Low Sensitivity Cascaded Quadruplet Microwave Filters

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Abstract: **The paper presents a modification of the chained filtering function for low sensitivity cascaded quadruplet microwave filters. The new filtering function uses generalized Chebyshev polynomials as a seed function. It is estimated and compared the responses' sensitivity of modified chained filter and conventional Chebyshev filter. It is proven that the attenuation of the chained filters is 6dB less than the Chebyshev filter.**

Keywords: **Microwave filter, chained filtering function, low sensitivity, coupling matrix.**

I. Introduction

The synthesis and design of cross-coupled microwave filters is a subject of intense research efforts due to their importance in modern wireless communication systems. Most of them are from the Chebyshev family and have symmetrical responses. This filter class is equiripple in the passband and realize pairs of prescribed transmission zeros (TZs).

A major topology of symmetrical response microwave filters is the cascaded quadruplet (CQ) topology. The coupled resonators are arranged in quadruplets and the coupling and routing diagrams for 4-, 6- and 8-order filter are shown on fig.1. CQ filters may realize $N/2$ prescribed finite TZs. Each quartet may be identified with the production of a TZ pair. If the cross coupling is negative a pair of attenuation poles will be created. If it is positive a real axis pole pair will result with group delay self equalization.

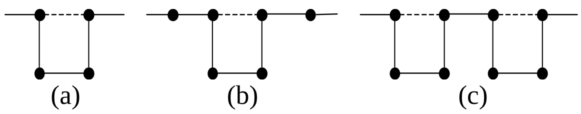

Fig.1 Coupling and routing diagrams for CQ filters of (a) 4-, (b) 6-, (c) 8- order filter

The calculation of the generalized Chebyshev polynomials for the filter synthesis is described in [1]. The matrix rotations order for reduction of the folded coupling matrix to a matrix corresponding to the CQ topology is outlined in [2]. In [3] is studied the sensitivity of different microwave filter topologies to the variation of the coupling coefficients.

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The filter's sensitivity depends on the frequency separation between the return-loss zeros. The wider is the frequency separation between the return-loss zeros, the lower is the sensitivity of the filter to the manufacturing errors. The increasing of the filter order leads to decreasing the frequency separation and consequently the filter sensitivity grows up

A solution of the problem with the sensitivity of microwave filters to the variation of the coupling coefficients is a proper grouping of the return-loss zeros. The result of the grouping is the reduction of the filter sensitivity. The idea of return-loss zeros grouping is exploited in [4]. However the using of generalized Chebyshev polynomials as a seed function leads to filters with multiple TZs. This approach constraints the microwave filter synthesis. The increasing of the filter order leads to growing up multiplicity of the TZs instead of increasing the number of realizable TZs.

This paper proposes modified chained filtering function for low sensitivity microwave filters with CQ topology. It is estimated the filter responses sensitivity to the coupling coefficient variation and the tuning accuracy of the coupled resonators. Chebyshev and modified chained filtering function filters are synthesized and it is estimated their sensitivity to verify the presented theory.

II. Modified Chained Filtering Function

The reflection and transmission coefficients of a lossless filter, comprised of *N* resonators, are presented as a ratio of polynomials. Generally, these polynomials are of *N-th* order.

$$S_{11}(s)=\frac{F_N(s)}{E_N(s)}\ ,\ S_{21}(s)=\frac{P_N(s)}{\varepsilon E_N(s)} \qquad (1)$$

where

$$\varepsilon=\frac{1}{\sqrt{10^{RL/10}-1}}\cdot\left.\frac{P_N(s)}{F_N(s)}\right|_{s=j} \qquad (2).$$

RL is the maximum return loss.

For a passive lossless circuit is valid the energy saving law: $S_{11}^2+S_{21}^2=1$. Consequently:

$$\left|S_{21}(s)\right|^2=\frac{1}{1+\varepsilon^2\Phi_N^2(s)} \qquad (3)$$

where $\Phi_N=\dfrac{F_N(s)}{P_N(s)}$ is the filtering function.

The Chebyshev approximation uses as a filtering function the generalized Chebyshev polynomials, expressed as [1]:

$$C_N(\omega)=\cosh\left[\sum_{k=1}^{N}a\cosh(x_k)\right] \qquad (4)$$

where $x_k = \frac{\omega - 1/\omega_k}{1 - \omega/\omega_k}$ and ω_k are the prescribed transmission zeros.

The filtering function for CQ filters of even order and pairs of symmetrical finite TZs is introduced as:

$$\Phi_{2N}(s) = C_N(s) C_N^*(s) = \frac{F_N(s) F_N^*(s)}{P_N(s) P_N^*(s)} \tag{5}$$

where $s = j\omega$, and $C_N^*(s)$ is the generalized Chebyshev polynomial $C_N(s)$ with complex conjugate coefficients (4).

The filter realizability is ensured by the fact that each function in the product is realizable.

It is necessary to sort the TZs in ascending order with alternatively changing signs when computing $C_N(s)$. For example if the prescribed TZs of the filter are ±1.8, ±2.5 and two zeros in the infinity, they should be sort in the following way: 1.8, -2.5, ∞. The polynomial $C_N(s)$ is synthesized as described in [1].

It is important to ensure that the transfer and reflection vectors are orthogonal in order to satisfy the unitary conditions for the scattering matrix [5]. It follows that the phase difference between the S_{21} vector and the average of the phases of S_{11} and S_{22} must be an odd multiple of $\pi/2$ rad.

The filter synthesis proceeds with computing the coupling matrix for a given filter topology from the proposed filtering function.

The reduction of the sensitivity for the modified chained filtering function filter leads to decreasing the filter attenuation in the stopband. Using the approximation:

$$C_N(\omega) \approx 2^{N-1}\omega^N \tag{6}$$

which is valid for large ω and for N-even, it is easily derived that attenuation of the modified filter is *6 dB* less than the Chebyshev filter with the same maximum level of return loss.

This fact must be taken into account when synthesizing a filter, in order to ensure the necessary attenuation in the stopband.

With the proposed modification of the chained filtering function filters, the zeros of the reflection coefficient groups in pairs and the frequency separation between them increases. The frequency separation of two zeros in a group depends on the position of the TZs. The further from the cut off frequency are the TZs, the smaller is the frequency separation in the group. When all prescribed TZs approach infinity, the two frequencies in the group theoretically coincide. In practice, the zeros in the group are very close, because of the inaccurate tuning of the resonators and the manufacturing errors.

III. MODIFIED CHAINED FILTERING FUNCTION FILTERS SENSITIVITY

The transmission and reflection coefficients for a lossless *N-th* order filter depend only on the coupling matrix [6]:

$$S_{21} = -2j[A]^{-1}_{n+2,1} \text{ and } S_{11} = 1 + 2j[A]^{-1}_{11} \tag{7}$$

The matrix $[A]$ is related to the coupling matrix $[M]$ by the relation $[A] = -j[R] + \omega[W] + [M]$. Here $[R]$ is a $(n+2)\times(n+2)$ matrix, whose only non-zero entries are $R_{11} = R_{n+2,n+2} = 1$. $[W]$ is similar to the $(n+2)\times(n+2)$ identity matrix, except that $W_{11} = W_{n+2,n+2} = 0$. $[M]$ is the $(n+2)\times(n+2)$ symmetric coupling matrix.

To analyze the filter sensitivity to the variation of the coupling coefficients, it is necessary to know the partial derivatives of S_{21} and S_{11} with respect to the entries in the coupling matrix. They are derived in [6]:

$$\frac{\partial S_{11}}{\partial M_{pq}} = -4jP_{pq}[A]^{-1}_{p1}[A]^{-1}_{q1} \quad \text{for } p \neq q \tag{8a}$$

$$\frac{\partial S_{11}}{\partial M_{pp}} = -2jP_{pp}[A]^{-1}_{p1}[A]^{-1}_{p1} \tag{8b}$$

$$\frac{\partial S_{21}}{\partial M_{pq}} = 2jP_{pq}\left([A]^{-1}_{(n+2),p}[A]^{-1}_{q1} + [A]^{-1}_{(n+2),q}[A]^{-1}_{p1}\right) \quad \text{for } p \neq q \tag{8c}$$

$$\frac{\partial S_{21}}{\partial M_{pp}} = 2jP_{pp}[A]^{-1}_{n+2,p}[A]^{-1}_{p1} \tag{8d}$$

Since the reflection and transmission coefficients involve the magnitude of the scattering parameters, the following expression is used to determine the partial derivative of the magnitude of a complex quantity Z in terms of the gradient of Z [3]:

$\frac{\partial |Z|}{\partial x} = \mathrm{Re}\left[\frac{|Z|}{Z}\frac{\partial Z}{\partial x}\right]$, where $\mathrm{Re}[.]$ means the real part of a complex number.

The impact of the variation of each coupling coefficient on the transmission and reflection coefficients is estimated by the formulas (8). In [3] are introduced the sensitivity estimates of S_{11} and S_{21} simplify the analysis:

$$K_1 = \sum_{i,j}\left|\frac{\partial |S_{11}|}{\partial M_{ij}}\right| \text{ and } K_2 = \sum_{i,j}\left|\frac{\partial |S_{21}|}{\partial M_{ij}}\right| \tag{9}$$

The sums (9) run over all entries of the coupling matrix. These estimates form the worst case, when all effects from the errors sum and lead to the maximum degradation in the filter responses. The parameters K_1 and K_2 are functions of the normalized frequency and it may be analyzed the frequency dependence of the filter sensitivity to the coupling coefficient variation.

The CQ filters are synchronously tuned and all diagonal elements in the coupling matrixes are zeros. The terms (8b) and (8d), corresponding to M_{ii} should be included in the sums (9), because the shifts in the resonant frequency alter the filters' responses. All sensitivities corresponding to the other zero entries in the coupling matrix are neglected. The cross correlation between two coupling coefficients is assumed to be zero.

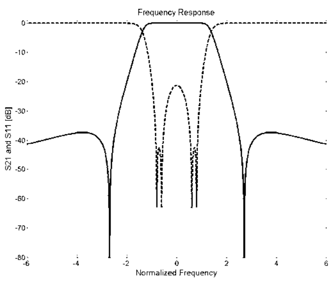
IV. RESULTS

4-th and 8-th order CQ filters with Chebyshev and modified chained filtering functions are synthesized to investigate their sensitivity. Using the procedures outlined above, it is computed the coupling matrixes for both type filters and the sensitivity estimates. The chained function cross-coupled filters have lower attenuation in the stopband than the cross-coupled Chebyshev filters.

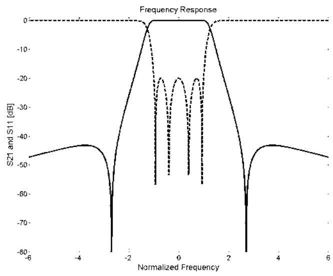
A .4-th order filter with a symmetrical pair of TZs placed on $\omega_{z1} = 2.7$ and $\omega_{z2} = -2.7$

Both filters have maximum return loss value of $-20dB$. Following the synthesis procedure outlined in [5], the coupling coefficients for Chebyshev filter are derived as: $M_{S1} = M_{4L}$=1.0289, $M_{12} = M_{34}$=0.8922, $M_{23} = 0.7345$, $M_{41} = -0.0866$. Using the modified chained filtering function and following the same synthesis procedure, the coupling coefficients are: $M_{S1} = M_{4L}$=1.1696, $M_{12} = M_{34}$=1.0483, $M_{23} = 0.8095$, $M_{41} =$ -0.1341.

The frequency responses of the filters are shown on fig.2. It is clearly seen from fig.2a, that the modified chained filtering polynomials leads to return loss zeros grouping. As closer to the cutoff frequency are the prescribed TZs, as further are the two RL zeros are within the group. Generally the frequency separation between the RL zeros groups is wider then the Chebyshev filter, which is obvious when comparing fig.2a and fig.2b. The maximum value of the return loss of the modified chained function filter is smaller than the prescribed value of $-20dB$, because of the RL zeros grouping. The attenuation in the stopband of the modified filter is $6dB$ smaller than the Chebyshev filter.



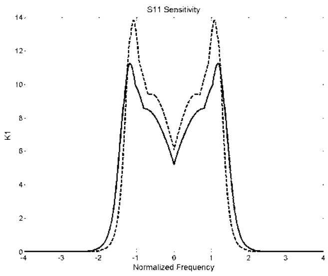
(a)



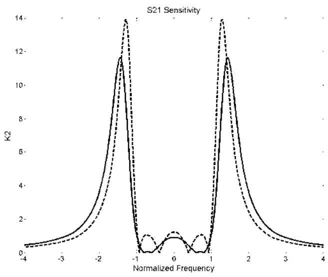
(b)

Fig.2. Reflection (dashed line) and transmission (solid line) coefficients versus normalized frequency for (a) Modified chained function filter and (b) Chebyshev filter

The sensitivity estimates, derived in accordance to formulas (9), are shown on fig.3.



(a)



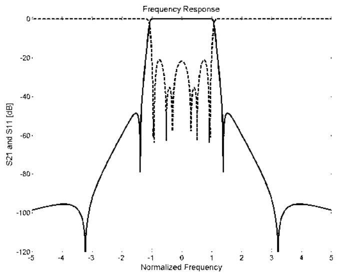
(b)

Fig.3 Variation of: (a) K_1 and (b) K_2 against the normalized frequency for Modified chained function filter (solid line) and Chebyshev filter (dashed line)

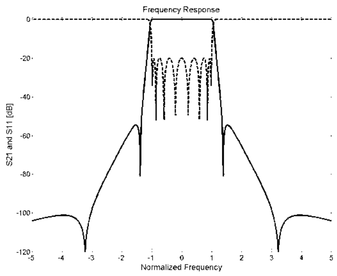
The improvement in the S_{11} sensitivity of the modified chained function filter is about 13% and for S_{21} with 10%, comparing to the Chebyshev filter. Out of the passband, the sensitivity of the Chebyshev filter responses is smaller than the modified chained filter.

B.8-th order filter with two symmetrical pair of TZs placed on $\omega_{z1,2} = \pm 1.39$ and $\omega_{z3,4} = \pm 3.22$

Two filters with maximum return loss value of $-20dB$ are synthesized, following the synthesis procedure outlined in [5], using the Chebyshev and modified chained filtering functions. The coupling coefficients for the Chebyshev filter are: $M_{S1} = M_{8L}$=0.9873, M_{12}=0.8159, $M_{23} = 0.6044$, $M_{34} = 0.5437$, $M_{45} = 0.5348$, $M_{56} = 0.5048$, $M_{67} = 0.7202$, $M_{78} = 0.7904$, $M_{41} =$ -0.0268, $M_{58} =$ -0.2041. The coupling coefficients for the modified chained filter are found as: $M_{S1} = M_{8L}$ =1.1011 , M_{12} =0.9200 , M_{23} = 0.6415 , M_{34} = 0.5668 , M_{45} = 0.5367 , M_{56} = 0.5043 , M_{67} = 0.7762 , M_{78} = 0.8830 , M_{41} = -0.0337 , M_{58} = -0.2606 . The frequency responses for both filters are shown on fig.4.



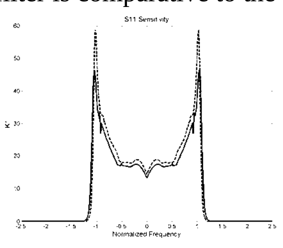
(a)



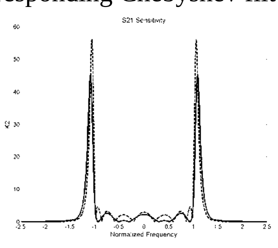
(b)

Fig.4. Reflection (dashed line) and transmission (solid line) coefficients versus normalized frequency for (a) Modified chained function filter and (b) Chebyshev filter

As it is seen from fig.4a, the RL zeros are gathered in four groups, each of two zeros. The frequency separation between them is comparatively wider than the Chebyshev filter, shown on fig.4b. The frequency separation between the zeros in the group depends on the position of the prescribed finite pairs of TZs. Further is the pair from the cutoff frequency, the closer are the RL zeros in the group. For high order filters, very closely placed pairs of TZs to the passband may vanish the effect of grouping the RL zeros. The sensitivity of that kind of filter is comparative to the corresponding Chebyshev filter.



(a)



(b)

Fig.5 Variation of: (a) K_1 and (b) K_2 against the normalized frequency for Modified chained function filter (solid line) and Chebyshev filter (dashed line)

Although the maximum return loss is set to $-20dB$, for the modified chained function filter this value is a bit smaller.

It is obvious from fig.5, that the sensitivity of the S_{11} and S_{21} for the modified chained function filter is lower than the Chebyshev filter. In the passband the improvement in the return loss and insertion loss estimates is respectively 10% and 13%. In the stopband the sensitivity of the Chebyshev filter is better than the modified filter.

V. Conclusion

The paper proposes a modification of the chained filtering function using generalized Chebyshev polynomials and grouping return loss zeros in order to increase the frequency separation between the groups. The result is improvement in the filter sensitivity to manufacturing errors and frequency shifts in the resonant frequencies. The filter's sensitivity to coupling coefficient variation is analyzed by means of sensitivity estimates. It is compared the Chebyshev filter and modified chained function filter sensitivity. It is seen from the analysis that in the passband the improvement of the sensitivity of the return loss is about 13% and for the insertion loss is about 12%. The Chebyshev filter sensitivity estimates have lower values in the stopband.

It is proven that the modified chained function filter' attenuation in the stopband is $6dB$ less than the Chebyshev filter. To ensure the necessary attenuation in the stopband it must increase the filter order. The coupling coefficients of the modified filter are 10% bigger than the Chebyshev filter coefficients.

The proposed method for approximation is applicable for millimeter wave filters, where the manufacturing errors are crucial, and for high sensitivity filter topologies.

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