Transmission Properties of Gradient Index Metamaterial Slabs

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Abstract – **We analyzed electromagnetic wave propagation through an interface between conventional dielectric and double-negative ("left-handed") metamaterial where the interface may be graded to an arbitrary degree. We investigated slab structures with linear, exponential and tangent hyperbolic spatial dependence of refractive index. To this purpose we utilized numerical procedure based on the modified transfer matrix method to calculate transmission. The obtained results may be of interest in designing optimized gradient index lenses applicable in ranges from microwave to optical, as well as in improvement of antireflection coatings.**

Keywords – **electromagnetic metamaterials, double negative materials, left-handed metamaterials, LHM, gradient index**

I. INTRODUCTION

A new paradigm in the area of electromagnetic materials is the left-handed metamaterials (LHM), also known as double negative (DNG) metamaterials [1]. These are artificial composites structured at subwavelength level which furnish a negative value of refractive index in a certain wavelength range. The direction of the Pointing vector in an LHM is opposite to that of the wavevector, i.e. the vectors of the electric and magnetic field and the wavevector form a left-oriented set, contrary to conventional materials ("right-handed" – RHM).

Typically a unit cell of an LHM consists of an element furnishing negative magnetic permeability (e.g. split-ring resonator) and an element furnishing negative dielectric permittivity (e.g. thin-wire inductive structures). Important works on this topic include Pendry's [2]-[5], while the first theoretical predictions were published in [6] and the first experimental confirmations were presented in [7].

The LHM structures offer a host of unique properties [1], and thus appear convenient for various applications not attainable with conventional materials. Different practical solutions have already been proposed, e.g. high-gain, electrically small antennas for the microwave [8], subwavelength resonant structures (resonant cavities much thinner than their operating wavelength) [9], superlenses or perfect lenses (lenses for subwavelength imaging of both far-field and evanescent near-field components of electromagnetic field which are not diffraction limited) [5], magnetic materials at THz and even optical frequencies [10], novel transmission lines [11], etc.

Continuously graded index structures offer a number of advantages over conventional elements with homogeneous and/or step index profile since they offer an additional degree of freedom in the design of the desired characteristics. Gradient index elements in conventional dielectrics have been analyzed and designed as early as in 1962 [12].

Electromagnetic metamaterials with refractive index continuously varying in space combine favorable properties of both the graded profiles and the LHM and thus promise increased practical usability in various applications, lensing and filtering being just a few of them. Thus these have been extensively studied – e.g. [1], [5], [13]-[16]. Ramakrishna described a spherical perfect lens composed of media with permittivity and permeability graded as ~1/r [15]. Smith et al [16] proposed the use of metamaterial lenses instead of conventional positive index ones for the coupling with radiative elements in high-gain antenna applications because of the reduced geometrical aberration profile in comparison to the conventional ones. Sang et al. analyzed the use of graded particles composites in the design of negative refractive index response [13]. Experimental studies of graded index LHM have been reported in e.g. [16].

In RHM, the approach to the calculation of reflection and transmission was published in [12] for the case of graded antireflection coatings. Approximate analytical solutions for electric field were done using the WKB approximation of the Helmholtz equation for some special gradient index profiles (e.g. linear or exponential dependences) [17]. As far as the authors are informed, no calculations of graded metamaterial structures were published until now.

In this work we analyze electromagnetic behavior of graded refractive index interfaces between RHM and LHM slabs using a modified transfer matrix technique (TMM). We analyze different realistic geometries including linear, exponential and tangent hyperbolic.

II. GRADIENT INDEX PROFILES

Fig. 1 shows a 1D structure consisting of a RHM/LHM/RHM sandwich where the refractive index between the RHM and LHM slabs is continually graded. We considered linear, exponential and tangent hyperbolic dependences in the graded zone.

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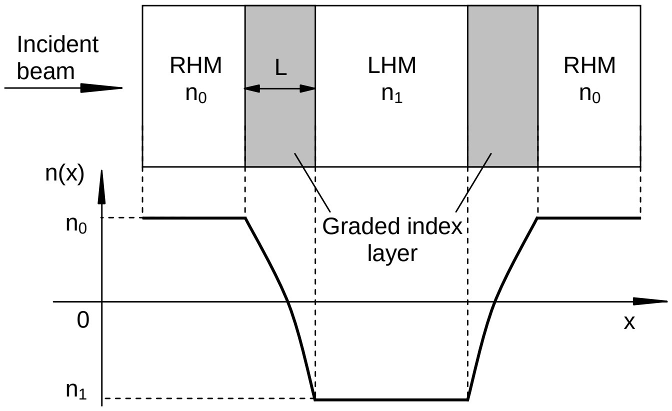

Fig. 1. Top: Structure consisting of a left-handed metamaterial slab (n_1) between two slabs of conventional lossy dielectric (n_0), where RHM-LHM interfaces are graded. Bottom: position dependence of the real part of refractive index

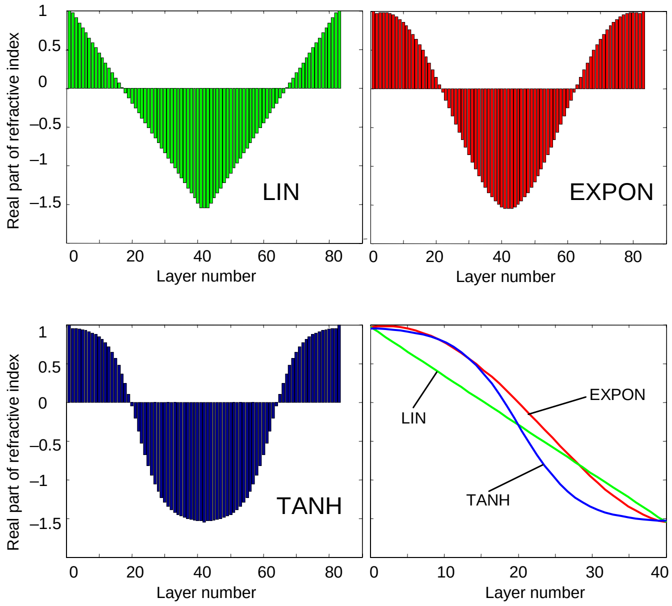

Fig. 2. The linear, exponential and tangent hyperbolic profiles used in transmittance calculation. Upper row and bottom left show the division used for gradient approximation

The position dependence of the real part of the refractive index for the linear gradient has the form

$$n(x) = n_0 + (n_1 - n_0)\frac{x}{L}, \tag{1}$$

in the exponential case it becomes

$$n(x) = n_0 \exp\left[\frac{x}{L}\ln\left(\frac{n_1}{n_0}\right)\right], \tag{2}$$

while the tangent hyperbolic dependence is

$$n(x) = \frac{n_0 + n_1}{2} - \frac{n_0 - n_1}{2}\tanh\frac{x}{L}. \tag{3}$$

The dependences (1)-(3) are shown in Fig. 2.

III. CALCULATION RESULTS AND ANALYSIS

We assumed that the beam is incident from and exiting to the same medium, which is vacuum or air, with purely real and positive refractive index equal to 1. We assumed that the index profile grading started from vacuum refractive index 1 to a negative index value of –1.5. We considered lossy medium in which the imaginary part of the refractive index was 0.001. Similar to a number of references (e.g. [9]), we disregarded dispersion effects. Approximation of the small losses and dispersionless metamaterial is common practice in literature, as far as one is solely interested in effects arising from negative value of refractive index. It is true that the majority of contemporary experimental results show that negative values of refractive index appear with a relatively large dispersion and in very narrow bandwidths. However, there are also known experimental results that lead to conclusion of possibility of obtaining larger frequency bandwidth for LHM behavior [1], [11].

The calculation was done by the transfer matrix method according to the procedure outlined in [17]. To this purpose we divided the calculation region into N=80 layers with a constant value of refractive index throughout each layer. The refractive index in the layers was determined using eqs. (1)-(3) as the value in the midline of each layer. For comparison, we also calculated the electromagnetic transmission through an abrupt interface in which refractive index changed from +1 to –1.5.

First we calculated transmission for different graded profiles for the case when subdivision strata had a quarter-wavelength thickness, $4\, n_i\, L_i = \lambda_0$. We chose this condition to obtain "well behaved" and periodic transmission spectra convenient for comparison with known results. The results (Figs. 3, 4) show that the graded multilayers have somewhat lower transmission compared to that of the abrupt interface. The result is similar to the situation with conventional positive-index materials.

Another result is that exponential and linear profiles furnish practically an identical spectral transmission. A similar result is obtained for the case when constant geometrical thickness strata are taken instead of those with constant optical quarter-wave thickness (Fig. 5). The absolute values of transmission were also similar for both of these situations. Of the calculated structures, tangent hyperbolic offered the lowest values of transmission. This particular behavior of transmission spectra can be easily understood when compared with known results for ordinary postitve index graded multilayer in this configuration.

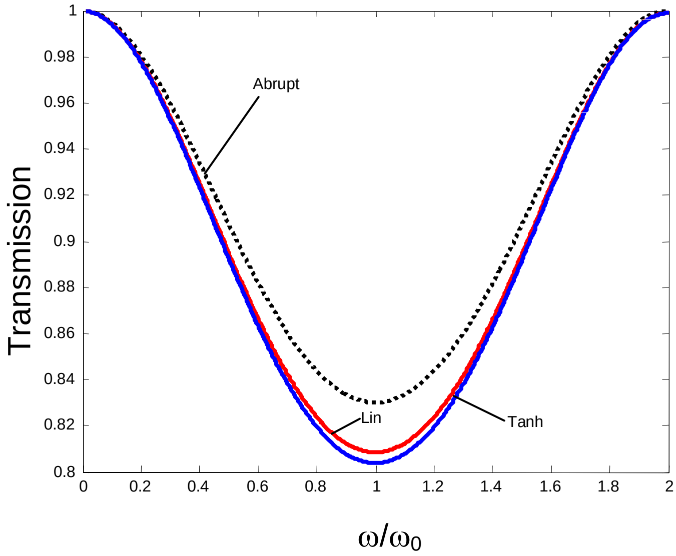

Fig. 3. Transmission of different multilayer structures with graded refractive index, quarter-wavelength thickness of subdivision strata

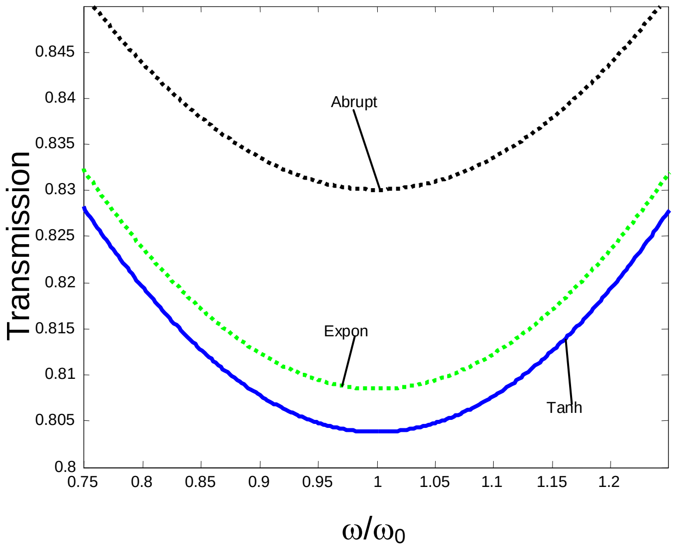

Fig. 4. Transmission of different multilayer structures with graded refractive index, quarter-wavelength thickness of subdivision strata

A more pronounced difference between diverse grading profiles can be obtained by proper choice of LHM parameters, but our intention was only to analyze graded structure transmissions as opposed to abrupt change of refractive index.

An overall conclusion is that the behavior of the spectral transmission of graded LHM interfaces is similar to the one in the case of conventional materials with positive refractive index.

It can be seen from Fig.6 that the transmission spectra for Tanh profile keep the minimal transmittance at about the same value for different optical thicknesses, but introduce oscillatory pattern in the transmission spectra as we change the normalization wavelength. It is interesting that the differences are more noticeable at higher frequencies.

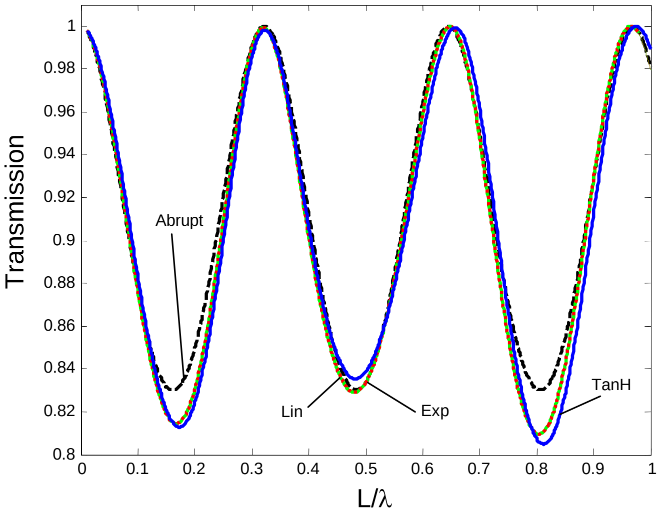

Fig. 5. Transmission of different multilayer structures with graded refractive index, constant geometrical thickness of subdivision strata

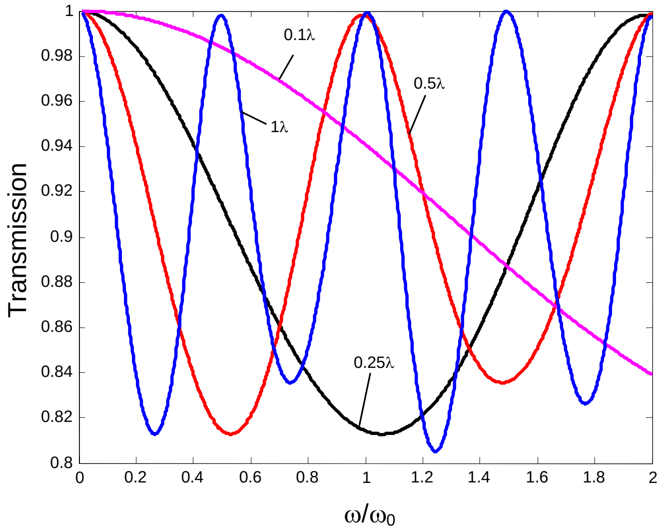

Fig. 6. Transmission of Tanh profile for different values of the graded region thickness. Each homogeneous layer has the same optical thickness, defined as $n_iL_i=\lambda_0 N$, where N=0.1, 0.25, 0.5, and 1.0.

Another interesting case for consideration is shown in Fig. 7 where we chose the graded layer to be only a small portion of the slab's fixed thickness. We chose again a Tanh profile for gradient index. We assumed that the slab thickness was quarter-wavelength and that the sum of all optical thicknesses of the strata in the graded region was $\lambda_0/4$, but that the number of layers varied. Thus the optical thickness of each layer was $n_iL_i=\lambda_0/4N$.

The transmission curve in Fig. 7 reveals information of the influence that graded interfaces have on transmission when this interface region has a much smaller optical thickness than the optical thickness of the whole slab. The difference from the case of the abrupt change of refractive index becomes less prominent with the refinement of the subdivision. This is expected behavior and in agreement with [13], [16] .

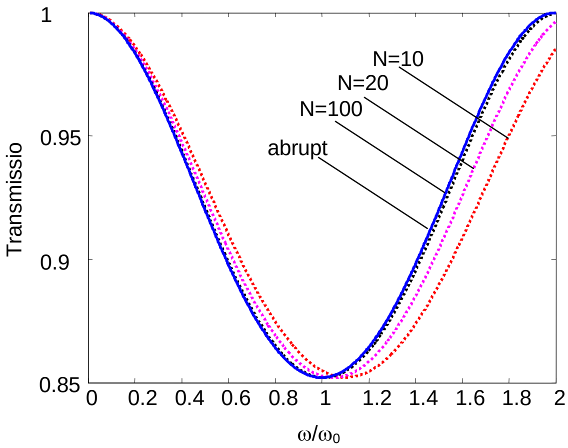

Fig. 7. Transmission of Tanh profile for different number of the equidivision for graded region . Each homogeneous layer has the same optical thickness. Central part of structure has quarter-wavelength optical thickness

IV. CONCLUSION

We considered electromagnetic wave propagation through slabs containing left-handed metamaterials with graded refractive index. Linear, exponential, hyperbolic and tangent hyperbolic dependences were analyzed by the transfer matrix method. The calculations were done for lossy media, while the dispersion was neglected.

The behavior of negative-index graded profiles is similar to the one of positive index material. Bearing in mind that at the same time negative index materials offer a significantly wider bandwidth in passband filtering structures than positive index materials, this points out to the possibility to utilize LHM structures instead of the conventional ones to obtain enhanced functionality. With this in mind, the gradient index metamaterial use may be anticipated in a wide range of applications, e.g. as an alternative to conventional gradient-index (GRIN) lenses and similar passive elements for electromagnetic beam shaping and directing, for high-efficiency antireflection structures, etc. Such use may be combined with the applications where magnetic response is required in the terahertz range.

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