Chaos Detection in Colpitts Oscillator

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Abstract – **Detection of chaotic state in nonlinear chaotic Colpitts oscillator has been studied. A new approach, based on concentration measure of the time-frequency representation, is proposed for estimation of the current circuit state and applied to Colpitts oscillator.**

Keywords – **chaos detection, nonlinear oscillator, time-frequency representations**

I. Introduction

Nonlinear systems (electrical, mechanical, biomedical, economical) that under the specific conditions exhibit chaotic behavior, attracted wide interest of researches in the last several decades [1]-[4]. One of important topics in this area is detection of the current system state. There are several techniques for chaos detection: Lyapunov exponents, Kolmogorov entropy, Poincare sections [5]. Lyapunov exponents are the most commonly used quantitative measure of chaos. Since Lyapunov exponents are calculation demanding, they are unusuable in the systems that in short interval, by varying one or several parameters, undergo different states. The other methods are applied in such cases.

In this paper, chaos detection in nonlinear oscillatory circuits, based on a time-frequency representation, is considered. Proposed detector estimates system state using a concentration measure of time-frequency representation. Detector has no information about oscilattor's structure and parameters. In this paper, detection is applied in case of the Colpitts oscillator, a simple circuit that is usually used in communications.

After the introduction, Colpitts oscillator is described in short in Section II. Proposed detector is studied in details and simulation results are shown in Section III. Finally, the conclusions and future research topics are given.

II. Colpitts Oscillator

Colpitts oscillator that we consider is shown in Fig. 1a. The circuit consist of a single bipolar junction transistor Q which is biased in its active region by appropriate choice of V_{EE}, R_{EE} and V_{CC}. The feedback network consists of an inductor L with series resistence R_L, and a capacitive divider composed of C_1 and C_2. If we suppose, as in [1], that bipolar transistor works in directly active region and cutoff, we can model transistor as a two-segment piecewise linear voltage controlled nonlinear resistor (Fig. 1b).

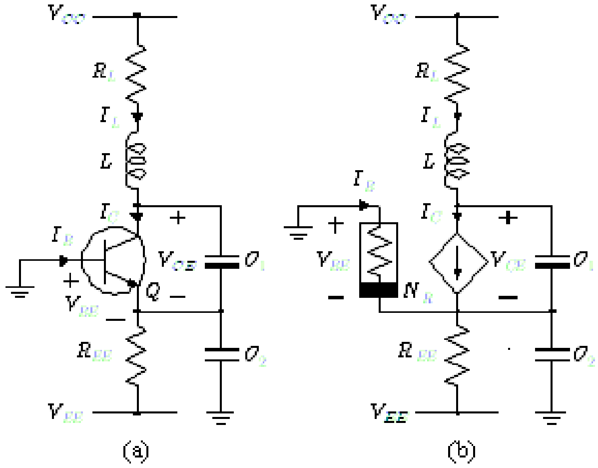

Fig. 1. (a) Colpitts oscillator with bipolar transistor. (b) Its equivalent circuit.

Thus, circuit can be described by system of three state equations:

$$C_1 \frac{dv_{CE}}{dt} = i_L - I_C \quad C_2 \frac{dv_{BE}}{dt} = -\frac{V_{EE}+v_{BE}}{R_{EE}} - i_L - I_B$$

$$L\frac{di_L}{dt} = V_{CC} - v_{CE} + v_{BE} - i_L$$

where N_R is characteristic of nonlinear resistor, given as:

$$I_B = \begin{cases} 0 & if \quad v_{BE} \le V_{TH} \\ \dfrac{v_{BE}-V_{TH}}{R_{ON}} & if \quad v_{BE} > V_{TH} \end{cases}$$

$$I_C = \beta_F I_B$$

V_{TH} is voltage threshold, R_{ON} is on-resistance for small signals and β_F is direct current gain.

This oscillator, with the suitable choice of parameters, exhibits chaotic behavior and can be exploited as a transmitter in chaotic-carrier communication systems [6].

Note that dynamic of the chaotic Colpitts oscillator linear conjugates to that of the asymmetric Chua's oscillator [7].

III. Proposed Detector

We consider behavior of Colpitts oscillator with parameters as in [1]: $C_1 = 54\text{nF}$, $C_2 = 54\text{nF}$, $L = 98.5\mu\text{H}$,

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$R_{EE} = 400\Omega$, $V_{EE} = -5\text{V}$, $V_{CC} = 5\text{V}$, $\beta_F = 255$, $R_{ON} = 100\Omega$, $V_{TH} = 0.75\text{V}$. Resistor R_L is linearly varied in the range from 67Ω to 5Ω. R_L is bifurcation parameter and route to chaos is period-doubling [2]. Time varying of parameter R_L causes change of the system state. Consequently, spectral content of the signal is changed and time-frequency representation is suggested as a natural tool for analysis. The short-time Fourier transformation (STFT) is applied in development of our detector, as the simplest and the most commonly used time-frequency representation [8]:

$$STFT(t, f) = \int_{-\infty}^{\infty} x(t+\tau)w(\tau)e^{-j2\pi f\tau}d\tau \qquad (2)$$

where $x(t)$ is the signal of the interest, (here, it is voltage or current), while $w(t)$ is window function.

Logarithm of the STFT magnitude of signal $v_{BE}(t)$ is considered and shown in Fig. 3a. In interval from 67Ω to 57Ω, oscillator exhibits periodic motion with the main frequency close to 88kHz. In the time-frequency plane the DC component, main frequency and components corresponding to main frequency multipliers can be seen. Logarithm of the STFT magnitude for $R_L = 59\Omega$ ($t = 2\text{ms}$) is shown in Fig 3b. Decrease resistance R_L causes period-doubling bifurcations. Periodic attractor with twice period occurs in the phase space. In the time-frequency plane, a subharmonic (which magnitude is lower than the one of the main harmonic) and its multipliers can be seen (Fig. 3c). After several period-doubling bifurcations system gets into chaos (Fig. 2b). In the time-frequency plane, between DC component and the main harmonic, there are many components of the same order of magnitude as dominant components. Logarithm of the STFT magnitude for $R_L = 48\Omega$ ($t = 5\text{ms}$) is shown in Fig. 3e. After that, chaos occurs, again. Finally, for $R_L = 9\Omega$, oscillator returns to periodic regime.

Note that similar results are obtained for signals $v_{CE}(t)$ and $i_L(t)$.

From previous consideration, we conclude that oscillator state can be estimated based on a concentration measure of signal's time-frequency representation between DC component and the main harmonic component. Thus, we create a concentration measure of the STFT as:

$$m(t) = \int_{0}^{f_m(t)} u_{\Omega(t)}(t; f)df \qquad (3)$$

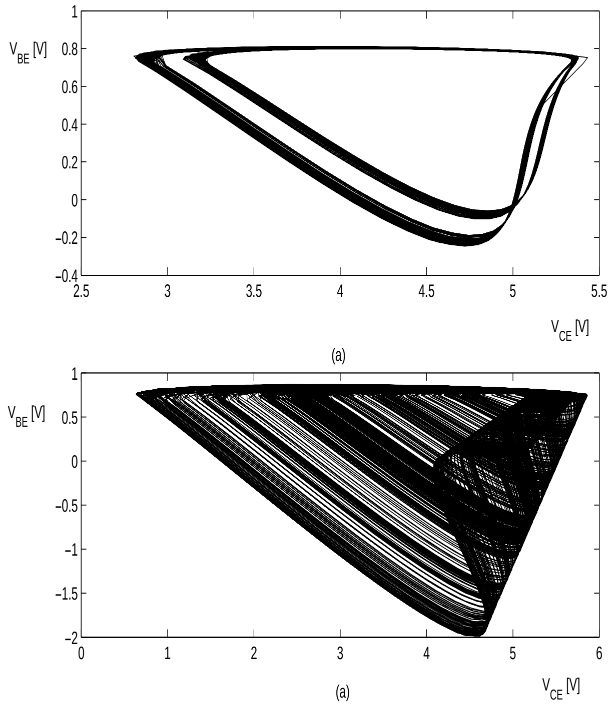

Fig. 2. (a) Period 2 attractor ($R_L = 57\Omega$). (b) Chaotic attractor $R_L = 35\Omega$.

where $\Omega(t)$ is threshold, $f_m(t)$ is frequency of the main spectral component. Function $u_{T(t)}(t; f)$ is given as:

$$u_{T(t)}(t; f) = \begin{cases} 1 & |STFT(t, f)| \geq \Omega(t) \\ 0 & \text{elsewhere.} \end{cases}$$

Procedure can be summarized in several steps.

I step - Calculation of the STFT using Eq. (2).

II step – Determination of the main spectral component $f_m(t)$ as a position of maximum in the STFT, excluding region around DC component:

$$f_m(t) = \arg\max_{f > \varphi} |STFT(t, f)| \qquad (4)$$

where φ is region of DC component (its width is several frequency samples).

III step – Threshold $\Omega(t)$ selection so that the STFT samples with magnitude higher than $\Omega(t)$ contain almost all signal's energy. Energy that remains outside of this region is very small:

$$(1-\varepsilon)\int_{0}^{\infty} |STFT(t, f)|^2 \, df = \int_{0}^{\infty} |STFT(t, f)|^2 \, u_{\Omega(t)}(t; f)df. \quad (5)$$

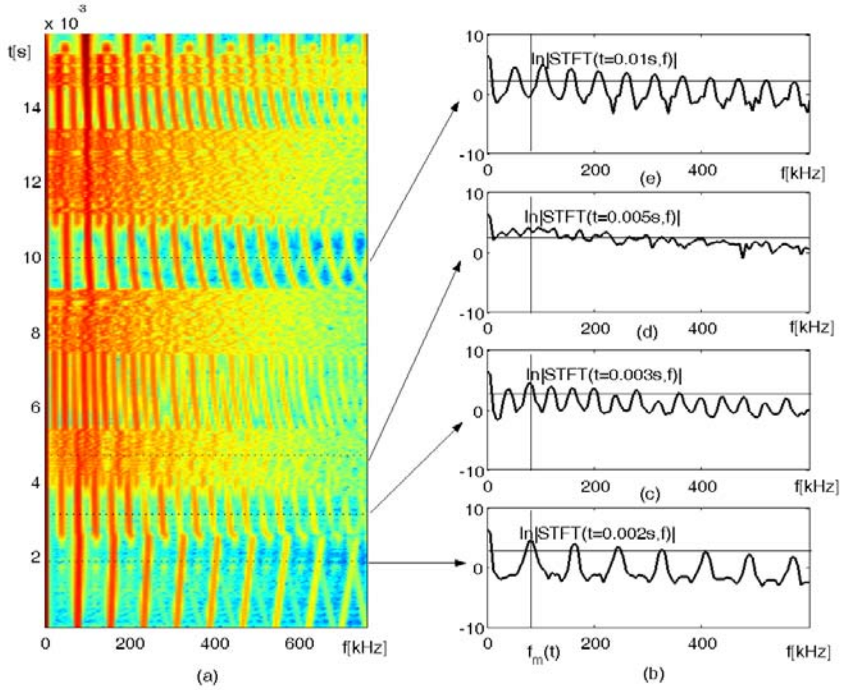

Fig. 3. (a) STFT. (b) Logarithm of the STFT for *t*=2ms - period 1 orbit. (c) Logarithm of the STFT for *t*=3ms - period 2 orbit. (d) Logarithm of the STFT for *t*=5ms - chaos. (e) Logarithm of the STFT for *t*=10ms - periodic window.

Note that our algorithm works well for relatively wide range of parameters ε. To determine threshold $\Omega(t)$, the magnitudes of the STFT samples, in considered instant, are sorted in descending order. Threshold is selected as a position where the rest of sorted sequence has energy smaller or equal to:

$$\varepsilon \int_0^{\infty} | STFT(t, f)|^2 \, df \, .$$

In Fig. 3 the threshold is shown as a solid horizontal line.

IV step – Detector response calculation according to relation (3). To avoid possible noise influence and other errors, detector response $m(t)$ is averaged within a small interval around considered instant:

$$m'(t) = \frac{1}{p} \int_{t-p/2}^{t+p/2} m(\tau) d\tau \, . \qquad (6)$$

V step – Determination of current state based on $m'(t)$:

$$\begin{aligned} m'(t) &\geq C(t) \quad \text{chaotic regime} \\ m'(t) &< C(t) \quad \text{periodic regime} \end{aligned} \qquad (7)$$

where $C(t)$ is detection threshold. In chaotic regime, it is expected that the STFT in entire region $[0, f_m(t)]$ is above the threshold. Thus, the expected value of $m'(t)$ in chaotic regime is close to $f_m(t)$. However, in periodic regime values of the STFT between the DC and the main spectral component are small. We assume that width of signal's components is known and determined by the used window function. Numerical calculation using Hanning window results with three nonzero frequency samples for frequency of sinusoidal component on frequency grid. Then, the expected value of $m'(t)$ in periodic regime, for the Hanning window of the width T, is $5/T$ (5 frequency samples, 3 of the main component and 2 of DC component, since one is in negative frequency region). Threshold is selected as the arithmetic mean of the expected values of detector response in periodic and chaotic regimes:

$$C(t) = \frac{f_m(t) + \frac{5}{T}}{2} \qquad (8)$$

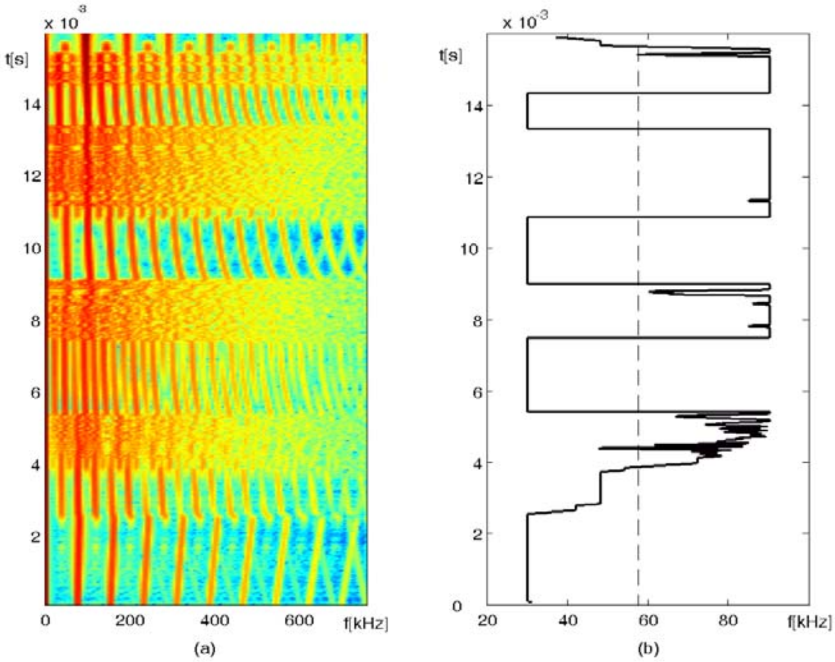

Fig. 4. (a) Time-frequency representation. (b) Thick line - detector response; dash line - threshold.

IV. SIMULATION RESULTS

In this section, the proposed detector is considered in the case of period-doubling route to chaos. Parameters of the Colpitts oscillator are the same as in previous section. We set $\varepsilon = 0.016$. Illustration of the time-frequency representation is repeated in Fig. 4a. Detector response is shown in Fig. 4b. Threshold is marked with dashed horizontal line. Regions where the detector response is above the threshold corresponding to chaos behavior. Our results are according to theory. Periodic windows are, also, correctly detected.

V. CONCLUSION

A simple chaotic state detector is presented. Using of the concentrating measure of the STFT samples enabled tracking and estimation time-varying behavior of the nonlinear oscillators. Colpitts oscillator is numerical analyzed, since it is well known in theory. Proposed estimator is tested on the other chaotic oscillators and obtained results are satisfactory.

In future work we will extend proposed detector to distinguish between various attractors.

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