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Turbo Coding Performances in Small Signal to Noise Ratio Environment

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Abstract - **This paper investigates several aspects of interleaver design as well as encoder and decoder structure and algorithms in small signal to noise ratio environment. Several interleaver structures are investigated, namely block, helical, hybrid, even-odd and random and compared with respect to overall system BER results. SOVA and MAP decoder algorithms are used with different interleaver lengths. Extended simulations have been made to evaluate the BER performances of different structures and the results are presented in section 5.**

Keywords - **turbocodes, interleaver, decoder algorithm, BER.**

I. INTRODUCTION

Turbo coding respresents one of the most efficient technique of channel coding. It leads to higher data transmission rates and it improves the quality of the communication service. Turbocodes can work real close to Shannon limit capacity overcoming any other technique of channel coding known so far. The strong capacity of correction shown by the turbocodes was recognized and accepted for almost every type of channel.

Interleavers play a very important role in achieving good performances with these structures. Interleaving is the process of rearranging the ordering of the data sequence in a one-to-one deterministic format. The reverse process is deinterleaving which restores the received sequence to its original order. Interleaving is also a practical technique to enhance the error correcting capability of coding.

In this article we are analyzing the performances of turbocodes using several types of interleavers (such as odd-even interleaver, hybrid interleaver, random interleaver and block interleaver) and two algorithms: MAP and SOVA.

II. TURBO CODING AND ITERATIVE DECODING

A turbo Encoder is formed by a parallel concatenation of two recursive systematic convolutional (RCS) encoders separated by a random interleaver:

The encoder structure is called parallel concatenation because the two encoders operate on the same set of input bits rather than one encoding the output of the other. Thus turbo codes are also referred to as parallel concatenated convolutional codes.

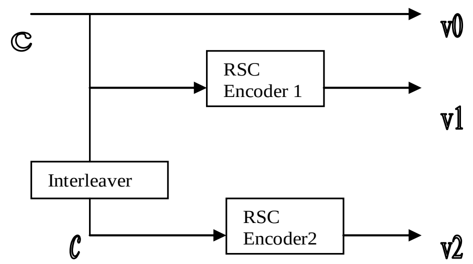

Fig.1. Encoder architecture

The interleaver in turbo coding is a pseudo-random block scrambler defined by a permutation of *N* elements with no repetitions. The roles of the interlevears are to generate a long block code from small memory convolutional codes; it decorrelates the inputs to the two decoders so that an iterative suboptimum decoding algorithm based on information exchange between the two component decoders can be applied.

The iterative decoder structure consists of two component decoders, serially concatenated via an interleaver, identical to the one used in the encoder, as shown in figure 2. The first decoder uses the received information bits r_0 and the parity bits generated by the first encoder r_1 in order to produce a soft output, which is interleaved and used to improve the estimate of the apriori probabilities for the second decoder. The other two inputs of the second decoder are the interleaved information sequence $\tilde{r}_0$ and the received parity sequence produced by the second encoder. This decoder produces a soft output also, that is de-interleaved and used by the first decoder to improve its apriori probabilities.

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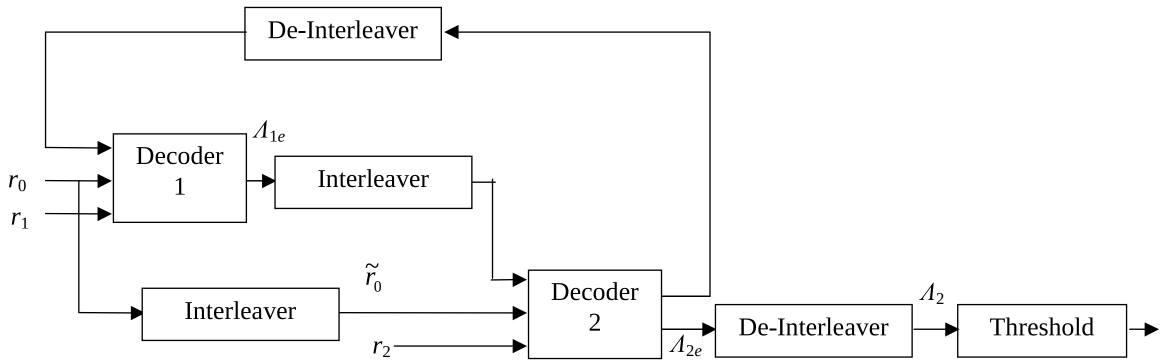

Fig.2. Iterative Decoder Architecture

The two decoders use the received sequences $r' = \left[\ldots \; t_{t,0},\, r_{t,1},\, t_{t+1,0},\, r_{t+1,1},\, \ldots \right]$ and $r'' = \left[\ldots \; \tilde{t}_{t,0},\, r_{t,2},\, \tilde{t}_{t+1,0},\, r_{t+1,2},\, \ldots \right]$ in order to compute the log-likelihood ratio for the overall code trellis

$$\Lambda(c_t) = \log\left[\frac{P(c_t = 1 \mid r', r'')}{P(c_t = 0 \mid r', r'')}\right] = = \log\left[\frac{\sum_{t,c_t=1} P(r' \mid c)P(r'' \mid c)P(c)}{\sum_{t,c_t=0} P(r' \mid c)P(r'' \mid c)P(c)}\right] \quad (1)$$

for all the paths in the code trellis, and makes the decision

$$c_t = \begin{cases} 1 \; ; & \Lambda(c_t) \geq 0 \\ 0 \; ; & \Lambda(c_t) < 0 \end{cases} \quad (2)$$

The log-likelihood ratio from (2) can be determined using MAP, log-MAP, Max-Log-MAP and SOVA algorithms ([5], [6]).

III. Various Interleavers

One of the most significant part of a turbo-code is the designing of the interleaver. Their size, structure and algorithm considerably affect both performances and complexity of the code. For low Signal-to-Noise Ratios (SNR's) the performances are determined mainly by the size of the interleaver, while for large SNR's the structure design becomes the key factor.

A.. The Even-Odd Interleaver is a block interleaver structure. It maps the odd indexed bits on even-indexed positions and vice-versa. It is mathematically described by

$$[\pi(i) + i] \bmod 2 = 0, \quad (\forall) i \in A \quad (3)$$

This structure is used to break long error patterns that are not uniformly distributed within the sequence.

B. The Hybrid Interleaver is based on the same structure as the even-odd interleaver but the input sequence is randomly mixed before the interleaver operation. So if the input sequence is

$$c = (c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9)` \quad (4)$$

and the randomly mixed sequence is

$$\tilde{c} = (c_2, c_5, c_8, c_3, c_7, c_6, c_9, c_1, c_4) \quad (5)$$

Then, the interleaved sequence will be

$$\tilde{c} = (c_2, c_5, c_8, c_6, c_7, c_3, c_9, c_1, c_4) \quad (6)$$

The performances achieved with this kind of interleaver are high for a lower SNR.

C. The Random Interleaver uses *N* input bits written and read in a random way.

IV. 4. SOVA and MAP: Iterative Decoding Methods

The purpose of this article is to analyze the performances of turbo codes using several types of interleavers (such as odd-even interleaver, hybrid interleaver, random interleaver and block interleaver) and two algorithms: MAP and SOVA. We'll present next the methods used.

A.. MAP Algorithm is based on the minimization of the error probability criteria. The decoder generates optimal estimates like symbol a posteriori
probabilities. For each transmitted symbol it generates a hard estimate and a soft output presented in the form of an a posteriori probability based on the received sequence *r*. A log-likelihood ratio is estimated like

$$\Lambda(c_t) = \log\frac{P_r\{c_t = 1 \mid r\}}{P_r\{c_t = 0 \mid r\}} \quad (7)$$

for $1 \leq t \leq \tau$, where τ is the length of the received sequence. This ratio is compared to a threshold leading to the hard estimate c_t:

$$c_t = \begin{cases} 1, & \Lambda(c_t) \geq 0 \\ 0 & \Lambda(c_t) < 0 \end{cases} \quad (8)$$

$\Lambda(c_t)$ is the soft information associated to the hard estimate c_t. This log-likelihood ratio can be used for the next step of the decoding process.

C. SOVA Algorithm is based on the same principle but it presents the advantage that it can be used on any input sequence (the MAP algorithm can be applied only to a sequence length limited). It estimates the soft information for each binary transmitted symbol based on the log-likelihood ratio $\Lambda(c_t)$ defined by

$$\Lambda(c_t) = \log \frac{P_r\{c_t = 1 \mid r_1^\tau\}}{P_r\{c_t = 0 \mid r_1^\tau\}} \qquad (9)$$

where r_1^τ is the received sequence and $P_r(c_t=i|r_1^\tau)$, i = 0,1 is the a posteriori probability of the transmitted symbol. This ratio is compared to a threshold leading to the hard estimate c_t

$$c_t = \begin{cases} 1, & \Lambda(c_t) \geq 0 \\ 0, & \Lambda(c_t) < 0 \end{cases} \qquad (10)$$

SOVA is implemented by a bidirectional and recursive method which implies the positive and negative recursive methods. For reducing the complexity of the decoding algorithm the negative recursivity and the soft decision can be achieved simultaneous.

V. SIMULATION RESULTS AND CONCLUSIONS

In this paper we investigated the BER performances of block, helical, hybrid, even-odd and random interleavers with MAP and SOVA decoders, interleaver lengths 200 and 400 and different encoders polynomials, for small *Eb*/*N*0. The simulation results are presented in figures 3-7.

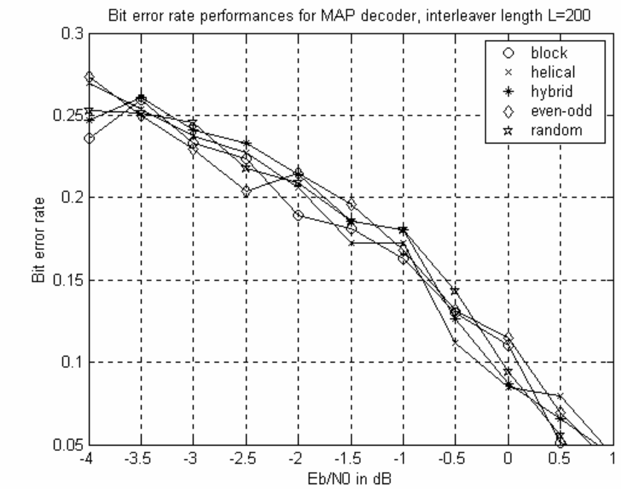

Fig. 3. BER performances for MAP decoder, L=200

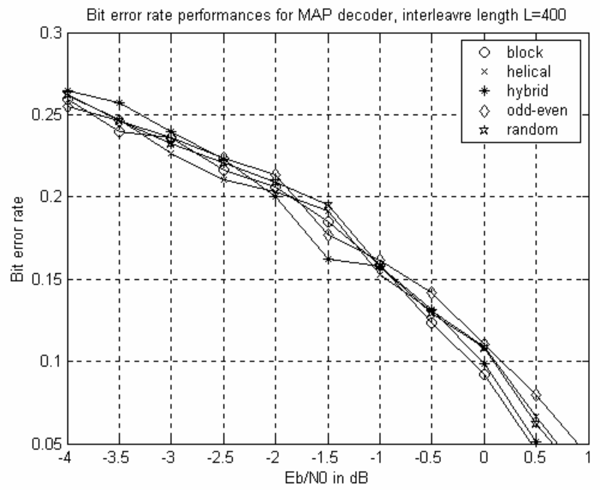

Fig. 4. BER performances for MAP decoder, L=400

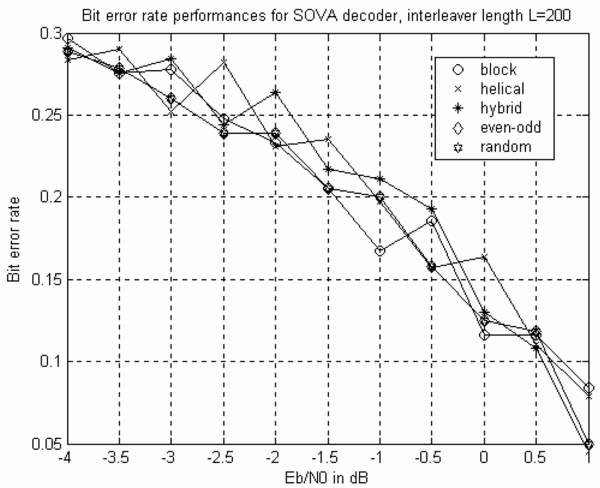

Fig. 5. BER performances for SOVA decoder, L=200

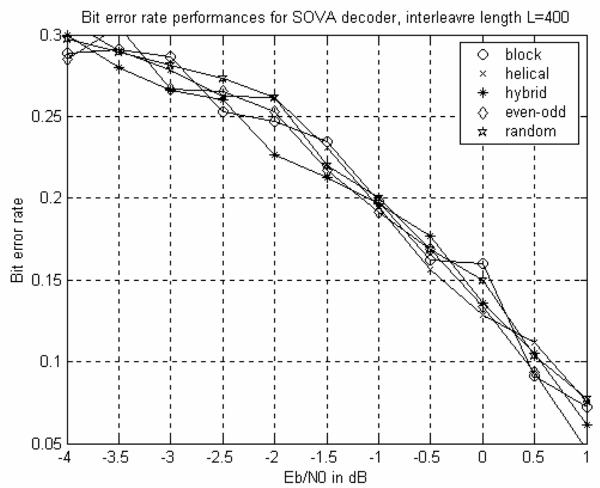

Fig. 6. BER performances for SOVA decoder, L=400

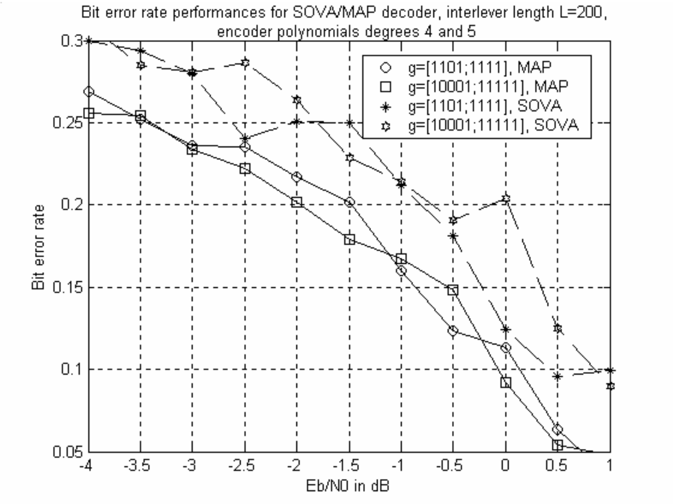

Fig.7- BER performances for SOVA/MAP decoder, interleaver length L=200 encoder polynomials degrees 4 and 5

This is a continuation of the work developed in [10]. From the results obtained several conclusions can be highlighted:

- the performances of the studied interleavers are very close to one another, differences being less than 0.05 in BER; the hybrid/random interleaver achieves still the best performances but the difference is no longer significant;
- the increase in the length interleaver brings a small improvement in BER but the difference is also very small (between 0.01 and 0.03), and it doesn't justify the implementation effort;
- the MAP performances are better than the SOVA ones, as shown in [6] with about 0.05 BER units. However the MAP decoder is much more complex than the SOVA one and the improvement still doesn't justify the effort;
- The memory encoder/decoder polynomials degree increase leads to better BER performances less than 0.7 units. This increase is more significant for the SOVA decoder. We consider that this increase does not justifies the exponential increase of the decoder complexity.

From those results we can conclude that if we have a system that operates at low *Eb*/*N*0 the best choice is to use the simplest structure possible (block interleaver, small framelengh, small degree polynomials, SOVA decoder).

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