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Composite Left-Handed/ Right-Handed TLM (Transmission Line Metamaterials) with Quasi-Periodically Ordered Unit Cells

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Abstract – **In this paper we consider quasi-periodic 1D structures composed from positive- and negative ("left-handed") refractive index material layers. Our consideration is based on the recently proposed approach to the implementation of left-handed metamaterial structures using L-C loaded transmission lines. We present transmission spectra analysis of Fibonacci-type ordered left-handed/right-handed composite.**

Keywords – **Electromagnetic Metamaterials, Double Negative Materials, Left-Handed Metamaterials, LHM, Transmission Line metamaterials, Backward Waves, Quasi-Periodicity, Fibonacci**

I. INTRODUCTION

A left-handed metamaterial (LHM) may be defined as an artificial medium which supports propagation of backward-traveling electromagnetic waves – those with anti-parallel phase and group velocity [1]. In other words, the direction of the Pointing vector in such material is opposite to that of the wavevector, i.e. $\vec{E},\vec{H},\vec{k}$ in an LHM form a left-oriented triplet, contrary to the conventional materials which thus may be dubbed "right-handed materials" – RHM.

The geometric features of an LHM have subwavelength dimensions and thus at the operating frequency such materials can be described by the effective medium approach. In a left-handed medium its effective magnetic permeability and dielectric permittivity are simultaneously negative, with a consequence that its refractive index is also negative (in order to preserve causality) [2]. The seminal papers on LHM include those Veselago's and Pendry's [3], [4], [5].

The left-handed structures possess many unique properties. Snell's law is reversed in them, as well as the Doppler shift and Cerenkov radiation [1]. The main applications utilizing LHM include the so-called superlenses [6] which enable imaging of both far-field and evanescent near-field components of electromagnetic field. Another one are subwavelength resonant cavities [7] (i.e. resonant cavities with dimensions much smaller than the operating wavelength). Different practical solutions stemmed from this, e.g. high-gain, electrically small antennas for the microwave [8], materials possessing magnetic properties at THz frequencies, different microwave transmission lines [9], directional couplers, resonators, filters, antireflection structures and many more [10]. In this very dynamic field new applications appear virtually every day. According to the *Science* journal, left-handed metamaterials were among the top ten scientific breakthroughs of the year in 2003 [11].

The first practical structures acting as left-handed materials included thin metallic wires [4] for creation of effective media with negative dielectric permittivity, and split ring resonators (SRR) for negative magnetic permeability [5]. These elements ('particles') are arranged in a unit cell, which is the smallest element of an LHM and whose multiplication furnishes the macroscopic or mesoscopic left-handed medium. Experiments confirming the functionality of such structures started with [12].

Left-handed media containing thin wire and SRR elements are necessarily resonant. Due to their resonant nature, negative values of refractive index are available only in a very narrow range of wavelengths. In addition to that, these media are always lossy.

An alternative approach to LHM was proposed in June 2002 in [13] and [14] and later published in [15], [16]. It utilizes the well-known duality between filters and distributed networks to produce left-handed materials based on transmission lines – the so-called transmission line metamaterials (TLM). There is a direct analogy between the voltage/current in a transmission line and the components of the electric and magnetic fields. Besides offering larger bandwidths and much smaller losses, the unit cells of TLM can be equipped with lumped circuit elements, allowing an additional degree of freedom in design. Additionally, [15] proposed the fabrication of dynamically tunable TLM by using lumped varactors instead of capacitors. TLM are especially suitable for RF and microwave devices.

A more practical application of TLM is LHM/RHM combination. A name coined for it was CRLH (composite Right/Left Handed) materials [17]. Such transmission lines behave as LHM at low frequencies and as RHM at high frequencies.

Filtering applications of LHM almost naturally bring to mind their combination with quasiperiodic structures which enable different practical applications with unique properties [18]. In spite of that, no papers appeared until today which deal with quasiperiodic transmission line metamaterials.

This paper analyses quasiperiodic CRLH transmission line structures. The parameters of the structures were analyzed by

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the transfer matrix method. Fibonacci-sequence scheme was chosen to study transmission and reflection properties for waves propagating along structure based on TL metamaterial.

II. THEORY

In an ideal case, conventional (RHM) materials are equivalent to a distributed L-C network with series inductance and parallel capacitance. A transmission-line based LHM is then obviously equivalent to a dual distributed network with series capacitance and shunt inductance. This is a high-pass filter structure and supporting backward wave propagation. In this paper we use concept of CRLH composite right-handed/left-handed transmission line metamaterial, as described in [17]. This choice is justified within framework of realizable structures that are already experimentally produced [15]. A unit cell of a realistic TLM structure includes parasitic series inductance and shunt capacitance (Fig. 1 a).

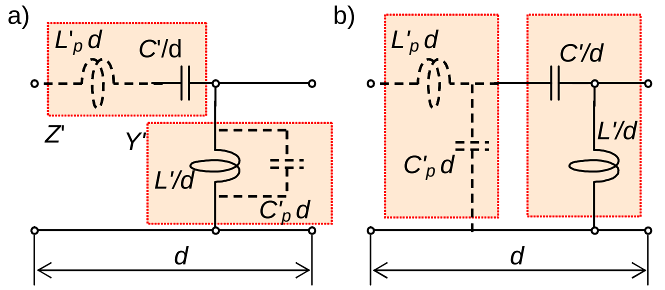

Fig. 1. a) An equivalent circuit for a unit cell with a length *d* of lossless transmission line CRLH metamaterial with corresponding per unit length parameters (primed). Index "p" denotes the parasitic inductance and capacitance (dashed lines). b) the same equivalent circuit for the balanced case.

The per-unit length impedance Z' and admittance Y' for the 1D unit cell in Fig. 1 are given as

$$Z'(\omega) = j\left[\omega L'_p - \frac{1}{\omega C'}\right], \quad Z'(\omega) = j\left[\omega C'_p - \frac{1}{\omega L'}\right] \qquad (1)$$

where 'prime' stands for 'per unit length'. The complex propagation constant γ is defined as $\gamma = \alpha + j\beta = (Z'Y')^{1/2}$.

Here β denotes the real propagation constant (attenuation constant). Phase velocity is v_p = ω/β. If $L'_p C' = L'C'_p$, we have the so-called balanced case. In this case the equivalent circuit for the unit cell presented in Fig. 1b is valid.

The propagation along the line is described by the well-known telegrapher's equation (e.g. [15]). The dispersion of the attenuation constant β for the balanced case is described by

$$\beta(\omega) = \omega\sqrt{L'_p C'_p} - \frac{1}{\omega\sqrt{L'C'}} \qquad (2)$$

Since the propagation constant of a material is defined as $\beta = \omega(\mu\varepsilon)^{1/2}$, the refractive index of a TLM is given by

$$n = c / v_p = c\beta / \omega \qquad (3)$$

From (2) and (3) it can be seen that at low frequencies the TLM refractive index is negative, and at high frequencies it is positive (Fig. 2). This means that below a certain frequency a CRLH transmission line behaves as a left-handed material and at higher frequencies it is conventional RH medium. The transition between the two regions occurs at *n*=0. This characteristic frequency is $\omega_0 = [LC]^{-1/2}$. Although β is zero at ω_0, which corresponds to an infinite guided wavelength ($\lambda_g = 2\pi / |\beta|$), wave propagation still occurs since in this case γ is always purely imaginary.

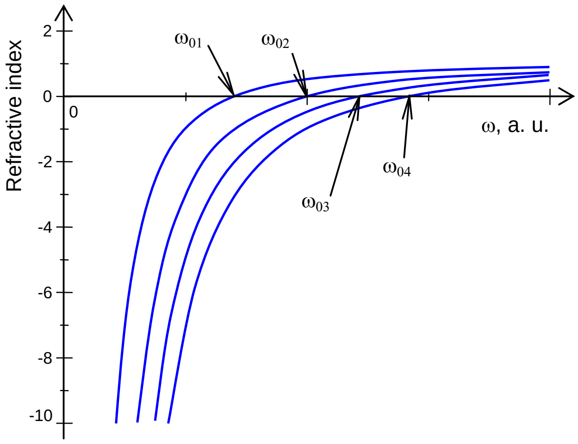

Fig. 2. Refractive index dependence on frequency for a balanced CRLH metamaterial.

A quasi-periodic structure often met in literature is the Fibonacci lattice, which belongs to substitution lattice sequences [18]. The transmission and reflection of these structures were widely studied in conjunction with electromagnetic spectral properties of quasi-crystals, superlattices, optical and microwave multilayers.

We give a general description of Fibonacci sequence and then map abstract objects to their concrete instances in transmission line metamaterial realization scheme. We proceed further by applying Fibonacci sequence scheme in order to study transmission and reflection properties for waves propagating along structure based on TL metamaterial.

A substitution lattice can be defined as a combination of physical settings with the corresponding rules of change. A physical setting is mapped on strings of formal symbols belonging to a predefined finite set. The initiator is a string of symbols from a finite set, called alphabet, from which the process starts. The rules of change are mapped on the rewriting rules for symbols in words, starting from the initiator, which are applied on each symbol in a word. The system evolution is described through a finite sequence of symbols.

We start with an alphabet $\Xi = \{A, B\}$, and denote Ξ^* as the set of all finitely long words that can be written in this alphabet. The rewriting rule is denoted by ξ and represents mapping from Ξ to Ξ^* by specifying action that ξ has on each letter of any word in transforming it to its image letter. The Fibonacci sequence can be described with rewriting rules $A \rightarrow \xi(A)=AB$, $B \rightarrow \xi(B)=A$. In instancing those sequences to the electromagnetic multilayer structures we define two refractive indices (n_A, n_B) and geometrical lengths (d_A, d_B) that correspond to (A) and (B). The number of elements increases according to the Fibonacci number, $F_n=F_{n-1}+F_{n-2}$ ($F_0=F_1=1$), and the ratio between the number of different elements A and B is equal to the golden mean number $\left(1+\sqrt{5}\right)/2$. Fibonacci generations are B, A, AB, ABA, $ABAAB$, etc.

Further we define that the constitutive layers have an equal optical thickness which is that of a quarter-wavelength slab $n_A d_A=n_B d_B=\lambda_0/4$. Our structure consist from layers with refractive index n_A and vacuum (or air) layers $n_B=1$. The incident and the output region are also air.

III. RESULTS AND DISCUSSION

We calculated the spectral properties of CLRH transmission line metamaterials using the transfer matrix technique (TMM) (e.g. [19]). To be able to compare the spectral transmission of quasi-periodic (Fibonacci) transmission lines for the case of conventional (A: RHM; B: RHM) to those of the "left-handed" (A: LHM; B: RHM) structures, we assumed that the refractive index n_A was constant in the frequency range under consideration. Since TLM have much wider bandwidth than the SRR-based LHM, this assumption is valid (although the same assumption is often applied for all structures, including those SRR-based – e.g. [7]; for an in-depth consideration see [1]). Fig. 3 shows the comparison of 12th generation Fibonacci sequences between n_A=1.5 and n_A=–1.5.

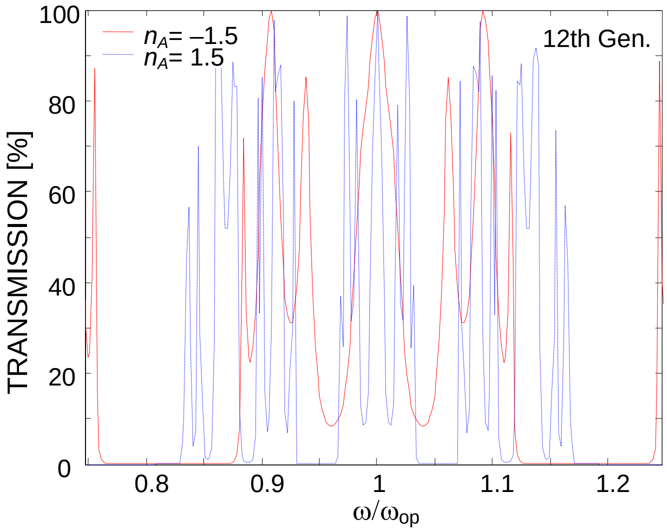

Fig. 3. Comparison of transmission spectra of 1D transmission lines with 12th generation Fibonacci-sequence layers. Solid: material A is left-handed, n_A=–1.5. Dashed: material A is right-handed, n_A=1.5. In both cases B is conventional, n_B=1.

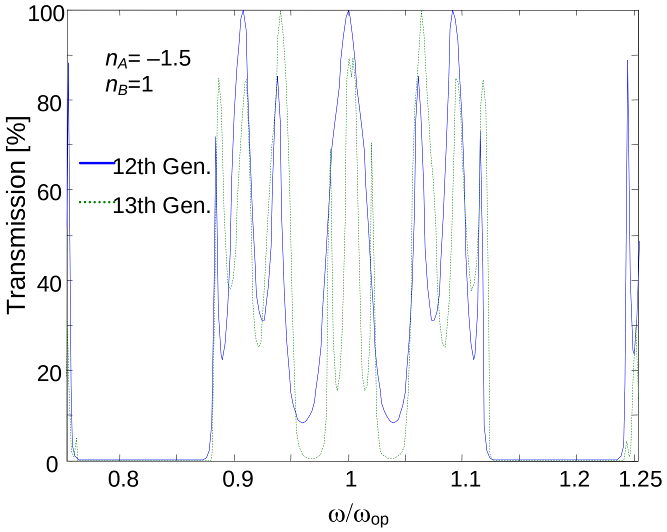

Fig. 4. Spectral transmission of 1D CLRH transmission line metamaterial with Fibonacci-sequence layers. Solid: 12th generation, dashed: 13th generation. Material A is left-handed, n_A=–1.5, B is conventional, n_B=1.

It can be seen that for the CLRH situation the spectra are wider and without sharp oscillations characteristic for the RHM. However, the mode localization property and self-similarity of the spectral images are fully retained. The comparison of two successive generations (Fig. 4) shows that the sequential splitting of the localized modes also exists, the same as in the RHM-RHM case.

Further we include the refractive index dispersion defined by a realistic set of transmission line metamaterial parameters [17]. For $L_p=L$=1 nH, $C_p=C$=1 pF and a characteristic length scale of d=1 mm, we obtain a refractive index in the form $n(\omega)=9.5(1-\omega_0^2/\omega^2)$ where ω_0=31 GHz. The operating frequency ω_{op}=28.8 GHz. The refractive index dispersion is shown in Fig. 5.

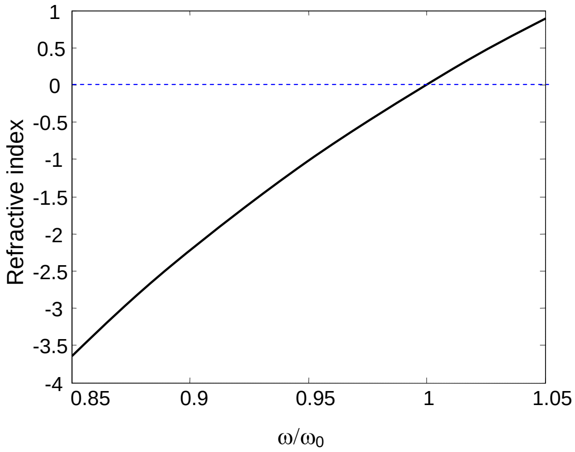

Fig. 5. Refractive index dispersion for LHM part of the CLRH transmission line for $L_p=L$=1 nH, $C_p=C$=1 pF, d=1 mm.

Fig. 6 shows the calculated spectral transmission of the TLM using the realistic transmission line negative refractive index dispersion.

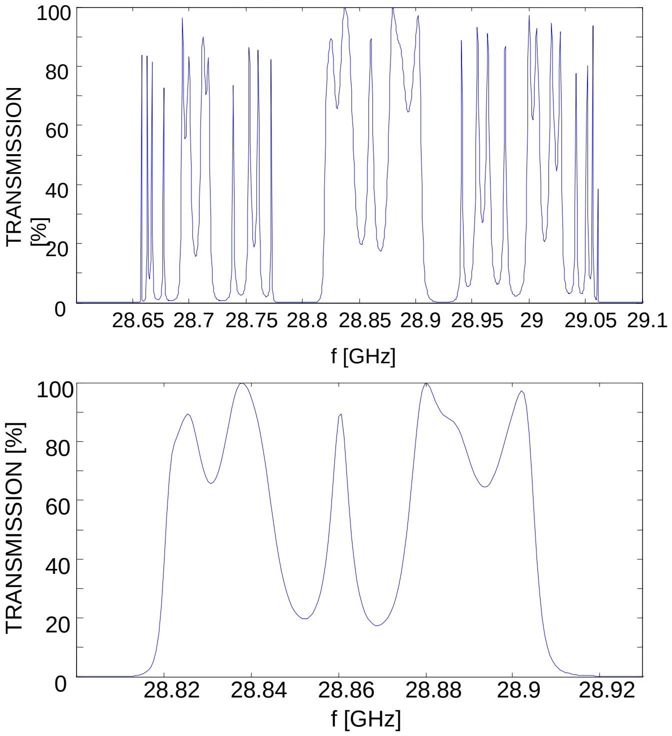

Fig. 6. Top: spectral transmission of 13th generation Fibonacci-sequence CLRH transmission line metamaterial for negative refractive index dispersion shown in Fig. 5. Bottom: the same dependence for a narrower frequency range 28.80–28.93 GHz.

Spectral self-similarity and mode localization can still be seen in Fig. 6, confirming the applicability of the proposed scheme for filtering applications.

IV. CONCLUSION

We considered electromagnetic wave propagation through quasi-periodically ordered transmission lines containing left-handed metamaterials. The quasi-periodicity was in the form of Fibonacci series. Our calculations were performed using the transfer matrix approach.

All the basic properties characterizing spectral properties of quasi-periodic structures in conventional materials are retained, i.e. the self-similarity property, sequential splitting when increasing generation and the appearance of narrow and high multiple peaks. Left-handed metamaterials addition to the transmission lines act by smoothing the peaks, while at the same time retaining all the mentioned benefits of quasi-periodic geometry. In addition to that, transmission lines containing negative index materials offer wider bandwidths and a number of other unique properties. We conclude that it may be expected that the combination of transmission line metamaterials and quasiperiodic structure could show enhanced functionality in filtering and other applications.

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