A Simplified Model for Coupled Magnetic and Thermal Fields in High Frequency Applications

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Abstract – **The work presents a simplified model for coupled magnetic and thermal fields in solid conductors in the context of the finite element method.**

Our target example is the induction heating at high frequency where the depth penetration of the magnetic field in a conductor workpiece is very small. This fact is exploited in the sense of reduction of the analysis domain and computational effort. In a numerical model based on finite element method (FEM) this effect can be exploited by the use of a special boundary condition, known as the surface impedance condition. In this way we don't waste run-time of a program based on FEM.

The mathematical model for magnetic field is based on Maxwell equations in vector magnetic potential formulation for axisymmetric fields. The model for the heat transfer is the heat conduction equation. A numerical model based on the finite element method is developed.

Keywords – **Coupled fields; finite element method; numerical simulation; induction heating**.

I. Introduction

Induction heating is used extensively in the metal industry. It is an illustrative engineering example for the thermal conductivity problem in which the heat is generated by ohmic losses from eddy currents induced in conducting materials, such as metals.

Induction heating is a convenient method for bulk-heating metals to a set temperature. It replaces furnaces, which tend to be large and which have the disadvantage of long start-up and shutdown times, so that their effectiveness is low. The induction heater is relatively small in size and is immediately available for use. It is clean and relatively efficient.

The special feature of induction heating is that the heat is produced in the material by currents flowing in it. In induction heating, the heat is generated by the passage of currents through the material but they are induced from a separate source.

Whatever the configuration, induction heating always involves a varying magnetic field. An induction-heating device can be assimilated to a transformer. The metal to be heated becomes the secondary circuit of a transformer. In a simplified model, the primary coil it always constructed of copper, since it is important to keep losses to a minimum so that the heating is done to maximum efficiency.

From the general theory, the induced currents will effectively flow in about a skin depth δ – depending on frequency of the source current and the two properties of the material - resistivity and relative permeability.

In this section we present some algorithmic skeletons for numerical simulation of the induction heating in metal cylinders using linear and non-linear models. The example is a model for the induction-heating furnace without iron core.

In the electromagnetic problems the field domain consists of more regions that differ by the electric properties. Thus, in induction heating the analysis domain consists in the following region:

- A conducting region where the eddy currents are induced
- A conducting region with externally imposed currents and the magnetic permeability of the free space
- A non-conducting region with low permeability magnetic materials without any kind of currents

In each region we can use different potential formulations and the coupling of the three regions is ensured by the interface conditions: the continuity of *H x n* and *B.n.*

The choice of the potential formulation in each region is a difficult task and depends on the problem particularities. In several cases the problem characteristics and the computation accuracy required allow a decoupling and a cascade solution. If this approach is not possible an approach which is frequently applied is to solve each equation separately and to recouple them in some way.

II. Mathematical Models for Coupled Fields in Induction Heating

Many areas of electrical engineering require the solution of problem in which the electromagnetic field equations are coupled to other partial differential equations, such as those describing thermal, fluid flow or stress behaviour. These phenomena are described by equations that are coupled. The coupling between the two fields – electromagnetic and thermal - is a natural phenomenon and only in a simplified approach the field analysis can be treated as independent problem.

In several cases, it is possible a decoupling and a cascade solution of the coupled equations. One approach is to solve each field equation separately, recouping then the set in some way.

Another attractive and efficient approach of solving coupled differential equations is to consider the set as a single system. In this way a single linear algebraic system for the whole set of differential equations is obtained after

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discretization, and is solved to a single step. If one or more equations are non-linear, non-linear iterations of the whole system are required.

Generally, in the mathematical and numerical simulation of transient induction heating problems, it is necessary to solve two systems of equations: one for the electromagnetic field and the other for the thermal field. These systems are strongly coupled and non-linear, since the thermal sources represent the electromagnetic field effects and the magnetic and thermal properties of the material change with the temperature. This inherent coupling of the two mathematical models leads to an increase in the numerical model dimensions due to the vectorial nature of the electromagnetic quantities.

The mathematical models of the coupled fields are:

- the transient heat conduction equation with boundary conditions of convective and radiation type
- Maxwell equations in potential formulations (especially in multi-potential formulations)

The constructive particularities of the electromagnetic device must be accounted for in a numerical model in order to develop efficient computer program. For example, the main particularities of the induction heating are:

- The thermal time constants are bigger than the electric time constants.
- The heat source is the power dissipated by the eddy currents so that the accuracy of the computation of the eddy current density influences the problem accuracy
- The workpieces are generally composed by only a single material so that it is not necessary to include discontinuities in the electric conductivity
- The variations of the electromagnetic properties (permeability, electric conductivity) are continuous.

The particularities of the problem can be exploited in some directions although a unified approach is a desirable requirement for an expert system in the area of coupled problems. The following directions can be exploited in our target example:

- The electromagnetic computation can be done as a sequence of sinusoidal electrical steady-state
- The most important electromagnetic field parameter from the viewpoint of the thermal field is the current density so that the computation accuracy of this quantity is very important
- The lack of discontinuities in electric conductivity allows the use of magnetic vector potential formulation
- The computation accuracy of J depends of the simulation method. For example, if J is derived by a "*rot*" operation as J= rot H, the accuracy of J is not the same accuracy as H. If the density is directly computer as J=-jωσA, then the accuracy of J is the same accuracy as A.

The mathematical model for the heat transfer is the heat conduction equation:

$$(c\gamma)\frac{\partial T}{\partial t} = \nabla(k\nabla T) + q \qquad (1)$$

$$T(x,0) = f(x) \quad x \in \Omega \qquad (2)$$

$$-k\frac{\partial T}{\partial n} = g(x,t) \quad x \in \Gamma, \quad t \succ 0 \qquad (3)$$

The significances of the quantities are: T(r,z,t) is temperature in the point with co-ordinates (r, z) at the time t; k is the tensor of thermal conductivities; q is the heat source (the ohmic losses due to the eddy-currents); g is a known function that leads to convection, Neuman or radiation condition.

The electromagnetic field model is Maxwell's equation in formulation with the magnetic vector potential A:

$$\nabla \times (\nu\nabla \times \overline{A}) = \overline{J} \qquad (4)$$

with J – the total current density:

$$\overline{J} = -\sigma\left(\frac{\partial \overline{A}}{\partial t} + \nabla V\right) \qquad (5)$$

Induced losses are:

$$P = \frac{1}{2\sigma}\int_S |\overline{J}|^2 dS$$

For the magnetic field there are two basic types of boundary conditions on the boundary Γ of the space domain Ω (Dirichlet and Neumann conditions):

$$\overline{A}(x,y,t)\Big|_\Gamma = \overline{A_D}(x,y,t)$$

$$\upsilon\frac{\partial \overline{A}}{\partial n}\Bigg|_\Gamma = \overline{H} \times \overline{n} = \overline{H_n}(x,y,t)$$

where the subscript n has the significance of the normal component of the field variable.

In harmonic time applications, with V a scalar electric potential, the electric the current density is:

$$\overline{J^*} = -j\sigma\omega\overline{A^*} - \sigma\nabla V^*$$

and the field model becomes:

$$\nabla \times (\nu\nabla \times \overline{A^*}) + j\sigma\omega\overline{A^*} + \sigma\nabla V^* = 0 \qquad (6)$$

where the superscript * denotes the complex value.

III. A SIMPLIFIED NUMERICAL MODEL

The computation of the eddy current distribution in a conductor using 3D model is a difficult and computationally expensive problem. To reduce the computation complexity the concept of the surface impedance is used. This approach is appropriate in those applications where the skin depth of the eddy currents induced is small compared with the thickness of the conductor. The idea of this approach is simple: the conductor is replaced by an appropriate boundary condition.

For many practical eddy current problems as the induction heating devices the magnetic flux penetration is confined to surface layers. Thus, a sinusoidal magnetic field with the amplitude B_m at the conductor surface and parallel to the surface has an exponential decay in the conductor:

$$B = B_m.\exp\left(\frac{-x(1+i)}{\delta}\right)$$

In a point on surface of the conductor, the surface impedance is defined by the components of the electric and magnetic fields:

$$Z_t = \frac{E_t}{H_s}$$

where subscripts t and s denote the tangential direction for the two fields. The tangential magnetic field, H_s, is due to excitation.

If the skin depth δ is small, it is not economic to do a discretization of the solid conductor. This idea is exploited in many numerical models by considering a special boundary condition known as the surface impedance condition. In an A-formulation for two-dimensional magnetic problems the boundary is:

$$\frac{\partial A}{\partial n} = -\frac{(1+i)}{\delta \mu_r} A = -\beta A$$

This condition in a finite element model leads to the evaluation of a term as:

$$\int_\Gamma N_i \frac{\partial A}{\partial n} d\Gamma = -\beta \left(\int_\Gamma N_i N_j d\Gamma\right) A_i$$

In the modelling of the induction heating we can use this concept by decomposition of the whole domain into two subdomains.

As target example we consider a long cylindrical workpiece excited by a close-coupled axial coil (figure 1). The problem is an axisymmetric heating device. The coil is assimilated with a massive conductor. A quarter of the whole domain is considered.

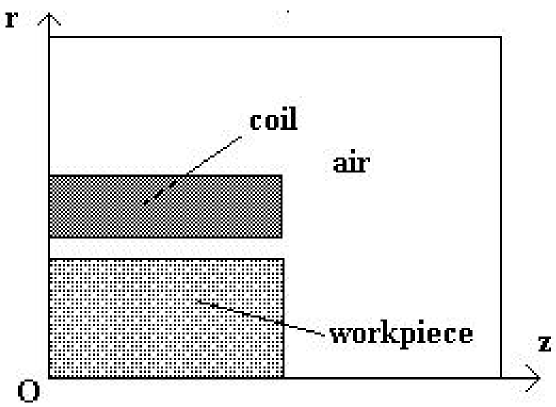

Fig.1. The whole analysis domain

The main advantages of the use of the surface impedance are:

- Reduction of the size of the problem domain
- Reduction of the solution time for the numerical model

Practically, the whole domain of the problem can be decomposed into 2 subdomains:

- A subdomain Ω_1 contains current source (the coil) and the air
- A subdomain Ω_2 consists in the workpiece (a metal cylinder) and source-free

The solid conductor may be modelled using the Poynting vector that replaces the solid conductor. In the figure 2 the analysis domain is plotted.

Numerical algorithms were developed using this concept. The first stage of any algorithm involves the evaluation of the

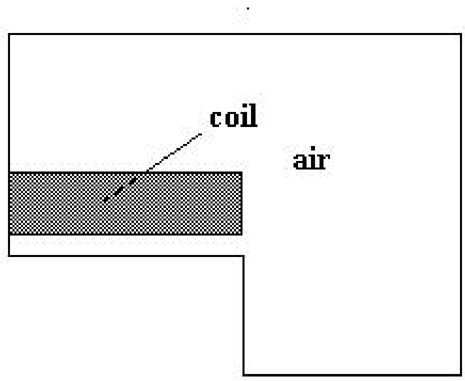

Fig.2. The reduced analysis domain

ohmic losses developed in the conducting part.

Concerning the target example, a *simplified algorithmic skeleton* consists in the decomposition of the whole domain in two disjoint subdomains Ω_1 and Ω_2 with the following properties:

- A subdomain Ω_1 for electromagnetic field analysis that does not include the workpiece (replaced by the surface impedance);
- A subdomain Ω_2 for the thermal field that include the workpiece with the heat generated by the electromagnetic field on the workpiece surface.

The algorithm consists into 2 stages:

- Computation of the electromagnetic field in the subdomain 1 by replacing the subdomain 2 with a surface term
- Computation of the thermal field consisting of the solid conductor only but having a common boundary with the first subdomain.

The boundary condition for the thermal on the common boundary is derived from the electromagnetic solution. In this algorithm we consider that the heat penetrates the workpiece by the outer surface. This assumption is valid at high frequency or for the cases where the dept penetration of the magnetic field is very small. We can use Poynting vector to get real and reactive components of power, that is:

$$\overline{S} = \overline{P} + j\overline{R} = \overline{E} \times \overline{H}^*$$

with H^* the conjugate complex of H.

The power in workpiece is given by *P x S* , with S the surface area, and P the real part of the Poynting vector. This value is used in the computation of the boundary condition for the analysis domain.

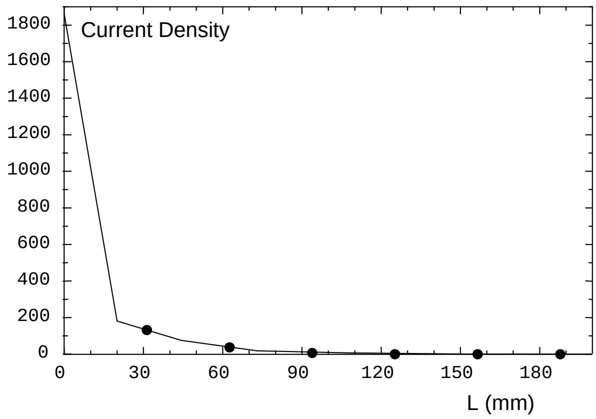

Fig.3. The current density in the workpiece

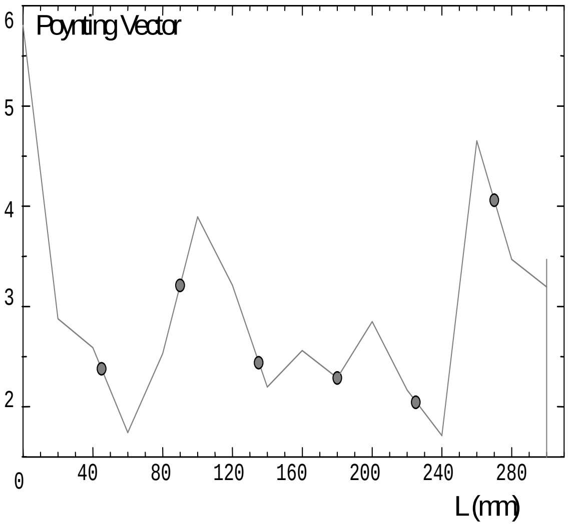

Fig.4. Values of Poynting vector at workpiece surface

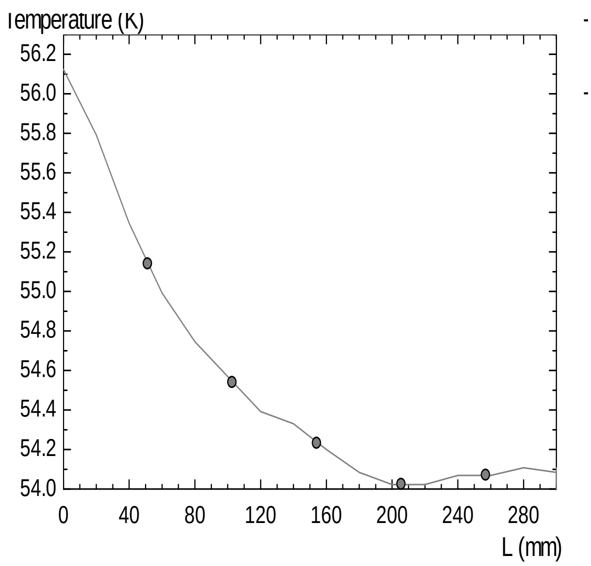

Fig.5. Final temperature on the external surface (tf=500s)

IV. NUMERICAL RESULTS

The numerical models were obtained by the finite element method for axisymmetric fields. For numerical simulation we used the program Quickfield [4]. In our target example we considered a workpiece of the metal with the conductivity 1.10^7 [S/m]. The frequency was 1 MHz. The copper coil has the conductivity of $6.3\ 10^7$ [S/m]. We made more simulations for a frequency range from 1 KHz to 1 MHz. The Figure 3 shows the current density [A/mm^2] in the workpiece along the centerline (the axis Or). The depth of penetration is small and can vary with the temperature.

RMS power flow through the external axial surface of the workpiece (Poynting vector flow) is computed with the formula:

$$P = \int (\overline{S}.\overline{n})ds$$

The variation of the vector Poynting to the workpiece surface is plotted in the figure 4 and can be used for heating computation in the workpiece.

At a frequency of 1000 Hz the final temperature is shown in the figure 5 for a time interval equal to 500 s.

V. CONCLUSIONS

In this paper we presented a simplified model for the numerical simulation of the induction heating of cylinders in large furnaces. The model was based on the concept of the surface impedance with direct impact on the computing resources. At high frequency the penetration depth of the magnetic field in a metal is very small so that we can eliminate the workpiece domain in the magnetic field computation. The model is based on the simplifying assumption that all the heat entered at the surface. This assumption is valid if the frequency is very high for the depth of heating to be small compared with the radius of the workpiece. The finite depth of penetration changes the temperature in the workpiece considerably.

REFERENCES

[1] Cârstea, D. “CAD tools for magneto-thermal and electric-thermal coupled fields”. *Research Report in a CNR-NATO Grant*. University of Trento. Italy, 2004.

[2] Cârstea, D., Cârstea, I. *CAD in electrical engineering.* Editor: Sitech, Craiova. 2000.

[3] Davies, E.J. *Conduction and Induction Heating.* IEE Power Engineering Series 11, Peter Peregrinus Ltd. London, U.K. 1990.

[4] *** QuickField program Version 5.2. User’s guide, 2004. Tera Analysis.