System Sensitivity and Identification Error Correlation for Discrete-time Dynamic Systems

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Abstract- **This paper presents system sensitivity and identification error correlation for discrete-time dynamic systems. Parametric sensitivity of discrete-time dynamical systems is determined using simulation model. Using the same model and sum method, error of parametric identification is calculated. Finally, correlation between these two set of values is determined.**

Keywords- **Sensitivity, Identification, Correlation**

I. Introduction

It's well known fact that linear system's parameter sensitivity depends on system working frequency. For low frequencies, sensitivity is highest with respect to lowest parameter indexes. In case of high frequencies, sensitivity is highest with respect to highest parameter indexes.

On the other hand, it's well known that during inertial systems identification, it is easiest to estimate coefficients with lowest indexes. During estimation of coefficients with higher indexes, great error occur.

These two facts reveal existence of correlation between system sensitivity and identification error i.e. higher sensitivity implicates lower error and lower sensitivity implicates higher identification error.

In this paper, method for estimating correlation coefficient between process sensitivity and identification error is given. For sensitivity determining, system sensitivity model is used [1], [2]. For process identification, sum method is used. Identification error is calculated as difference between real system responce and responce from model obtained during system identification. For correlation coefficient between process sensitivity and identification error determining, known methods are used [2].

II. Determining Correlation between System Sensitivity and Identification Error

A. System Sensitivity

Parameter sensitivity is determined with sensitivity functions which are defined with [1]:

$$u_{a_i}(k) = \frac{\partial y(k, a_0, a_1, ..., a_n)}{\partial a_i}, i = 0,1,...,n \qquad (1)$$

where y represents system output and $a_1, a_2, ..., a_n$ are system's parameters. Logarithm sensitivity functions can also be defined:

$$u_{l,a_i}(k) = \frac{\partial y(k, a_0, a_1, ..., a_n)}{\partial \ln(a_i)}, i = 0,1,...,n$$

Figure 1 shows a model for simultaneous measurement of sensitivity functions for linear n-th order discrete system which can be described with equation:

$$a_n \Delta^n y(k) + a_{n-1}\Delta^{n-1} y(k) + ... + a_1 \Delta y(k) + a_0 y(k) = x(k) \qquad (2)$$

Sensitivity vector can be formed on the basis of measured sensitivities of single parameters:

$$\mathbf{u}_{Ma} = (u_{Ma_0} u_{Ma_1} ... u_{Ma_n}) \qquad (3)$$

where:

$$\mathbf{u}_{Ma_i} = \max\left| u_{a_i}(k) \right|$$

B. Sum method

Consider discrete time dynamic system in form (2) with boundary conditions [4]:

$$y(0) = \Delta y(0) = ... = \Delta^{(n-1)} y(0) = 0$$

$$\Delta y(\infty) = ... = \Delta^{(n-1)} y(\infty) = 0$$

Using transformations elaborated in [5], we obtain equations for parameter estimation:

$$a_0 = \frac{x(\infty)}{y(\infty)}$$

$$a_1 = \frac{1}{y(\infty)} a_0 \sum_{k_1=0}^{N} [y(\infty) - y(k_1)] \Delta t$$

$$a_2 = \frac{1}{y(\infty)} *$$

$$* \{ a_1 \sum_{k_1=0}^{N} [y(\infty) - y(k_1)] \Delta t - a_0 \sum_{k_1=0}^{N} \sum_{k_2=k_1}^{N} [y(\infty) - y(k_2)] \Delta t^2 \}$$

$$...$$

$$a_m = \frac{1}{y(\infty)} \sum_{j=1}^{m} (-1)^{j+1} a_{m-j} S_j$$

where:

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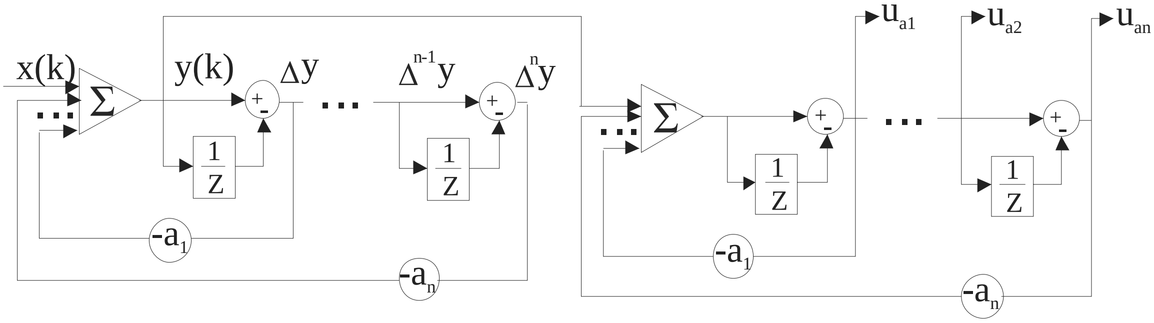

Fig. 1 Model for simultaneous measurement of sensitivity functions

$$S_j = \sum_{k_1=0}^{N} \sum_{k_2=k_1}^{N} \dots \sum_{k_j=k_{j-1}}^{N} [y(\infty) - y(k_1)]\Delta t^j; \tag{4}$$

$$m = 1, 2, \dots, N;\; j = 1, 2, \dots, m$$

where Δt represents discretisation period.

Equations (4) alowe identification of arbitrary order discrete systems, in form of linear diference equation based on measured responce values: $y_1, \dots, y_n$.

It's best to measure responce values in sampling moments. N should be large enough to satisfy:

$$y(N) > y(\infty)$$

Using equations above, vector of estimated parameter values can be found:

$$\hat{a} = (\hat{a}_1 \dots \hat{a}_n) \tag{5}$$

C. Correlation

Correlation between system sensitivity and identification error can be calculated using Persons correlation formula:

$$r = \frac{\sum_{i=0}^{n} (u_i - \bar{u})(\hat{a}_i - \bar{\hat{a}})}{\sqrt{\sum_{i=0}^{n} (u_i - \bar{u})^2} * \sqrt{\sum_{i=0}^{n} (\hat{a}_i - \bar{\hat{a}})^2}} \tag{6}$$

III. Experimental Determining of Correlation

Sensitivity functions are determined based on Fig.1 model for a fifth order system. These functions are measured in points u_{ai} $(i=1,\dots,n)$. Figures 2 and 3 show results for low and high frequency cases.

System coefficients are identified with sum method for fifth order system case. Starting system is:

$$0,0012\Delta^5 y + 0,0032\Delta^4 y - 0,114\Delta^3 y + +0,955\Delta^2 y + 2,08y + y = x \tag{7}$$

Using equations (4), next system is obtained:

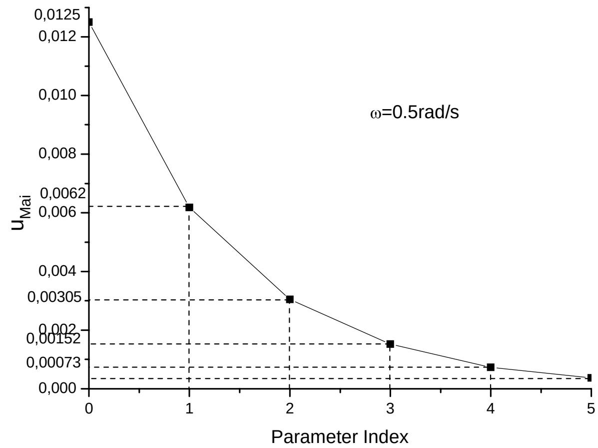

Fig. 2 U_{Mai} with respect to parameter index for low frequency case

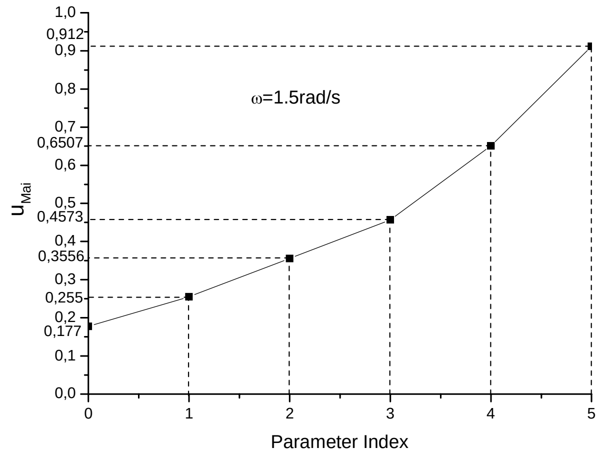

Fig. 3 U_{Mai} with respect to parameter index for high frequency case

$$-0,0123\Delta^5 y + 0,0127\Delta^4 y - 0,017\Delta^3 y + +1,0098\Delta^2 y + 2,0397\Delta y + 1,004y = x \tag{8}$$

Fig. 4 shows error in percents of obtained system parameters (8) with respect to real parameters (7).

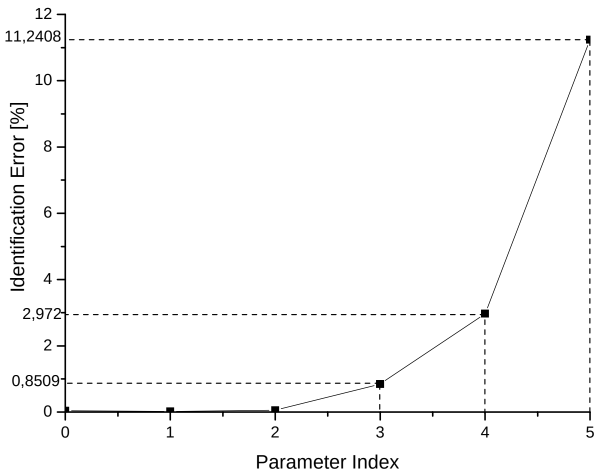

Fig. 4 Identification error with respect to parameter index for a low frequency case

System sensitivity and identification error correlation is calculated for the case of low frequency (figures 2 and 4). Using (6) and data logarithms, we obtain:

$$r = -0,98679$$

Obtained value shows high level of correlation between analyzed values. It means that parameters with respect to which the system is more sensitive, are easier to identify, and vice versa.

IV. CONCLUSION

Correlation between linear discrete system sensitivity and identification error is determined in this paper. Sensitivity model constructed using Δ operator is used for sensitivity functions determining. For low frequency, sensitivity is highest with respect to lowest parameter indexes. In case of high frequency, sensitivity is highest with respect to highest parameter indexes. Experimental results show high level of correlation between sensitivity and identification error.

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