# General Analysis of Resonant Inverters with Reverse Diodes, Operating at Various Modes

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*Abstract* – In this paper is obtained a general analysis of resonant inverters with reverse diodes (RIRD), operating at various modes.

This work will show the possibility that the results from the analysis of RIRD at under resonance mode and continuous current can be spread out and used for the over resonance mode and continuous current.

This generalization is useful even at methodical point of view when introducing with the RIRD operation, and to perform comparative analysis of the qualities of the power circuit at the both modes.

*Keywords* – Resonant inverters with reverse diodes, Analysis, Under resonance mode, Over resonance mode.

#### I. INTRODUCTION

The analysis of resonant inverters with reverse diodes RIRD when the current is ahead of the inverter voltage (under resonance mode) is presented at [1, 2, 6, 7, 8]. In [2, 8] are obtained the expressions of the inverter current and the voltage across the commutating capacitor and with their assistance the required for the design the RIRD at this mode quantities are worked out.

The analysis of RIRD when the current is behind of the inverter voltage (over resonance mode) is presented at [3, 4, 5, 7]. There are also the expressions of the inverter current and the voltage across the commutating capacitor, which express the required quantities to design the RIRD, operating in this mode.

With the assumption that the active and the reactive components are quite ideal a same equivalent circuit is valid. The inverter current at the examined cases is described by one ant the same common expression.

At methodical point of view it is much comfortable to use the analysis in established operation at under resonance mode, as at the first part of the semi wave the energy flow is form the power supply to the load, but at the second part the flow is backwards, which is just opposite for the over resonance mode.

This work will show the possibility that the results from the analysis of RIRD at under resonance mode and continuous current can be spread out and used for the over resonance mode and continuous current.

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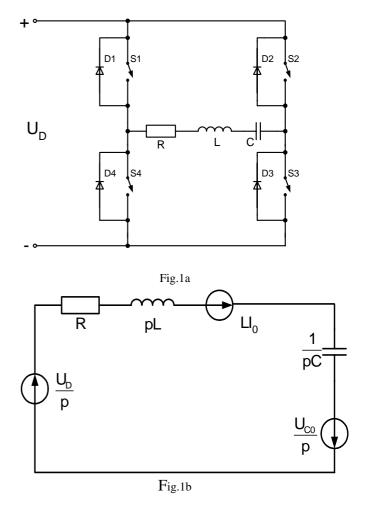
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### **II.** BASIC EQUATIONS

The schematic of the RIRD is shown at fig.1a, but at fig.1b the corresponding equivalent circuit. It is valid for the both examined modes, as the different initial conditions are affected by the corresponding polarities of their energy sources.



The waveforms of the inverter voltage and the inverter current for the both cases are shown at fig.2 and fig.3. The analysis of the processes in established regime, operating at under resonance mode [1, 2] and the obtained by this analysis expressions for the inverter current and the voltage across the commutating capacitor follow:

$$i(\vartheta) = \frac{2k_{od}U_{d}}{\omega_{0}L} \Upsilon e^{-\delta\frac{\vartheta}{\omega}} \sin\frac{\pi}{\lambda}(\vartheta + \psi), \qquad (1)$$

$$u_{\rm C}(\vartheta) = U_{\rm d} - 2k_{\rm od}U_{\rm d}\Xi e^{-\delta\frac{\vartheta}{\omega}}\sin\frac{\pi}{\lambda}(\vartheta+\phi), \qquad (2)$$

where  $\theta = \omega t$ ,  $\omega$  - control (gate) frequency,  $R = R_{(1)}$  is the equivalent load resistance, L and C are the equivalent commutating inductance and capacitance, U<sub>d</sub> is the power supply DC voltage,  $\omega_0 = \sqrt{\frac{1}{LC} - \delta^2}$  is the natural frequency of the serial resonant circuit,  $\delta = \frac{R}{2L}$  is its attenuation coefficient,  $v = \frac{\omega}{\omega_0}$  is frequency coefficient,  $\lambda = \frac{\pi\omega}{\omega_0}$  normalized to the control frequency angle of conduction of the controllable switches,  $\Upsilon = \sqrt{\left(1 - a\frac{\delta}{\omega_0}\right)^2 + a^2}$ ,

$$\Xi = \sqrt{1 + \left(\frac{\delta}{\omega_0} - a - a\left(\frac{\delta}{\omega_0}\right)^2\right)^2}, \text{ h and a are:}$$
$$h = \frac{\frac{1}{\pi} \ln\left(\frac{k}{k-1}\right) \sin \frac{\pi}{\nu} + \cos \frac{\pi}{\nu} + \left(\frac{k-1}{k}\right)^{\frac{1}{\nu}}}{\left(\frac{k-1}{k}\right)^{\frac{1}{\nu}} \left(\frac{1}{\pi} \ln\left(\frac{k}{k-1}\right) \sin \frac{\pi}{\nu} - \cos \frac{\pi}{\nu}\right) - 1},$$
$$a = \frac{\sin \frac{\pi}{\nu}}{\frac{1}{\pi} \ln\left(\frac{k}{k-1}\right) \sin \frac{\pi}{\nu} - \cos \frac{\pi}{\nu} - \left(\frac{k}{k-1}\right)^{\frac{1}{\nu}}}.$$

The hesitation coefficient k has the appearance  $k = \frac{1}{1 - e^{-\frac{\delta \pi}{\omega_0}}}$  and  $k_{od} = \frac{1}{1 - h \cdot \left(\frac{k-1}{k}\right)^{\frac{1}{v}}}$  is quantity which

characterizes the serial RLC circuit, and it is called hesitation coefficient of the RIRD, operating at under resonance mode,  $\lambda$  a  $\lambda$  1

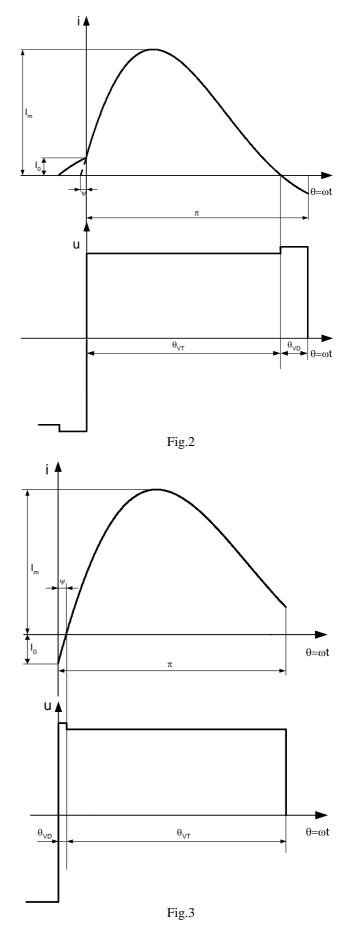
but 
$$\psi = \frac{\pi}{\pi} \arctan \frac{\delta}{1 - a \frac{\delta}{\omega_0}}$$
,  $\phi' = \frac{\pi}{\pi} \arctan \frac{\delta}{\omega_0} - a - a \left(\frac{\delta}{\omega_0}\right)^2$ .

The average current, consumed from the DC power source is  $I_d$  is expressed by:

$$I_{d} = -\frac{4k_{od}\delta U_{d}}{\omega_{0}R\pi F}\Upsilon\Delta, \text{ where}$$
(3)

$$\Delta = e^{-\frac{\delta\pi}{\omega}} \sin\left(\frac{\pi}{\lambda} (\pi + \psi) + \alpha\right) - \sin\left(\frac{\pi}{\lambda} \psi + \alpha\right),$$
$$F = \sqrt{\left(\frac{\delta}{\omega}\right)^2 + \left(\frac{\pi}{\lambda}\right)^2} \text{ and } \alpha = \operatorname{arctg}\left(\frac{\pi}{\lambda} / \frac{\delta}{\omega}\right).$$

The expressions for the average current through the switch  $I_{\rm av}$  and  $I_{\rm dav}$  through the diodes follow:



$$I_{av} = \frac{k_{od}U_{d}}{\omega_{0}L\pi F} \Upsilon \left( e^{-\frac{\delta}{\omega}(\lambda - \psi)} \sin \alpha + \sin(\frac{\pi}{\lambda}\psi + \alpha) \right), \tag{4}$$

The average current through the diodes  $I_{dav}$  is determined by the expression:

$$I_{dav} = I_{av} - \frac{I_d}{2} .$$
<sup>(5)</sup>

For determination of the RMS value of the load voltage U it is used the equation for the balance of the active power at the input and the output, which has the following appearance at various type of load:

$$P_{d} = U_{d}I_{d} = UI = \frac{U^{2}}{R} - \text{active load},$$
(6)

$$\begin{split} P_{d} &= U_{d}I_{d} = UI\cos\phi_{T} = \frac{U^{2}}{R}\cos^{2}\phi_{T} \text{ - active-inductive load,} \\ P_{d} &= U_{d}I_{d} = UI\cos\gamma = \frac{U^{2}}{R_{(1)}}\cos^{2}\gamma \text{ - parallel compensated} \\ & \text{load,} \end{split}$$

where I is the RMS value of the inverter current,  $R_{(1)}$  is the resistance at the first harmonic of the inverter current when parallel compensated load,  $\phi_T$  is the power factor of the active-inductive load, but  $\gamma$  is angle of displacement of the parallel compensated load circuit.

Finally the RMS value of the load voltage is:

$$U = \sqrt{U_{d}RI_{d}} - \text{when active load,}$$
$$U = \frac{1}{\cos \varphi_{T}} \sqrt{U_{d}RI_{d}} - \text{when active-inductive load,}$$
(7)

$$U = \frac{1}{\cos \gamma} \sqrt{U_d R I_d}$$
 - when parallel compensated load,

where the quantity  $I_d R$  is substituted with the determined by expression (3), which is function of the coefficient k and v.

The rest of the quantities, required for the design are presented in [2].

The expressions for the inverter current and the voltage across commutating capacitor when RIRD operates at over resonance mode are determined in [3, 4, 5] and they are:

$$i(\theta) = \frac{2K_{odn}U_{d}}{\omega_{0}L_{k}} \Upsilon_{n} e^{-\delta\frac{\theta}{\omega}} \sin\frac{\pi}{\lambda} (\theta - \psi_{n}), \qquad (8)$$

$$u_{\rm C}(\theta) = U_{\rm d} - 2K_{\rm odn}U_{\rm d}\Xi_{\rm n}e^{-\delta\frac{\theta}{\omega}}\sin\frac{\pi}{\lambda}(\theta + \varphi_{\rm n}), \qquad (9)$$

where

$$\Upsilon_{_{n}}=\sqrt{\left(1+a_{_{n}}\frac{\delta}{\omega_{_{0}}}\right)^{2}+a_{_{n}}^{2}}\ ,$$

$$\Xi_{n} = \sqrt{1 + \left(\frac{\delta}{\omega_{0}} + a_{n} + a_{n}\left(\frac{\delta}{\omega_{0}}\right)^{2}\right)^{2}} \quad \text{and}$$

$$K_{n} = \frac{1}{\omega_{0}} = \frac{1}{\omega_{0}} \text{ is quantity characterizing}$$

 $K_{odn} = \frac{1}{1 - h_n e^{\frac{-\delta\pi}{\omega}}} - \frac{1}{1 - h_n \left(\frac{k - 1}{k}\right)^{\frac{1}{\nu}}}$ the serial RLC circuit, and it is called hesitation coefficient of

RIRD, operating at over resonance mode, but h<sub>n</sub> and a<sub>n</sub> are :

$$h_{n} = \frac{\frac{1}{\pi} \ln\left(\frac{k}{k-1}\right) \sin\frac{\pi}{\nu} + \cos\frac{\pi}{\nu} + \left(\frac{k-1}{k}\right)^{\frac{1}{\nu}}}{\left(\frac{k-1}{k}\right)^{\frac{1}{\nu}} \left(\frac{1}{\pi} \ln\left(\frac{k}{k-1}\right) \sin\frac{\pi}{\nu} - \cos\frac{\pi}{\nu}\right) - 1},$$

$$a_{n} = \frac{\sin\frac{\pi}{\nu}}{\left(\frac{k}{k-1}\right)^{\frac{1}{\nu}} + \cos\frac{\pi}{\nu} - \frac{1}{\pi} \ln\left(\frac{k}{k-1}\right) \sin\frac{\pi}{\nu}}, \text{ but}$$

$$\psi_{n} = \frac{\lambda}{\pi} \arctan\frac{a_{n}}{1 + a_{n}\frac{\delta}{\omega_{0}}} \text{ and } \phi_{n} = \frac{\lambda}{\pi} \arctan\frac{1}{\frac{\delta}{\omega_{0}} + a_{n} + a_{n}\left(\frac{\delta}{\omega_{0}}\right)^{\frac{1}{\nu}}}$$

are the initial phase displacements of the inverter current and the voltage across the total commutating capacitor.

The consumed from the DC power supply current and the average values through the switches and the reverse diodes as function of the general parameters of the introduced in the analysis equivalent circuit of the AC circuit – the coefficients k and v are:

$$I_{d} = -\frac{2K_{odn}U_{d}\Upsilon_{n}}{\pi\omega_{0}L_{k}F}\Delta_{n}, \text{ where}$$
(10)  
$$\Delta_{n} = e^{-\frac{\delta\pi}{\omega}}\sin\left(\alpha + \frac{\pi}{\lambda}(\pi - \psi_{n})\right) - \sin\left(\alpha - \frac{\pi}{\lambda}\psi_{n}\right),$$
  
$$I_{av} = \frac{-K_{odn}U_{d}\Upsilon_{n}}{\pi\omega_{0}L_{k}F}\left(e^{-\frac{\delta}{\omega}\pi}\sin\left(\alpha + \frac{\pi}{\lambda}(\pi - \psi_{n})\right) - e^{-\frac{\delta}{\omega}\psi_{n}}\sin\alpha\right)(11)$$
  
$$I_{dav} = I_{av} - \frac{I_{d}}{2}.$$
(12)

The RMS value of the load voltage is determined by the same expressions (7) just the same as at under resonance mode.

From the comparison of the expressions, which describe the currents and the voltages at the both modes it is seen that the generalized hesitation coefficients  $k_{od}$  and  $k_{odn}$  are quite equal, and the initial phase angles of the inverter current and the voltage across the equivalent commutating capacitor have equal magnitudes and different signs, which

express the different phase displacement between the inverter current and inverter voltage at the both modes. The same is valid for the coefficient "a", which expresses the initial conditions of the inverter current related to the inverter voltage in both cases.

Then from expression (1) for the inverter current and the voltage across the commutating capacitor (2) at under frequency mode when substitute " $\Psi$ ", " $\phi$ " and "a" respectively by "- $\Psi$ ", "- $\phi$ " and "-a" the corresponding expressions at over resonance mode are obtained (8) and (9).

This gives us a reason to use the results from analysis of RIRD at under resonance mode as equal results to the analysis at over resonance mode. For this purpose it is required for all the expressions, describing the operation of RIRD and under resonance mode the frequency coefficient "v" to be greater than unity, when over resonance mode is present.

To determine the average values of the currents through the switches and the diodes, could be used their expressions at under resonance mode (4) and (5), if the integral boundaries are substituted in accordance with fig.3.

## **III.** CONCLUSION

This general look over the processes in the RIRD for the both modes and continuous current, allows the well known and spread results for under resonance mode to be used to evaluate the inverter behavior when the load or the gate frequency varies and of course for its design.

This generalization is useful even at methodical point of view when introducing with the RIRD operation, and to perform comparative analysis of the qualities of the power circuit at the both modes.

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