Amplifier Noise Model with Thevenin Input Source

Pesha D. Petrova[1]

Abstract – **An amplifier noise model is described. The equivalent noise input voltage is derived for the case where the source is represented by a Thevenin source. The correlation between noise sources is taken into consideration. The effect of both series and shunt impedance at the input on the equivalent noise is analyzed. Amplifier noise simulations are performed. MATLAB simulation results are presented.**

Keywords – **amplifier noise model, Thevenin source, series impedance, shunt admittance, MATLAB simulation**

I. Introduction

Noise behavior is an important characteristic of electronic circuits, including analog amplifiers, as it usually determines the fundamental limit of the performance of circuits.

The significance of the noise performance of each analog circuit is the limitation it places as the smallest input signals the circuit can handle before the noise degrades the quality of the output signal. For this reason, the noise performance is usually expressed in terms of equivalent input noise signal, which gives the same output noise as the circuit under consideration. This fact allows replacing the real noisy network by noiseless network and equivalent input noise signal [1], [2]. This approach of noise modeling can be successfully applied to the analog amplifiers.

A general noise model of an amplifier can be obtained by representing all internal noise sources to the input. In order for the reflected sources to be independent of the source impedance, two noise sources are required – a series voltage source and a shunt current source [3]. Both Thevenin's and Norton's theorems may be used to develop noise models for amplifiers. The equivalent noise input voltage and current can be determined if the source is represented by a Thevenin or by a Norton equivalent.

In many cases the correlation between the noise sources is neglected. In fact, the noise generators are always correlated and the correlation causes the addition input noise voltage or input noise current. In other words, the correlation between noises sources effects on the circuit equivalent input noise. Therefore, for precision noise analysis of the amplifiers, it is always necessary to take into consideration the effect of the correlation coefficient that can vary between -1 and 1. This is very important prerequisite to develop more effective and accurate amplifier noise models.

[1]Pesha D. Petrova is with the Department of Communication Techniques and Technologis, Technical University of Gabrovo, 4, H. Dimitar Str., 5300 Gabrovo, Bulgaria, E-mail: daneva@tugab.bg

II. Amplifier Noise Model with Thevenin Source

The amplifier noise model with a Thevenin noise input source is shown in Fig.1. In this model V_S is the source voltage, $Z_S = R_S + jX_S$ is the source impedance, V_{tS} is the thermal noise voltage generated by the source, and V_n, I_n are the noise sources representing the noise generated by the amplifier.

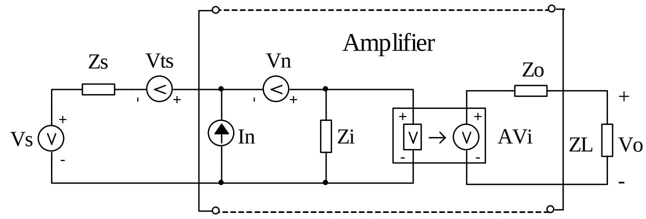

Fig.1. Vn – In amplifier model with Thevenin source.

The amplifier output voltage is given by

$$V_O = \frac{AZ_i}{Z_S + Z_i}\frac{Z_L}{Z_O + Z_L}[V_S + (V_{tS} + V_n + I_n Z_S)] \quad (1)$$

where A is the voltage gain, Z_i is the input impedance, Z_O is the output impedance, and Z_L is the load impedance. The equivalent noise input voltage V_{ni} can be considered as the voltage in series with V_S that generates the same noise voltage at the output as all noise sources in the circuit [3]. It consists of the terms in parenthesis in Eq. (1) and is given by

$$V_{ni} = V_{tS} + V_n + I_n Z_S \quad (2)$$

In fact, this is the noise across Z_i considering Z_i being an open circuit, i.e. it is the Thevenin input voltage. From the Eq. (2) it follows that V_{ni} can be calculated independently of the amplifier.

The mean-square value of V_{ni} is solved for as follows

$$\begin{aligned} v_{ni}^2 &= \overline{(V_{tS} + V_n + I_n Z_S)(V_{tS}^* + V_n^* + I_n^* Z_S^*)} = \\ &4kT\,\mathrm{Re}(Z_S)\Delta f + v_n^2 + 2v_n i_n \,\mathrm{Re}(cZ_S^*) + i_n^2 |Z_S|^2 \end{aligned} \quad (3)$$

where k is the Boltzman's constant, T is the temperature in Kelvin's, Δf is the bandwidth in Hertz, and $c = c_r + jc_i$ is the correlation coefficient between V_n and I_n and it is assume that V_{tS} is independent of both V_n and I_n.

If Z_i is considered to be a short circuit, the short-circuit or Norton input current is given by

$$I_{i(sc)} = \frac{1}{Z_S}\left[V_S + \left(V_{tS} + V_n + I_n Z_S\right)\right] \quad (4)$$

It is obvious that the term in parenthesis of Eq. (4) is V_{ni} given by Eq. (2) and it follows that V_{ni} can be solved for by either calculating the open-circuit input voltage or the short-circuit input current.

A. Effect of a Series Impedance at the Input

Fig.2 presents the input circuit of an amplifier with impedance Z_1 added in series with a Thevenin source.

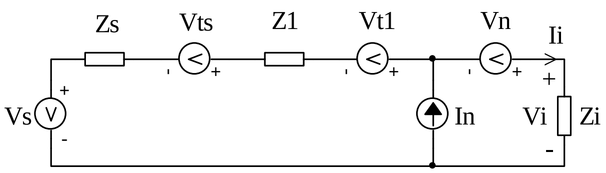

Fig.2. Amplifier input circuit with Thevenin source and series impedance added at input.

The noise source V_{t1} models the thermal noise generated by Z_1. The equivalent noise voltage in series with the source can be solved for by first solving for the open-circuit or Thevenin input voltage, i.e. the input voltage considering Z_i being an open circuit. The Thevenin input voltage in this instance can be expressed as

$$V_{i(oc)} = V_S + V_{ni} \quad (5)$$

where V_{ni} is the equivalent noise voltage in series with the source. It can be represented as

$$\begin{aligned} V_{ni} &= V_{tS} + V_{t1} + V_n + I_n\left(Z_S + Z_1\right) = \\ &= V_{tS} + V_{nS} + I_{nS} Z_S \end{aligned} \quad (6)$$

where V_{nS} and I_{nS} are the new values of V_n and I_n on the source side of Z_1. It follows from this equation that

$$V_{nS} = V_{t1} + V_n + I_n Z_1 \quad (7)$$

$$I_{nS} = I_n \quad (8)$$

It can be concluded that the addition of series impedance at the input of an amplifier changes the V_n noise but does not change the I_n noise. If Z_1 is lossless, it generates no noise itself so that V_{t1} = 0. The mean-square values and the correlation coefficient for V_{nS} and I_{nS} are obtained as

$$v_{ns}^2 = 4kT\,\mathrm{Re}\left(Z_1\right)\Delta f + v_n^2 + 2 v_n i_n\,\mathrm{Re}\left(c Z_1^*\right) + i_n^2\left|Z_1\right|^2 \quad (9)$$

$$i_{nS}^2 = i_n^2 \quad (10)$$

$$c_s = \frac{c v_n i_n + i_n^2 Z_1}{v_{ns} i_{ns}} \quad (11)$$

The mean-square equivalent noise input voltage, therefore, is

$$\begin{aligned} v_{ni}^2 &= 4kT\,\mathrm{Re}\left(Z_S + Z_1\right)\Delta f + v_n^2 + \\ &+ 2 v_n i_n\,\mathrm{Re}\left[c\left(Z_S^* + Z_1^*\right)\right] + i_n^2\left|Z_S + Z_1\right|^2 \end{aligned} \quad (12)$$

It can be seen from this expression that Z_1 = 0, i.e. Z_1 a short circuit, has no effect on the noise. Hence, series impedance should have a magnitude much less than the magnitude of the source impedance if it is not to increase the noise.

B. Effect of a Shunt Admittance at the Input

Fig. 3 illustrates the input circuit of an amplifier with admittance Y_2 added in parallel with the source.

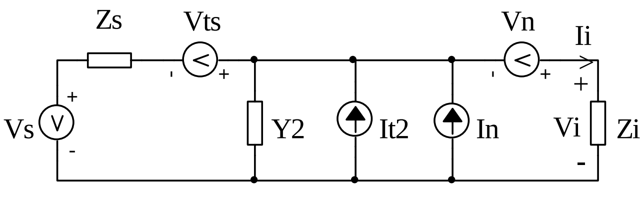

Fig.3. Amplifier input circuit with Thevenin source and shunt admittance added at input.

The noise source I_{t2} models the thermal noise generated by Y_2. As is shown above, the equivalent noise voltage in series with the source can be solved for by first solving for the short - circuit or Norton input current, i.e. the input current corresponding Z_i to be a short circuit. The input current for this case is

$$\begin{aligned} I_{i(sc)} &= \frac{V_S + V_{tS}}{Z_S} + V_n\left(\frac{1}{Z_S} + Y_2\right) \\ &+ I_{t2} + I_n = \frac{1}{Z_S}\left(V_S + V_{ni}\right) \end{aligned} \quad (13)$$

where V_{ni} is the equivalent noise voltage in series with the source and

$$\begin{aligned} V_{ni} &= V_{tS} + V_n\left(1 + Z_S Y_2\right) + \left(I_{t2} + I_n\right)Z = \\ &= V_{tS} + V_{nS} + I_{nS} Z_{SS} \end{aligned} \tag{14}$$

V_{nS} and I_{nS} are the new values of V_n and I_n on the source side of Y_2. It follows from this equation that

$$V_{nS} = V_n \tag{15}$$

$$I_{nS} = I_{t2} + V_n Y_2 + I_n \tag{16}$$

It follows that the addition of shunt admittance at the input of an amplifier changes the I_n noise but does not change V_n noise. If Y_2 is lossless, it generates no noise itself so that $I_{t2} = 0$. The mean-square values and the correlation coefficient for V_{nS} and I_{nS} may be expressed as

$$v_{ns}^2 = v_n^2 \tag{17}$$

$$i_{ns}^2 = 4kT\,\mathrm{Re}(Y_2)\Delta f + v_n^2\left|Y_2\right|^2 + 2v_n i_n\,\mathrm{Re}(cY_2) + i_n^2 \tag{18}$$

$$c_s = \frac{c v_n i_n + v_n^2 Y_2^*}{v_{ns} i_{ns}} \tag{19}$$

The mean-square equivalent noise input voltage is given by

$$\begin{aligned} v_{ni}^2 &= 4kT\,\mathrm{Re}\left(Z_S + \left|Z_S\right|^2 Y_2\right)\Delta f + v_n^2\left|1 + Z_S Y_2\right|^2 + \\ &+ 2v_n i_n\,\mathrm{Re}\left[c\left(1 + Z_S Y_2\right)Z_S^*\right] + i_n^2\left|Z_S\right|^2 \end{aligned} \tag{20}$$

It can be seen from this expression that $Y_2 = 0$, i.e. Y_2 an open circuit, has no effect on the noise. It can be concluded that shunt impedance should have a magnitude much greater than the magnitude of the source impedance if it is not to increase the noise.

III. Simulation Results

Analogue networks usually consist of series and parallel components at the input to the amplifier. One method of analyzing the effect of these components on the amplifier noise is by transforming the noise sources from the amplifier input back to the source by use the above relations. To demonstrate this approach the input circuit of an amplifier, shown in Fig. 4, is analyzed.

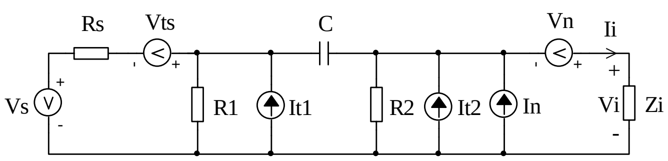

Fig.4. Amplifier input circuit.

Let v_{na}, i_{na} and c_a be the new values to the left of R_2. They are given by

$$v_{na} = v_n \tag{21}$$

$$i_{na} = \left[\frac{4kT\Delta f}{R_2} + \frac{v_n^2}{R_2^2} + 2v_n i_n\,\mathrm{Re}\left(\frac{c}{R_2}\right) + i_n^2\right]^{1/2} \tag{22}$$

$$c_a = \frac{c v_n i_n + \dfrac{v_n^2}{R_2}}{v_{na} i_{na}} \tag{23}$$

The new values v_{nb}, i_{nb}, and c_b to the left of a capacitor C are described by expressions

$$v_{nb} = \left[v_{na}^2 + 2v_{na} i_{na}\,\mathrm{Re}\left(c_a Z_C^*\right) + i_{na}^2\left|Z_C\right|^2\right]^{1/2} \tag{24}$$

$$i_{nb} = i_{na} \tag{25}$$

$$c_b = \frac{c_a v_{na} i_{na} + i_{na}^2 Z_C}{v_{nb} i_{nb}} \tag{26}$$

Let v_{nc}, i_{nc} and c_c be the new values to the left of R_2. They are given by

$$v_{nc} = v_{nb} \tag{27}$$

$$i_{nc} = \left[\frac{4kT\Delta f}{R_1} + \frac{v_{nb}^2}{R_1^2} + 2v_n i_n\,\mathrm{Re}\left(\frac{c_b}{R_1}\right) + i_{nb}^2\right]^{1/2} \tag{28}$$

$$c_c = \frac{c_b v_{nb} i_{nb} + \dfrac{v_n^2}{R_1}}{v_{nc} i_{nc}} \tag{29}$$

The equivalent noise voltage in series with the source is

$$v_{ni} = \left[4kTR_S\Delta f + v_{nc}^2 + 2v_{nc} i_{nc}\,\mathrm{Re}(cR_S) + i_{nc}^2 R_S^2\right]^{1/2} \tag{30}$$

To investigate the relations between the amplifier noise and its input component parameters Matlab simulations with $v_n = 2nV$, $i_n = 1.5pA$, $c = 0.2 + 0.1$, $\Delta f = 1Hz$ have been performed.

In Fig. 5 the effect of the resistance R_2 on the noise current to the left of R_2 for two temperature values is presented. It can be concluded from these results that the increase of parallel impedance R_2 reduces the noise current to the left of R_2, i.e. the noise current on the source side of R_2 is inversely proportional to the parallel impedance.

Fig. 6 shows the simulated noise voltage to the left of the capacitor C versus frequency for capacitance varying from 6.3nF to 15nF. It is obvious that the noise voltage is greater at lower frequencies that at higher frequencies. Furthermore, the capacitance increase, i.e. the series impedance decrease reduces the noise voltage.

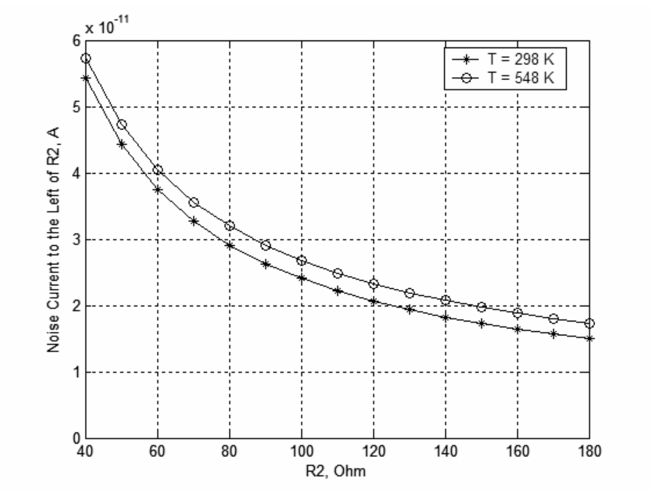

Fig.5. Noise current to the left of parallel resistor R_2 versus R_2 for two temperature values.

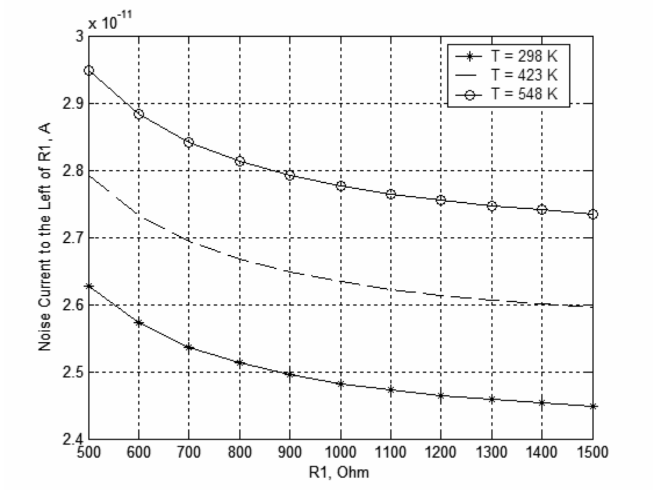

Fig.7. Noise current to the left of parallel resistor R_1 versus R_1 for three temperature values.

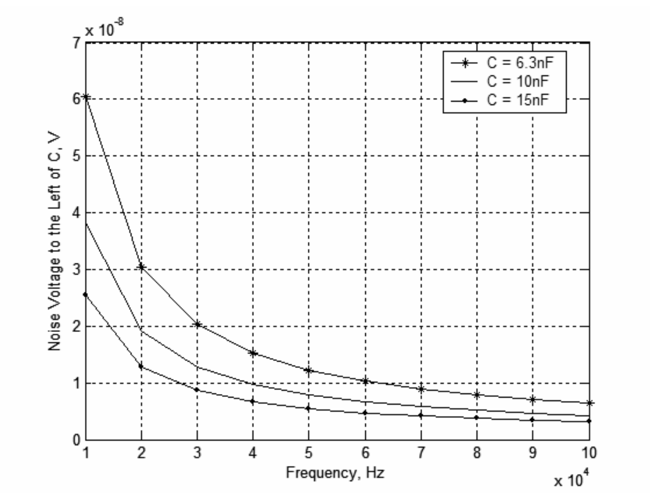

Fig.6. Noise voltage frequency response to the left of series capacitor C for capacitance varying from 6.3nF to 15nF.

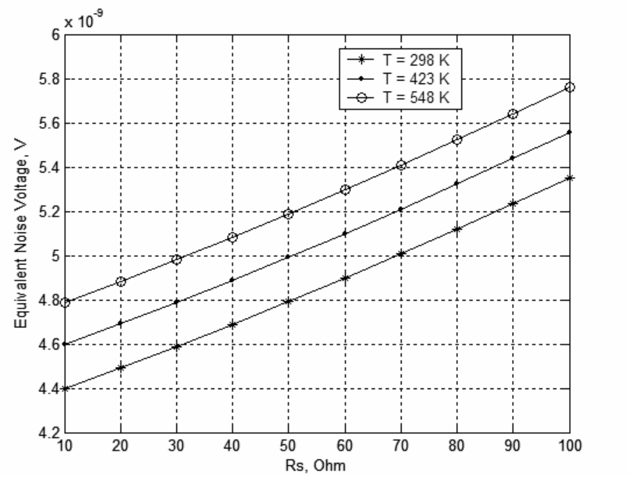

Fig.8. Dependency of equivalent input noise voltage on source resistance for three temperature values.

In Fig. 7 is presented the noise current contributed by the parallel resistor R_1 for three temperature values. It can be seen the noise current changes in the same manner as the noise current in Fig. 5, but the temperature effect is larger.

Fig.8 demonstrates the simulated equivalent noise voltage in series with the source varying both, source resistance R_S and temperature value T. It can be said that the equivalent input noise voltage variation for this case is nearly linear, while it reduces nonlinear with R_1, R_2, and C increase.

The results in Fig.8 are obtained with f=100kHz, C=10nF, R_1=1kΩ, R_2=100Ω.

The expressions for equivalent input noise voltage due to the series and parallel components presented here constitute a compact and consistent set of equations, very useful for design purposes. The results show that the decrease in noise voltage can be obtained at the expense of either reduced series impedance or a reduced parallel admittance. The simulation results can be used to choose the suitable input amplifier component values that provide a minimum noise.

IV. CONCLUSION

An approach for amplifier noise modeling and analysis is developed. Noise simulation results support an availability of the approach proposed. The results allow a wide range of designers to analyze accurately and in a simple manner the effect of series and parallel input amplifier components on the equivalent input noise in series with the source and to minimize the noise.

REFERENCES

[1] AC Analysis and Noise Analysis for Analog Circuits, University of California, Berkeley, 2003.

[2] Herry W. Ott, Noise Reduction Techniques in Electronic Systems, Edition 2, ISBN: 0471850683, 1991.

[3] Jieh-Tsorng Wu, Noise Analysis and Modeling, National Chiao-Tung University, 2001.