Overlapping Domain Decomposition for Coupled Magnetic and Thermal Fields

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Abstract – **The work presents a domain decomposition approach for coupled magnetic and thermal fields at low frequency applications in electromagnetic devices. The analysis domain for the two fields is divided into overlapping subdomains where the decomposition pattern is imposed by physical considerations. In this way a substantial reduction of the computing resources is obtained. As target example we consider the electric furnace based on the induction-heating phenomenon. The model is an axisymmetric field both for magnetic field and the thermal field. The mathematical model for magnetic field is based on time-harmonic Maxwell equations in vector magnetic potential formulation for axisymmetric fields. The model for the heat transfer is the heat conduction equation. A numerical model based on the finite element method is developed.**

Keywords – **Coupled fields; domain decomposition; finite element method; numerical simulation; induction heating**.

I. Introduction

An electric engineering problem can be formally divided into electric and magnetic sub problems, with disjoint or overlapping subdomains. In each subproblem we can use different mathematical models with interface conditions on the common pseudo boundaries between electric and magnetic subdomains. In other words we must choose between two options:

- An artificial subdivision which is the general principle of the domain decomposition
- A natural division imposed by the problem particularities

An artificial subdivision of the whole field domain does not change the solution of the field problem but a natural subdivision can lead to a better convergence. The pseudo boundary surface may be selected to follow some natural surfaces, such as the material interface.

The pseudo boundary conditions are treated in terms of the mathematical models developed in each subdomain. For example in the electromagnetic problems we can use different potential formulations (scalar and/or vector potentials).

The differences appear at the pseudo boundary. In principle the treatment of the pseudo boundary condition is the same but in vector potential formulations we have a vector quantity. Let us consider the magnetic field problem in the induction heating for the target example. Let us suppose that the domain is divided into 2 disjoint subdomains Ω_1 and Ω_2 with the common pseudo boundary S_B. The natural interface condition for this problem is:

$$\upsilon_1(\bar{n}\times\bar{B})\Big|_1=\upsilon_2(\bar{n}\times\bar{B})\Big|_2$$

The subscripts 1 and 2 refer to the two regions that the pseudo boundary separates and the normal direction on the interface surface is assumed to point from subdomain 1 into subdomain 2. Physically, the interface condition means that the tangential magnetic flux density has a discontinuity if the permeability differs in the two media:

$$\frac{\left|\overline{B_t}\right|_1}{\left|\overline{B_t}\right|_2}=\frac{\mu_1}{\mu_2}$$

where the subscript “t” indicates the part of the vector B which lies in the tangential plane at the interface surface. If the permeability is a scalar, the direction of the tangential vector is the same on both sides of the interface.

The natural boundary condition at an exterior boundary S of the whole domain can be considered for zero-reluctivity space as:

$$\overline{B_t}=0$$

These boundary conditions are natural and therefore, they need not be explicitly imposed on any approximate solution.

If we consider the A-formulation and FEM for a 2D domain Ω with the boundary Γ, the Galerkin's method leads to the following equation for steady-state magnetic field:

$$\int_{\Omega^e}\nu(\nabla A\cdot\nabla N-J\cdot N)d\Omega-\oint_{\Gamma}\upsilon N.\frac{\partial A}{\partial n}d\Gamma=0$$

with A the unknown vector potential and N the weighting vector function.

At a boundary point between two subdomains, A and ∂A/∂n are supposed unknown in each subdomain. The FEM gives an equation for each subdomain so that two equations are necessary to have a complete system. At the interface between two magnetic media those conditions are the continuity of the vector potential A and of the tangential magnetic field $H_t=\nu.\partial A/\partial n$.

In this work we limit the discussion to the through heating by induction. The process is critically dependent on the uniformity of the temperature in the workpiece so that a

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computer-aided design of the induction devices is a major requirement of the design [4].

II. MATHEMATICAL MODELS FOR COUPLED FIELDS IN INDUCTION HEATING

Induction heating is a convenient method for bulk-heating metals to a prescribed temperature. In our target example we consider a long iron cylindrical workpiece immersed in the field of a cylindrical coil (solenoid) supplied with alternating current. In figure 1 an axial section in the device is shown. The coil (3) is formed by rectangular bars and is surrounded by the air (2). The coil is normally water-cooled and has refractory lines.

The problem of induction heating in this example is an axisymmetric problem. Some standard assumptions are considered that lead to a 2D-scalar problem:

- The device has a rotational symmetry
- Current density J has only the azimuthal component in the θ-direction
- Materials have isotropic physical properties
- The source is a coil with wires of rectangular cross-section

This application can be modelled using mathematical models for axisymmetric fields because both magnetic field and thermal field have geometrical and physical symmetry. In other words we can use the differential models in cylindrical coordinates for axisymmetric fields.

The mathematical model for the electromagnetic field using A-formulation is a 2D-scalar model in (r-z) plane [1]:

$$\frac{\sigma}{r}\frac{\partial (rA)}{\partial t} - \nabla\left[\frac{\upsilon}{r}\nabla(rA)\right] = J_S \quad (1)$$

or for the harmonic-time case:

$$\frac{\partial}{\partial r}\left[\frac{\upsilon}{r}\frac{\partial (rA)}{\partial r}\right] + \frac{\partial}{\partial z}\left[\frac{\upsilon}{r}\frac{\partial (rA)}{\partial z}\right] - j\sigma\omega\frac{\upsilon}{r}(rA) = -J_S \quad (2)$$

Equation (1) represents the electromagnetic field description, with σ – the electric conductivity, ν – the magnetic reluctivity and J_s - the excitation current density. It is solved with specified initial and boundary conditions that depend on the problem.

The heat transfer problem in homogeneous orthotropic material is described in terms of the temperature T ([1],[2]):

$$\frac{\partial}{\partial t}[(c\gamma)(T).T] + \nabla[-k \quad (T).\nabla \quad T] = \rho(T).J^2(A) \quad (3)$$

The significances of the quantities are: T(r,z,t) is temperature in the point with co-ordinates (r, z) at the time t; k is the tensor of thermal conductivities; γ is the specific mass; c is the specific heating and J is the global current density. The current density J includes both the driving and skin effect. The term J can be computed as

$$J = \nabla \times H$$

In the inner and outer thermal insulating layers of the coil the driving term is missing.

The boundary conditions are convective and radiation conditions, that is:

$$-k\frac{\partial T}{\partial n} = h(T - T_\infty) \quad (4)$$

$$-k\frac{\partial T}{\partial n} = \alpha_r(T^4 - T_\infty^4) \quad (5)$$

where h and α_r are the transfer coefficients at the boundaries.

The model is non-linear because the specific heat c and the thermal conductivity k are dependent on the temperature. The coupling between the magnetic and thermal fields is represented through both the heat sources in (3) and the dependency on the temperature of the electromagnetic properties from (1).

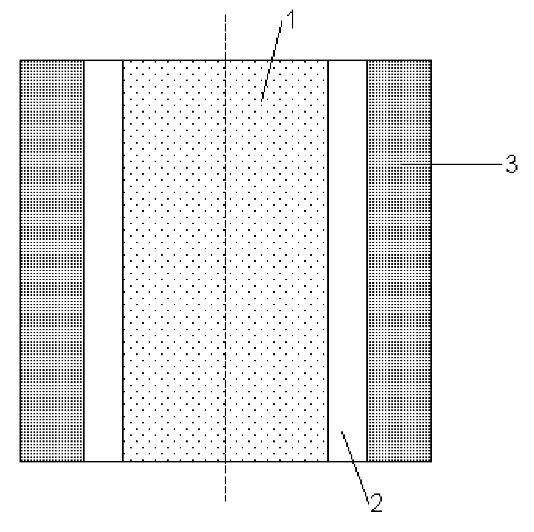

Fig.1. An axial section in device

Conventional coils are constructed from hollow copper pipe and are water-cooled to remove the heat generated in the conductors. An alternative approach is to use litz-wire in the construction of the coil, thereby ensuring that the losses in the coil are reduced. Nearly all through-heating coils use thermal insulation between the coil and the workpiece. This protects the winding and its insulation from the radiant heat and acts as a muffle around the workpiece. The coil conductors are characterised by the lack of eddy currents in a simplified model.

III. A NUMERICAL MODEL BASED ON THE OVERLAPPING DOMAIN DECOMPOSITION

The domain decomposition can be used only for magnetic field computation [3], or it can be used in coupled model by a strategy that we present in this section. The analysis domain plotted in the figure 1 can be decomposed in two subdomains that are not disjoint. The physical considerations are followed in this decomposition. For many eddy-current problems the magnetic flux penetration into a conductor without internal sources of the magnetic field is confined mainly to surface layer. This is the skin effect. The skin depth δ depends on the material properties μ, ω and σ so that for the small depths all of the effects of the magnetic field is confined to a surface layer [4].

In steady-state low-frequency eddy current problems in magnetic materials, the mathematical model is the diffusion equation (1). The skin effect can be exploited in two directions:

- To reduce the space domain in analysis with a fine mesh close to conductor surfaces
- To reduce the material volume since a significant proportion of the conductor is virtually unused

The penetration depth is given by the formula [4]:

$$\delta = \sqrt{\frac{2}{\omega\sigma\mu}} \qquad (6)$$

In other words for problems with the skin depth very small all the effect of the field is confined to a surface layer. Designer engineers use the formula (6) considering the permeability and the conductivity as constants. In reality the two physical parameters change during heating. The changes in the value of δ affect the loss in the material and depend on the process (conduction or induction). For example, if the conductivity decreases by x, the depth increases by √x, that is the current penetrates deeper into the metal. If the magnetic material heats, its resistivity (the inverse of the conductivity) rises but its relative permeability remains substantially constant up to the Curie point. In this point it drops suddenly to unit.

For low frequency applications the field depth is large. For an industrial frequency we can compute the depth in workpiece and limit the magnetic field analysis at this depth. In the thermal field analysis we reduce the field domain at the workpiece. The two domains are overlapping on a surface defined by the penetration depth of the magnetic field in workpiece. For a range of the frequencies and the temperatures we can compute the range of the depth penetration. In order to cover the whole domain of frequencies we select the maximum value of the depth. The depth penetration depends on the material properties so that the boundary of the field in conductor can change in time.

In the figure 1 the field domain in a finite element model is plotted with the horizontal axis Oz and vertical axis Or. For the thermal field we use a Neumann boundary condition, combining a given boundary heat flux with convection and radiation:

$$-k\left.\frac{\partial T}{\partial n}\right|_{\Gamma} = q_n + h(T - T_\infty) + \alpha(T^4 - T_\infty^4)$$

with T_∞ - the surrounding medium temperature, q_n the heat flux and n the outward unit normal on the boundary.

IV. NUMERICAL RESULTS

Figure 1 shows a long cylindrical workpiece excited by a close-coupled axial coil. The problem is an axisymmetric heating device. The coil is assimilated with a massive conductor. In the numerical model we can consider or ignore the eddy currents in the coil. If we ignore the eddy-currents in the coil the electric conductivity is equal to zero and the domain for thermal field is reduced to the workpiece volume.

The geometry and the physical properties are defined as in the tables I, II and III.

TABLE I
Coil conductor's properties

Relative permeability	1
Conductivity	$6.3\ 10^7$ S/m
Thermal conductivity	140 W/K.m
Specific density	380 J/Kg.K
Metal density	8700 Kg/m^3

TABLE II
Workpiece's properties

Relative permeability	500
Conductivity	$1\ 10^7$ S/m
Thermal conductivity	100 W/K.m
Specific density	200 J/Kg.K
Material density	7800 Kg/m^3

TABLE III
Air's properties

Relative permeability	1
Conductivity	0 S/m
Thermal conductivity	0.01 W/K.m

The depth penetration for a low frequency was 4 mm. The amplitude of the current in the coil is 40 000 A.

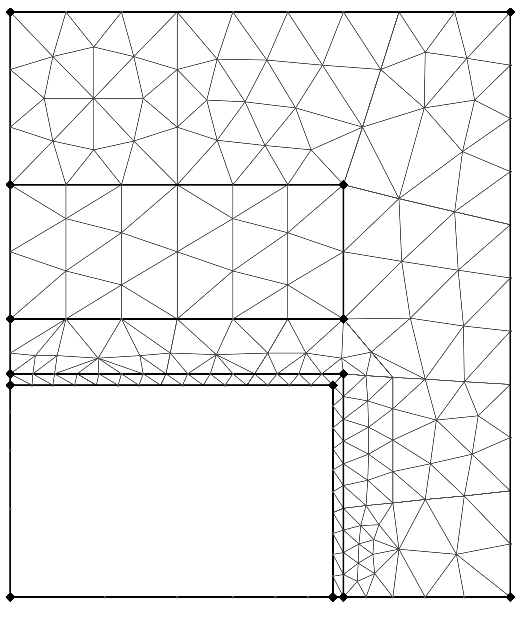

Fig.2. The analysis domain for magnetic field

In the figures 3-5 the results of the numerical simulation are plotted using QuickField [5]. The finite element method was used for the numerical modelling both magnetic field and thermal field. In fig. 3 the temperature of the workpiece surface is plotted. Starting with the initial temperature 40 ^{0}C, the variations of the temperature in two points are plotted in the figure 5.

In fig. 4 the final distribution of the temperature on the axis of the workpiece in the center direction is plotted.

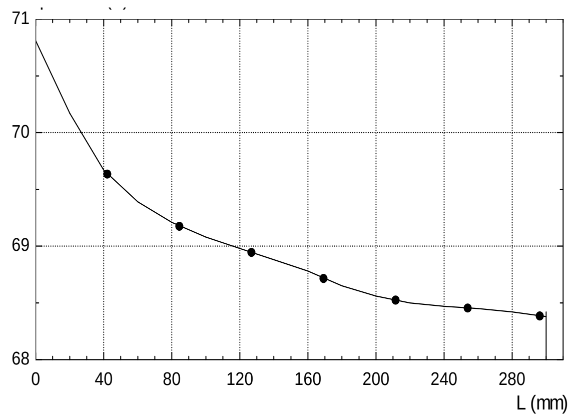

Fig.3. The final temperature on the external surface of the workpiece

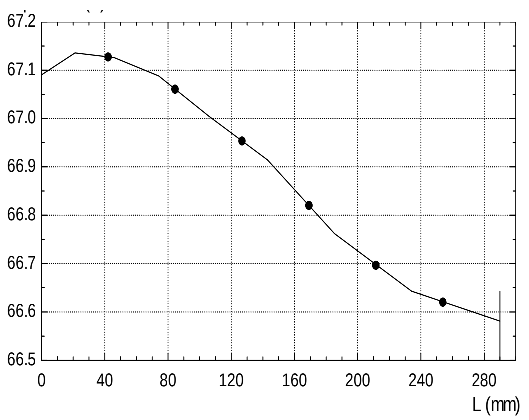

Fig.4. The final temperature on the axis of the workpiece

Some relevant aspects must be considered in the design of the CAD software for coupled magneto-thermal problems:

- The thermal source term in the heat equation can be defined by the time-mean of the ohmic power loss. The motivation is simple: the time constant of the magnetic phenomenon is small compared to the diffusion time of the heat transfer.
- A cascade solution may be more efficient than a fully coupled model. In some applications there is a strict coupling between magnetic and thermal equation at each time instant, but in many situations we can split the analysis of the magnetic field by the analysis of the thermal field.
- It can be used a predefined temperature profile of a material for updating the magnetic field at specified temperatures. For example, at Curie temperature the material properties change dramatically. After this critical point the magnetic field equation must be updated.
- The analysis domain can be divided in more subdomains with different solvers for the coupled problems.

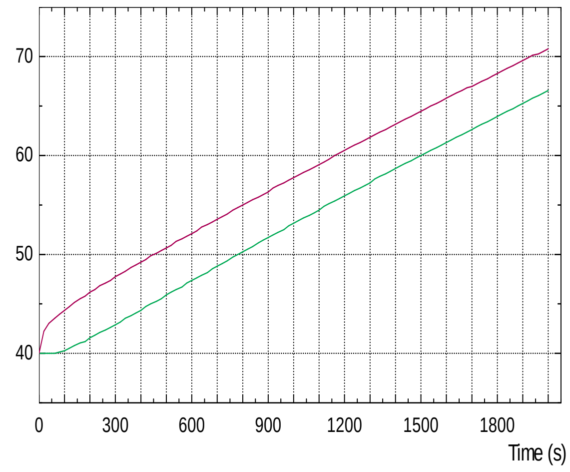

Fig.5. The temperatures in the center and in the corner of the workpiece (the above curve)

V. CONCLUSIONS

In this paper we presented a numerical model for the induction heating of cylinders in large furnaces. The model was based on the simplifying assumptions but we did not ignore the non-linear effect of the temperature distribution. The radiation effect and convection were considered on the workpiece surface.

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