Typical of the Inverters Transitive Processes

Hristo P. Hinov[1]

Abstract – **The free oscillations method has been applied to the series inverter. The equations have been canonized and the inverter parameters have been reduced to two. The inverter functions have been stylized. The whole operation process is described only by the function of the first half-period, modified by the current start point. The dependence between the sequential start points has been derived. A recurrent dependence has been applied for the formulation of the stationary mode. The mode accumulations are presented in a power series. The development of the transient process is presented depending on the inverter parameters and frequency. The operations have been summarized for both elementary and complex inverters. Author's conclusions: The free oscillations method is applicable for the mathematical interpretation of the inverter. The first half-period, starting from zero initial conditions, is multiplied and functionally presents all subsequent half-periods in the operation process of the inverter. The start point is the result of the completed half-periods and determines the mode level (voltage and current) of the current half-period. The position of the start point depends on the relationship between the natural frequency and the timer frequency. The transient process is represented by a fractal (broken line), whose diminishing segments are at equal angles. The broken line tends to the start point of the stationary mode.**

Key words – **inverter, free oscillation**.

I. Introduction

The functional presentation of electromagnetic processes taking place in inverters is traditionally influenced by the concept of forced oscillations [1], [2], [3]. The references analyze the inverter reaction under the influence of the square pulses from the commutated supply voltage.

The present article develops the alternative approach, according to which the inverter is treated as a freely oscillating system within each half-period. The periodic commutations in the inverter keep activating the temporary solutions to the differential equations, which is the mathematical interpretation of free oscillations.

The aim is to study the transient process and formulate the inverter stationary mode.

Inverters are sources of high-frequency energy and consist of controllable electronic switches and an oscillatory system (resonant circuits). The switches are usually transistors connected in parallel to diodes, which form a four-arm bridge. The transistors are controlled by a timer determining the generated frequency. The timer closes the switches along one bridge diagonal and opens the switches along the other action diagonal at intervals $T_t/2$ – called half-periods /HP/. The of the switches causes free oscillations. The free oscillations energy is realized in a separate part of the same oscillatory system.

[1]Prof. Hristo P. Hinov is with the Faculty of Electronic Engineering, Technical University of Gabrovo, H. Dimitar Str.4, 5300 Gabrovo, Bulgaria,
E-mail: hino@abv.bg.

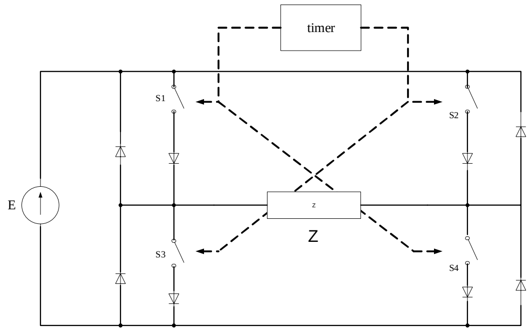

Fig.1. Object Circuit

Fig.1 presents the object circuit. The oscillatory system Z is determined by the number and type of the characteristic equation roots.

If Z is characterized by the only (real) root, then the inverter is determined as "elementary".

The popular series inverter has a pair of complex conjugate roots, but the analysis presented also refers to other configurations of Z, in which a complex pair of roots dominates.

The complex inverters, with a lot of root Z, can be presented as a combination of the preceding types or their processes can be interpreted by means of multidimensional matrix operators.

II. Electromagnetic process in a series inverter

The series inverter is suitable for demonstrating the base processes in inverters. Its two-terminal network Z consists of a capacitor, inductance and resistor, connected in series.

A. Equations and parameters

The differential equation of the series inverter is:

$$L(dI/dt) + RI + (1/C)\int Idt = E$$

The roots of the characteristic equation determine:

- $\omega = [(LC)^{-1} - (R/2L)^2]^{0.5}$ – natural frequency,
- $\varepsilon = (R/2L)/\omega \approx 1/2Q$ – atténuation. $(Q=(L/C)^{0.5}/R)$

A priority of the method applied is the substitution of a current angle called "phase" for the current time: $\theta = \omega t$.

The angle-phase symbolizes the current time in the inverter in the same way as the clock presents time by means of an angle (thirty degrees signify an hour).

The phase is a limited argument: $0 \le \theta \le \theta_s$. The maximum phase, called cycle: $\theta_s = \pi (T_t / T_0)$, is proportional to the timer period T_t, related to the natural period $T_0 = 2\pi/\omega$. The cycle θ_s is the angular route traveled by the "inverter clock hand" Eq. 1, for the timer interval Tt/2.

The four circuit parameters (C, L, R and period T_t) have been reduced to the two dimensionless quantities – attenuation ε and cycle θ_s. These are the degrees of freedom or the inverter coordinates. They are combined in the complex parameter:

$$\dot{p} = (-\varepsilon + j)\theta_s \qquad (1)$$

The solutions to the differential equation are presented in dimensionless quantities:

- Capacitor voltage $u(\theta) = -1 + u_s \exp(-\varepsilon\theta) \cos\theta$,
  Where: u = U/E, us random starting voltage.
- Current across inductance $i(\theta) = i_s \exp(-\varepsilon\theta) \sin\theta$,
  Where: $i = I (L/C)^{0.5}/E$, i_s random starting current.

B. Dynamics of the primary phase vector

The last two quantities are presented jointly by a complex function, the real part of which is voltage, and the imaginary part is the current [4], [5]:

$$F_1(\theta) = -1 + \exp((-\varepsilon+j)\theta) \qquad (2)$$

This is a rotating vector in a complex plane. It is primary because it describes the inverter start from zero initial conditions: $u_s=0$, $i_s=0$. It describes a spiral with center (-1, 0), traveling the angular route θ_s, and completes its half-period development in the final point

$$F_1 = -1 + \exp(\dot{p}).$$

The timer replaces the conduction switches of the bridge in the final point, causing the so-called commutation in the inverter. The commutation changes the source direction, which is manifested by changing the signs in the last equality.

The second half-period does not start from the zero, but from the inverse F_1:

$$S_2 = 1 - \dot{q} \qquad (3)$$

Where the complex number $\dot{q} = \exp(\dot{p}) = \exp((-\varepsilon+j)\theta_s)$, is the completed development of the exponential function in Eq. 2.

The described development and final of the first half-period deserve special attention, because each subsequent half-period reproduces them at a higher mode and energy level.

C. General form of the phase vector

The second half-period is presented by a phase vector (PV) similar to Eq. 2 but starting from a point, which is the result of the first half-period action:

$$F_2(\theta) = -1 + (1+S_2)\exp((-\varepsilon+j)\theta).$$

The general form of the phase vector is the generalization of the last equality, which is valid for a half-period of an arbitrarily high number:

$$F_n(\theta) = -1 + (1+S_n)\exp((-\varepsilon+j)\theta) \qquad (4)$$

The PV (Eq. 4) can be presented in the form: $F_n(\theta) = F_1(\theta) + S_n \exp((-\varepsilon+j)\theta)$

The phase vector of a random half-period in the operation process of the series inverter is determined by the primary PV ФВ – Eq. 2 and by the start point /SP/ S_n, formed by the preceding half-period.

Fixing the argument in Eq. 4 at its limit $\theta = \theta_s$ and performing the commutation inversion, the SP of the next half-period is obtained.

The law of the start point's reproduction is:

$$S_{(n+1)} = 1 - (1+S_n)\,\dot{q} \qquad (5)$$

Both the PV and point (Eq. 5) can be presented as the sum of the primary SP S_1 and the preceding one S_n, reduced by the complex $\dot{q}$: $S_{(n+1)} = S_1 + S_n\,\dot{q}$.

D. Direct definition of the stationary mode

Although the stationary mode of the inverter is a product of the preceding process, its direct determination is also possible, applying important recurrent dependences and properties.

For example, if the mode reproduction principle is applied in stationary mode, then each half-period will repeat the preceding one. To achieve such reproducibility it is sufficient to equalize two subsequent SP: $S_{(n+1)} = S_{(n)} = S$. The substitution is applied in Eq. 5, which determines the stationary mode of the series inverter by means of the start point definition:

$$\dot{S} = \frac{1-\dot{q}}{1+\dot{q}} \qquad (6)$$

If the substitutions applied in Eq. 3 are used, the hyperbolic form of the stationary mode will be obtained:

$$\dot{S} = th(-\dot{p}/2) \qquad (7)$$

E. Accumulations in the transient process

The different half-periods of the inverter operation are determined by there start points S_n. It is the varying parameter in Eq. 4, which characterizes the inverter transient process. Starting from $S_1 = 0$ and applying Eq. 5, the following start points are obtained:

$$S_2 = 1 - \dot{q};$$
$$S_3 = 1 - 2\dot{q} + \dot{q}^2;$$
$$S_4 = 1 - 2\dot{q} + 2\dot{q}^2 - \dot{q}^3;$$

The following power series is obtained for a half-period number of arbitrary magnitude:

$$S_n = 1-2\dot{q}+2\dot{q}^2-2\dot{q}^3+\ldots+2(-\dot{q})^{(n-2)}+(-\dot{q})^{(n-1)} \quad (8)$$

The signs in Eq. 8 are alternative, and the coefficients are pairs, except the first and the last term. For each elapsed half-period the end of the series is complemented by a term of a higher power. The coefficient of the last-but-one term is also complemented. These are the mode accumulations, which are a result of the preceding half-periods.

The series is complemented to an exact geometric progression and allows the application of a limit transition when $n\to\infty$, which is a second proof of the stationary mode law Eq. 6 and respectively – Eq. 7.

The development of Eq. 8 through the sequential half-periods provides a good vision for course of the transient process. It is expedient to reformulate Eq. 5, so as to emphasize the accumulation at the end of the series: $S_{(n+1)} = S_{(n)} + (1-\dot{q})(-\dot{q})^{(n-1)}$.

With each subsequent half-period two terms are added to Eq. 8, which complement the higher powers of the polynomial or series.

F. Characteristics of the accumulation

The alternative signs in the polynomial – Eq. 8 are a result of the commutation inversion described; however, prior to it another transfer has been performed – functional transfer. It is expressed in the completed development of the exponential function in Eq. 4 or the complex number $\dot{q} = \exp((-\varepsilon+j)\,\theta_s)$. It is not accidental that the inverter cycle is $\theta_s = \pi \pm \alpha$ [5] [6], which leads to an encoded minus of $\dot{q}$:

$$-\dot{q}_{(\alpha)} = \exp(-\varepsilon(\pi\pm\alpha)+j(\pm\alpha)) = r_{(\alpha)} \quad (9)$$

The imaginary part of Eq. 9, or the angular route, has a reduced argument at the expense of the derived minus. The polynomial Eq. 8 is restructured in:

$$S_n = 1+2r+2r^2+2r^3+\ldots+2r^{(n-2)}+r^{(n-1)} \quad (10)$$

The imaginary sign alternativeness is eliminated and the power series has the form of a normal geometric progression.

The complex character of Eq. 10 facilitates the visualization of the transient process – Fig. 2. The figure presents two open polygons, which form a broken line – fractal. The fractal sides decrease and are directed to a definite point – the limit. Each side is deviated from the preceding one at an angle of 30^0, added in the case of the upper fractal and subtracted for the lower one (declination $\pm\alpha$). The angles are the start points of the first ten half-periods, starting from $S_1=0$, $S_2=1+r$ etc. according to Eq. 10.

The two fractals express the transient processes in two inverters having equal attenuation (Q-factor = 3). One of the examples is characterized by a half-period angle $\theta_s = \pi+\alpha = 210^0$ – long cycle. The other inverter operates with a short cycle $\theta_s = \pi-\alpha = 150^0$. Equal declination is chosen for the two examples $\pm\alpha=30^0$.

The different cycles of the two examples are obtained from the same inverter, but at with different frequency modes. In the first example the "long cycle", timer dictates a frequency that is less than the natural frequency, $T_t > T_0 = 2\pi/\omega$. In the second - short cycle – the other way round, $T_t < T_0$, as a result of which frequency greater than the natural frequency is generated.

The complex number (Eq. 9) in the long-cycle inverter has a positive imaginary part and their power sum (Eq. 10) is in the first quadrant. Due to an analogous reason the start point of the short-cycle inverter is in the fourth quadrant. This difference leads to significant commutation and recuperation differences, which are the object of other articles [5] [6].

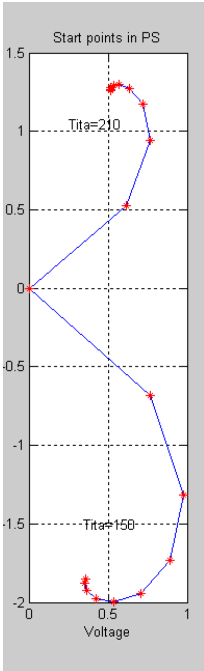

Fig.2 Start points

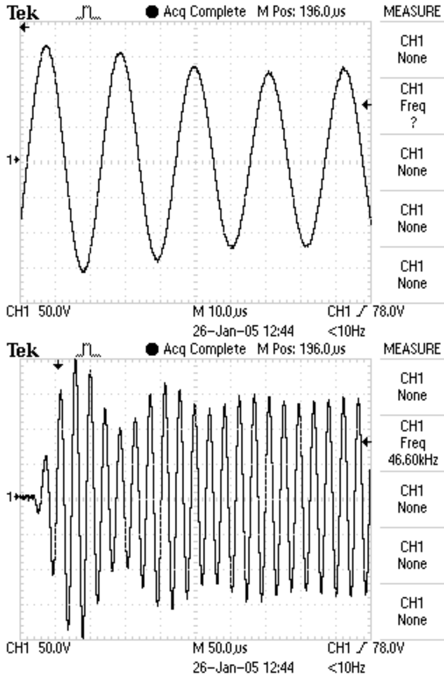

Fig.3 Transient process

The line segments (corresponding to voltages and currents) in the short-cycle inverter ($\theta_s=150^0$) are greater, because it has shorter time for realization (absorption) of the energy and it increases the accumulations in the reactive elements.

Fig.3 demonstrates the shell of the transient process typical for inverters with low attenuation. The stationary mode is reached when the fractal is circumferentially moved closer to the limit point, the so-called alternative convergence of Eq. 10.

G. Electromagnetic process in an aperiodic inverter

Inverters, whose oscillatory systems are characterized by one real root only, have just one reactive element and develop an aperiodic process.

Their phase vector is determined by a real quantity Eq. 1 $p=-\varepsilon\theta_s$. The power series (Eq. 8) of the transient process and its limit (Eq. 6) have the same form, but are real-scalar. They can be presented as a separated case of their complex analogues.

H. Electromagnetic process in a complex inverter

The inverter, whose oscillatory system is characterized by several roots, develops a complex oscillatory process. The free oscillations are a sum of the elementary oscillations that determine their roots. The operations described must be interpreted by matrix operators.

III. Conclusions

- The free oscillations method is applicable for the mathematical interpretation of the inverter. Its application allows the formulation of the stationary mode and the power of the inverter [4], [5], [6].
- The first half-period, starting from zero initial conditions, is multiplied and functionally determines all subsequent half-periods in the inverter operation process.
- The start point is the result of the completed half-periods and determines the mode level (voltage and current) of the current half-period.
- The position of the start point depends on the relationship between the natural frequency and the timer frequency.
- The transient process is represented by a fractila (broken line), whose diminishing segments are at equal angles. The fractila tends to the start point of the stationary mode.

References

[1] B.D.Bedford, R.G.Hoft, Principles of Inverter Circuits. J.Willey & Sons, New York, 1964, chapt.3.

[2] V. A. Chvanov, Dynamics of Autonomous Inverters with Direct Commutation, Moscow, Energiya, 1978 (in Russian)

[3] M. H. Rashid, Power Electronics, Circuits, Devices and Applications. *Pearson Prentice-Hall International, Inc.,2004*, chapt. 8.

[4] H. P. Hinov, Accumulations in a Series Inverter, *Proceedings of the Technical University, XVII-1994*, pp. 158-164.

[5] H. P. Hinov, V. V. Rankovska, Analysis of an Asymmetric Inverter in Phase Space, *ICEST, 16-19 June 2004*, pp. 403-406, Bitola, Macedonia.

[6] H. P. Hinov, Short Cycle Inverter, (forthcoming).