b b	(s_1,a)	(s_1,a)	(s_2,a)
a b a a	(s_2,b)	(s_1,a)	(s_2,a)
a b a b	(s_2,b)	(s_1,a)	(s_2,a)
a b b a	(s_2,b)	(s_1,a)	(s_2,a)
a b b b	(s_2,b)	(s_1,a)	(s_2,a)
b a a a	(s_1,a)	(s_2,a)	(s_2,b)
b a a b	(s_1,a)	(s_2,a)	(s_2,b)
b a b a	(s_1,a)	(s_2,a)	(s_2,b)
b a b b	(s_1,a)	(s_2,a)	(s_2,b)
b b a a	(s_2,a)	(s_2,b)	(s_2,b)
b b a b	(s_2,a)	(s_2,b)	(s_2,b)
b b b a	(s_2,a)	(s_2,b)	(s_2,b)
b b b b	(s_2,a)	(s_2,b)	(s_2,b)

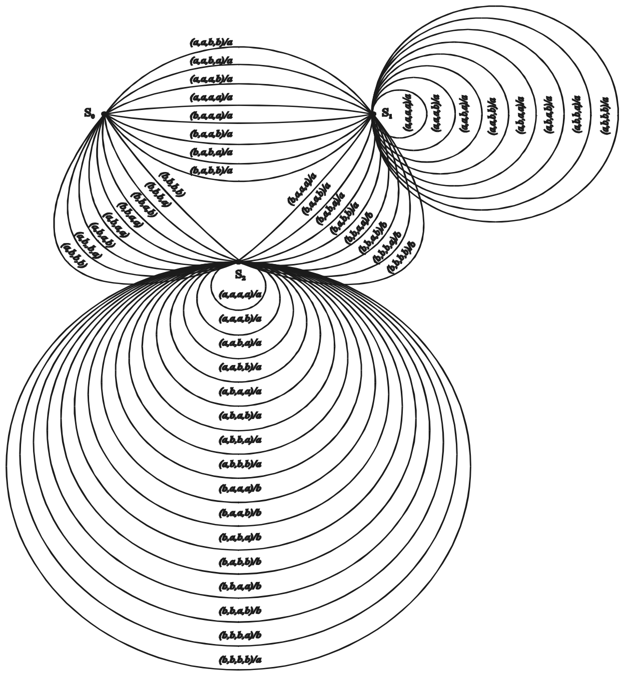

Fig. 1 (5,2)-semigroup automaton

III. FREE (5,2)-SEMIGROUPS AND (5,2)-SEMIGROUP AUTOMATA ON THEM

Let B be a nonempty set. We define a sequence of sets $B_0, B_1, \dots, B_p, B_{p+1}, \dots$ by induction as follows:

$B_0 = B$.

Let B_p be defined, and let A_p be the subset of B_p of all the elements u_1^{2+3s}, $u_\alpha \in B_p, s \geq 1$. Define B_{p+1} to be $B_{p+1} = B_p \cup A_p \times \{1,2\}$.

Let $\overline{B} = \bigcup_{p \geq 0} B_p$. Then $u \in \overline{B}$ iff $u \in B$ or $u = (u_1^{2+3s}, i)$ for some $u_\alpha \in \overline{B}, s \geq 1, i \in \{1,2\}$.

Define a length for elements of $\overline{B}$, i.e. a map $|\ | : \overline{B} \to N$ (N is a set of positive integers) as follows:

1^0 If $u \in B$, then $|u| = 1$;

2^0 If $u = (u_1^{2+3s}, i)$, then $|u| = |u_1| + |u_2| + \dots + |u_{2+3s}|$.

By induction on the length we are going to define a map $\varphi : \overline{B} \to \overline{B}$. For $b \in B$, let $\varphi(b) = b$. Let $u \in \overline{B}$ and suppose that for each $v \in \overline{B}$ with $|v| < |u|$, $\varphi(v) \in \overline{B}$ and

(1) If $\varphi(v) \neq v$, then $|\varphi(v)| < |v|$;

(2) $\varphi(\varphi(v)) = \varphi(v)$.

Let $u = (u_1^{2+3s}, i)$. Then, for each α, $\varphi(u_\alpha) = v_\alpha \in \overline{B}$ is defined, $|\varphi(u_\alpha)| \leq |u_\alpha|$ and $\varphi(\varphi(u_\alpha)) = \varphi(u_\alpha)$. Let $v = (v_1^{2+3s}, i)$.

(i) If for some α, $u_\alpha \neq v_\alpha$, then $|v_\alpha| < |u_\alpha|$, and so, $|v| < |u|$. In this case let $\varphi(u) = \varphi(v)$.

Because $|v| < |u|$, it follows that $\varphi(v)$ is defined, and moreover, (1) and (2) imply that

$|\varphi(u)| = |\varphi(v)| \leq |v| < |u|$, $\varphi(u) \neq u$ and

$\varphi(\varphi(u)) = \varphi(\varphi(v)) = \varphi(v) = \varphi(u)$.

(ii) Let $u_\alpha = v_\alpha$ for each α. Then $u = v$. Suppose that there is $j \in \{0,1,\dots,3s\}$ and $r \geq 1$, such that $u_{j+\nu} = (w_1^{3r+2}, i)$ for each $\nu \in \{1,2\}$ and let t be the smallest such j. In this case, let

$\varphi(u) = \varphi(u_1^t w_1^{3r+2} u_{t+4}^{3s+2}, i)$.

Because $\left|(u_1^t w_1^{3r+2} u_{t+4}^{3s+2}, i)\right| < |u|$ it follows that $\varphi(u)$ is well defined, and moreover, (1) and (2) imply that $\varphi(u) \neq u$, $|\varphi(u)| < |u|$ and $\varphi(\varphi(u)) = \varphi(u)$.

(iii) If $\varphi(u)$ can't be defined by (i) or (ii), let $\varphi(u) = u$. In this case, $\varphi(\varphi(u)) = \varphi(u) = u$ and $|\varphi(u)| = |u|$.

The above discusion and (i), (ii) and (iii) complete the inductive step, and so we have defined a map $\varphi : \overline{B} \to \overline{B}$. Moreover, we have proved the folloing:

Lemma: (a) For $b \in B$, $\varphi(b) = b$;

(b) For each $u \in \overline{B}$, $|\varphi(u)| \leq |u|$;

(c) For $u \in \overline{B}$, if $\varphi(u) \neq u$, then $|\varphi(u)| < |u|$;

(d) For each $u \in \overline{B}$, $\varphi(\varphi(u)) = \varphi(u)$. ■

Now, let $Q = \varphi(\overline{B})$. By Lemma (d),

$Q = \{u \mid u \in \overline{B}, \varphi(u) = u\}$.

Define a map $[\ ] : Q^5 \to Q^2$, by $[u_1^5] = (v_1^2)$ $\Leftrightarrow v_i = \varphi(u_1^5, i)$ for each $i \in \{1,2\}$.

Because $u_j \in Q$, it follows that $(u_1^5, i) \in \overline{B}$, and so $\varphi(u_1^5, i) \in Q$ for each $i \in \{1,2\}$. Hence $[\ ]$ is well defined.

Theorem: $(Q, [\ ])$ is a free (5,2)- semigroup with a basis B. ■

Let $(S, (B, \{\ \}), f)$ be a (5,2)-semigroup automaton.

Now, we define a sequence of maps $\psi_0, \psi_1, \ldots, \psi_p, \psi_{p+1}, \ldots$ for a sequence of sets $B_0, B_1, \ldots, B_p, B_{p+1}, \ldots$ by induction as follows:

$\psi_0 : B_0 \to B_0$ with $\psi_0(b) = b$, for each $b \in B_0$;

$\psi_1 : B_1 \to B_0$ with $\psi_1(b_1^n, i) = \{b_1^n\}_i$;

$\psi_2 : B_2 \to B_0$ with $\psi_2(u_1^n, i) = \{\psi_1(u_1) \ldots \psi_1(u_n)\}_i$

$\vdots$

$\psi_p : B_p \to B_0$ with

$$\psi_p(u_1^n, i) = \{\psi_{p-1}(u_1) \ldots \psi_{p-1}(u_n)\}_i$$

$\vdots$

Because $\overline{B} = \bigcup_{p \geq 0} B_p$, we define a map $\psi : \overline{B} \to B_0$ with $\psi(u) = \psi_p(u)$ for $u \in \overline{B}$ and $|u| \leq p$. Now we will prove that ψ is well defined. If

$u = (u_1^r (w_1^{2+3s}, i_1)(w_1^{2+3s}, i_2) u_{r+3}^{2+3t}, i)$,

$v = (u_1^r w_1^{2+3s} u_{r+3}^{2+3t}, i)$

and $\varphi(u) = \varphi(v)$, we have to prove that $\psi(u) = \psi(v)$. We have

$$\psi(u) = \psi_p(u) = \psi_p(u_1^r (w_1^{2+3s}, i_1)(w_1^{2+3s}, i_2) u_{r+3}^{2+3t}, i) =$$
$$= \{\psi_{p-1}(u_1) \ldots \psi_{p-1}(u_r) \psi_{p-1}(w_1^{2+3s}, i_1) \psi_{p-1}(w_1^{2+3s}, i_2) \psi_{p-1}(u_{r+3}) \ldots \psi_{p-1}(u_{2+3t})\}_i =$$
$$= \{\psi_{p-1}(u_1) \ldots \psi_{p-1}(u_r) \{\psi_{p-2}(w_1) \ldots \psi_{p-2}(w_{2+3s})\}_{i_1} \ldots \{\psi_{p-1}(w_1) \ldots \psi_{p-2}(w_{2+3s})\}_{i_2} \psi_{p-1}(u_{r+3}) \ldots \psi_{p-1}(u_{2+3t})\}_i =$$
$$= \{\psi_{p-1}(u_1) \ldots \psi_{p-1}(u_r) \psi_{p-1}(w_1) \ldots \psi_{p-1}(w_{2+3s}) \psi_{p-1}(u_{r+3}) \ldots \psi_{p-1}(u_{2+3t})\}_i.$$

Also,

$$\psi(v) = \psi_p(v) = \psi_p(u_1^r w_1^{2+3s} u_{r+3}^{2+3t}, i) =$$
$$= \{\psi_{p-1}(u_1) \ldots \psi_{p-1}(u_r) \psi_{p-1}(w_1) \ldots \psi_{p-1}(w_{2+3s}) \psi_{p-1}(u_{r+3}) \ldots \psi_{p-1}(u_{2+3t})\}_i.$$

Hence $\psi(u) = \psi(v)$. On the other hand, $Q = \varphi(\overline{B})$, so it follows that the restriction of ψ on Q is well defined.

Now again, we define a sequence of maps $\tau_0, \tau_1, \ldots, \tau_p, \tau_{p+1}, \ldots$ for a sequence of sets $B_0, B_1, \ldots, B_p, B_{p+1}, \ldots$ by induction as follows:

$\tau_0 : S \times B_0^4 \to S \times B_0$ with $\tau_0(s, x_1^4) = f(s, x_1^4)$;

$\tau_1 : S \times B_1^4 \to S \times B_1$ with

$$\tau_1(s, (u_{11}^{1\alpha_1}, i_1), (u_{21}^{2\alpha_2}, i_2), (u_{31}^{3\alpha_2}, i_3), (u_{41}^{4\alpha_4}, i_4)) =$$
$$f(s, \psi_1(u_{11}^{1\alpha_1}, i_1), \psi_1(u_{21}^{2\alpha_2}, i_2), \psi_1(u_{31}^{3\alpha_3}, i_3), \psi_1(u_{41}^{4\alpha_4}, i_4))$$

$\tau_2 : S \times B_2^4 \to S \times B_2$ with

$$\tau_2(s, (u_{11}^{1\alpha_1}, i_1), (u_{21}^{2\alpha_2}, i_2), (u_{31}^{3\alpha_3}, i_3), (u_{41}^{4\alpha_4}, i_4)) =$$
$$f(s, \psi_2(\overline{u}_{11}^{1\alpha_1}, i_1), \psi_2(\overline{u}_{21}^{2\alpha_2}, i_2), \psi_2(\overline{u}_{31}^{3\alpha_3}, i_3), \psi_2(\overline{u}_{41}^{4\alpha_4}, i_4))$$

$\vdots$

$\tau_p : S \times B_p^4 \to S \times B_p$ with

$$\tau_p(s, (u_{11}^{1\alpha_1}, i_1), (u_{21}^{2\alpha_2}, i_2), (u_{31}^{3\alpha_3}, i_3), (u_{41}^{4\alpha_4}, i_4)) =$$
$$f(s, \psi_p(\overline{u}_{11}^{1\alpha_1}, i_1), \psi_p(\overline{u}_{21}^{2\alpha_2}, i_2), \psi_p(\overline{u}_{31}^{3\alpha_3}, i_3), \psi_p(\overline{u}_{41}^{4\alpha_4}, i_4))$$

$\vdots$

Now we define a map τ for the sequence of maps $\tau_0, \tau_1, \ldots, \tau_p, \tau_{p+1}, \ldots$ by $\tau : S \times \overline{B}^4 \to S \times \overline{B}$, so that $\tau|_{B_p} = \tau_p$ and

$$\tau(s, (u_{11}^{1\alpha_1}, i_1), (u_{21}^{2\alpha_2}, i_2), (u_{31}^{3\alpha_3}, i_3), (u_{41}^{4\alpha_4}, i_4)) =$$
$$\tau_p(s, (u_{11}^{1\alpha_1}, i_1), (u_{21}^{2\alpha_2}, i_2), (u_{31}^{3\alpha_3}, i_3), (u_{41}^{4\alpha_4}, i_4)) =$$
$$f(s, \psi_p(u_{11}^{1\alpha_1}, i_1), \psi_p(u_{21}^{2\alpha_2}, i_2), \psi_p(u_{31}^{3\alpha_3}, i_3), \psi_p(u_{41}^{4\alpha_4}, i_4)) =$$
$$f(s, \psi(u_{11}^{1\alpha_1}, i_1), \psi(u_{21}^{2\alpha_2}, i_2), \psi(u_{31}^{3\alpha_3}, i_3), \psi(u_{41}^{4\alpha_4}, i_4)).$$

Because ψ is well defined, it follows that τ is well defined. On the other hand, $Q=\varphi(\overline{B})$ so $\overline{\varphi}$ denotes the map $\overline{\varphi}: S\times Q^4 \to S\times Q$ defined by

$$\overline{\varphi}(s,(u_{11}^{1\alpha_1},i_1),(u_{21}^{2\alpha_2},i_2),(u_{31}^{3\alpha_3},i_3),(u_{41}^{4\alpha_4},i_4))=$$
$$=\tau(s,(u_{11}^{1\alpha_1},i_1),(u_{21}^{2\alpha_2},i_2),(u_{31}^{3\alpha_3},i_3),(u_{41}^{4\alpha_4},i_4))=$$
$$= f(s,\psi(u_{11}^{1\alpha_1},i_1),\psi(u_{21}^{2\alpha_2},i_2),\psi(u_{31}^{3\alpha_3},i_3),\psi(u_{41}^{4\alpha_4},i_4)).$$

Moreover, $(S,(Q,[\ ]),\overline{\varphi})$ is a (5,2)-semigroup automaton, where $(Q,[\ ])$ is a free (5,2)-semigroup with a basis B.

IV. RECOGNIZABLE (5,2)-LANGUAGES

Any subset $L^{(5,2)}$ of the universal language $Q^*=\bigcup_{p\geq 1} Q^p$, where Q is a free (5,2)-semigroup with a basis B, is called **a (5,2)-language (formal (5,2)-language)** on the alphabet B.

A (5,2)-language $L^{(5,2)}\subseteq Q^*$ is called **recognizable** if there exists:

(1) a (5,2)-semigroup automaton $(S,(B,\{\ \}),f)$, where the set S is finite;

(2) an initial state $s_0\in S$;

(3) a subset $T\subseteq S$ such that

$$L^{(5,2)}=\{w\in Q^* \mid \overline{\varphi}(s_0,(w,1),(w,2))\in T\},$$

where $(S,(Q,[\ ]),\overline{\varphi})$ is the (5,2)-semigroup automaton constructed above, for the (5,2)-semigroup automaton $(S,(B,\{\ \}),f)$.

We also say that the (5,2)-semigroup automaton $(S,(B,\{\ \}),f)$ **recognizes** $L^{(5,2)}$, or that $L^{(5,2)}$ **is recognized** by $(S,(B,\{\ \}),f)$.

Example 3: Let $(S,(B,\{\ \}),f)$ be a (5,2)-semigroup automaton given in Example 2. We construct the (5,2)-semigroup automaton $(S,(Q,[\ ]),\overline{\varphi})$ for the (5,2)-semigroup automaton $(S,(B,\{\ \}),f)$.

A (5,2)-language $L^{(5,2)}$, which is recognized by the (5,2)-semigroup automaton $(S,(Q,[\ ]),\overline{\varphi})$, with initial state s_0 and terminal state (s_1,a) is

$$L^{(5,2)}=\{w\in Q^* \mid w=w_1w_2...w_q,\quad q\geq 5,$$ where

$$w_l=\begin{cases}(u_1^n,i),\ n\geq 5, u_\alpha\in Q\\ (a^*b^*)^*\end{cases},\ l\in\{1,2,...,q\},$$ and:

a) If $i=1$, then:

a1) $(u_1^n,1)=a$, where

$$\psi_{p-1}(u_1)...\psi_{p-1}(u_n)=a(a\cup b)(a^tb^la^h)^*,$$

a2) $(u_1^n,1)=b$, where

$$\psi_{p-1}(u_1)...\psi_{p-1}(u_n)=b(a\cup b)(a^tb^la^h)^*$$

b) If $i=2$, then:

b1) $(u_1^n,2)=a$, where

$$\psi_{p-1}(u_1)...\psi_{p-1}(u_n)=(a\cup b)a(a^tb^la^h)^*,$$

b2) $(u_1^n,2)=b$, where

$$\psi_{p-1}(u_1)...\psi_{p-1}(u_n)=(a\cup b)b(a^tb^la^h)^*,$$

and $\psi_p(w_1)...\psi_p(w_q)=(a\cup b)a(a\cup b)^2(a^tb^la^h)^*$, for $t+l+h=3k$, $t,l,h\in\{0,1,2,...\}$, $k\geq 1$, $q=3k+4$ }.

4.1⁰ Let $L^{(5,2)}$ be a (5,2)-language on the set B recognized by (5,2)-semigroup automaton $(S,(Q,[\ ]),\overline{\varphi})$. Let $(S,(Q,[\ ]),\overline{\varphi})$ be a (5,2)-semigroup automaton with initial state s_0 and a set of terminal states $T\times C\subseteq S\times B$. Then $x\widetilde{L}^{(2,1)}c\subseteq L^{(5,2)}$ for each $x\in Q$ and for any language $L^{(2,1)}$, which is recognized by the semigroup automaton $(S\times Q,(Q^2,*_c),\psi)$ with an initial state $s'_0=(s_0,x)$, a set of terminal states $T\times C$, where $\psi: S\times Q\times Q^2\to S\times Q$ is a transition function defined by $\psi((s,x),y_1^2)=\overline{\varphi}(s,x,y_1^2,c)$ for $c\in Q^p$ and p is the least non-negative integer, such that $2+p\equiv 0(\mathrm{mod}\,3)$, and $\widetilde{L}^{(2,1)}=\{\widetilde{w}\mid w\in L^{(2,1)}\}$.

Proof: $L^{(5,2)}$ is a recognizable (5,2)-language on the set B by the (5,2)-semigroup automaton $(S,(Q[\ ]),\overline{\varphi})$ with an initial state s_0 and a set of terminal states $T\times C\subseteq S\times B^{n-t}$, so

$$L^{(5,2)}=\{w\in Q^* \mid w=w_1w_2...w_{3q+4},\ q\geq 1 \text{ and}$$
$$\overline{\varphi}(s_0,(w_1^{3q+2},1),(w_1^{3q+2},2),\psi_{p-1}(w_{3q+3}),\psi_{p-1}(w_{3q+4}))=$$
$$\overline{\varphi}(s_0,\psi_{p-1}(w_1),(w_2^{3q+3},1),(w_2^{3q+3},2),\psi_{p-1}(w_{3q+4}))=$$
$$\overline{\varphi}(s_0,\psi_{p-1}(w_1),\psi_{p-1}(w_2),(w_3^{3q+4},1),(w_3^{3q+4},2))\in T\times C\ \}.$$

By Proposition 2.2⁰, $(S\times Q,(Q^2,*_c),\psi)$ is a semigroup automaton. It recognizes a language $L^{(2,1)}$ with a set of initial states $s'_0=(s_0,x)$ and a set of terminal states $T\times C$, so it is of the form

$$L^{(2,1)}=\{w\in(Q^2)^* \mid \psi(s'_0,w)\in T\times C\}.$$

Let $w\in L^{(2,1)}$. It follows that $w\in(Q^2)^*$ and $\psi(s'_0,w)\in T\times C$. But $s'_0=(s_0,x)$, so

$$\overline{\varphi}(s_0,x,(\widetilde{w},1),(\widetilde{w},2),c)=\overline{\varphi}(s_0,x,w,c)=$$
$$=\psi((s_0,x),w)=\psi(s'_0,w)\in T\times C$$

.

Thus $x\tilde{w}c \in L^{(5,2)}$, i.e. $x\tilde{L}^{(2,1)}c \subseteq L^{(5,2)}$. ■

4.2⁰ Let $L^{(2,1)}$ be a recognizable language on the set B by a semigroup automaton $(S,(B,\| \;\|),\xi)$ with an initial state $s_0 \in S$ and a set of terminal states $T \subseteq S$, and $(S,(B,\{\;\}),f)$ be an (5,2)-semigroup automaton constructed by a semigroup automaton $(S,(B,\| \;\|),\xi)$. Let $f : S \times B^4 \to S \times B$ is a transition function defined by $f(s,x_1^4) = (\xi(s,\|x_1^3\|),x_4)$. Then $L^{(2,1)}a \subseteq L^{(5,2)}$, for each $a \in B$, where $L^{(5,2)}$ is a recognizable (5,2)-language on the set B by the (5,2)-semigroup automaton $(S,(Q,[\;]),\overline{\varphi})$ with an initial state $s_0 \in S$ and a set of terminal states $T \times \{a\}$.

Proof: A language $L^{(2,1)}$ is recognizable by a semigroup automaton $(S,(B,\| \;\|),\xi)$ with initial state $s_0 \in S$ and a set of terminal states $T \subseteq S$, so

$$L^{(2,1)} = \{w \in B^* \mid \xi(s_0,w) \in T\}.$$

By Proposition 2.1⁰, $(S,(B,\{\;\}),f)$ is a (5,2)-semigroup automaton. We construct an (5,2)-semigroup automaton $(S,(Q,[\;]),\overline{\varphi})$, where $Q = \varphi(\overline{B})$ and $\overline{\varphi} : S \times Q^4 \to S \times Q$ is a transition function defined by

$$\overline{\varphi}(s,(u_{11}^{1\alpha_1},i_1),(u_{21}^{2\alpha_2},i_2),(u_{31}^{3\alpha_3},i_3),(u_{41}^{4\alpha_4},i_4)) =$$
$$= f(s,\psi_p(u_{11}^{1\alpha_1},i_1),\psi_p(u_{21}^{2\alpha_2},i_2),\psi_p(u_{31}^{3\alpha_3},i_3),\psi_p(\overline{u}_{41}^{4\alpha_4},i_4)).$$

It follows that a recognizable (5,2)-language $L^{(5,2)}$ on the set B by (5,2)-semigroup automaton $(S,(Q,[\;]),\overline{\varphi})$, with initial state $s_0 \in S$ and a set of terminal states $T \times \{a\}$ is of the form

$$L^{(5,2)} = \{w \in Q^* \mid w = w_1 w_2 \ldots w_{3q+4},\; q \geq 1 \text{ and}$$
$$\overline{\varphi}(s_0,(w_1^{3q+2},1),(w_1^{3q+2},2),\psi_{p-1}(w_{3q+3}),\psi_{p-1}(w_{3q+4})) =$$
$$= \overline{\varphi}(s_0,\psi_{p-1}(w_1),(w_2^{3q+3},1),(w_2^{3q+3},2),\psi_{p-1}(w_{3q+4})) =$$
$$= \overline{\varphi}(s_0,\psi_{p-1}(w_1),\psi_{p-1}(w_2),(w_3^{3q+4},1),(w_3^{3q+4},2)) \in T \times C\}.$$

Let $w \in L^{(2,1)}$, $|w| \geq 3q+3$ i.e. $w = w_1^{3q+3}$, $q \geq 1$ and $a \in B$. Then

$$\overline{\varphi}(s_0,(w_1^{3q+2},1),(w_1^{3q+2},2),\psi_{p-1}(w_{3q+3}),a) =$$
$$= \overline{\varphi}(s_0,(w,a)) = (\xi(s_0,w),a) \in T \times \{a\}.$$

Thus $wa \in L^{(5,2)}$, i.e $L^{(2,1)}a \subseteq L^{(5,2)}$. ■

V. CONCLUSION

The results was given in this paper, are of the scientific interest, because there was defined a (5,2)-languages as a consequence of the generalization of the semigroup automata in case (5,2). Also, here was given the conection between (2,1)-languages and (5,2)-languages.

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