Inverter Model and Decomposition in the 3D Space

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Abstract - **The magnetic coupling inverter is a typical object of power electronic. The differential equations from the third order are canonicated. The residuum's law has been applied for the first time, towards such object like inverter and it has been decomposed to its components.**

Keywords **– inverter, phase model.**

I. Introduction

The generated energy transferring through magnetic coupling is an often met approach in power electronics. The magnetic coupling is in the base of induction heating, where the inductor and the heated metal work as a transformer, except as a transformer for galvanic isolation, or for load fitting.

The influence of the magnetic coupling at inverters is often ignored [1] with the hypothesis that it does not change the oscillating process. The mathematical interpretation shows increasing the order of the differential equations and processes, typical for the complex inverters.

The aim of the present analysis is a decomposition of the object and the processes in it to their components. The analytic methods, connected with the theorem for the residuum's has been applied, and a three-dimensional phase space model has been built up.

II. Circuit, equations, solutions

The inverter with magnetic coupling (IM) – Fig. 1, has been met both independently and built in more complex structures. It consists of two magnetically coupled circles, primary and secondary.

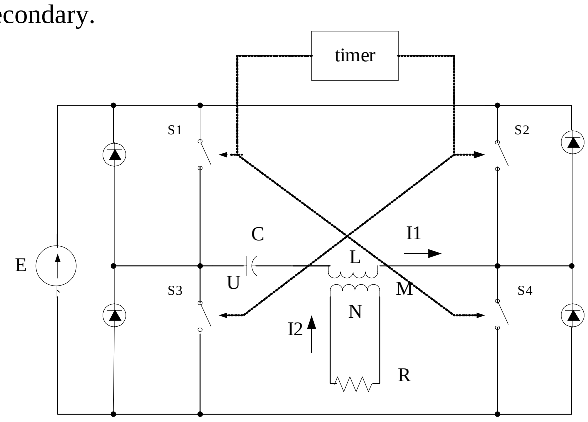

Fig. 1. Base circuit

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The differential equations are the following:

$$\begin{aligned} C(dU/dt) &= I_1 \\ L(dI_1/dt) + U - M(dI_2/dt) &= E \\ M(dI_1/dt) &= N(dI_2/dt) + RI_2 \end{aligned} \tag{1}$$

where the capacitor voltage *U* and the currents *I*, *H* are functions of the time *t*. After dividing by the derivatives and applying the substitutions:

Dispersing $\Sigma^2 = 1 - k^2$, connected with the coefficient of magnetic coupling $k = M/(LN)^{0,5}$;

Phase argument $\theta = \Omega t$, where $\Omega = (CL)^{-0,5}$;

Fading decrement $\varepsilon = (R/N)/\Omega$;

Dimensionless voltage $u = U/E$;

Dimensionless currents $i_1 = (I_1/E)(L/C)^{0,5}$, $i_2 = (I_2/E)(N/C)^{0,5}$.

The argument θ is limited by the boundaries of the half-period $0 \le \theta \le \theta_s$. The boundary is $\theta_s = (T_t/T_0)$, it is proportional of the time period T_t, divided by its own period $T_0 = 2\pi/\varpi$. Then Eq. (1) assumes the following:

$$\begin{aligned} du/d\theta &= i_1 \\ \Sigma^2 di_1/d\theta &= -u - \varepsilon k i_2 + 1 \\ \Sigma^2 di_2/d\theta &= -ku - \varepsilon i_2 + k \end{aligned} \tag{2}$$

The characteristic equation is as follows:

$$\Sigma^2 p^3 + \varepsilon p^2 + p + \varepsilon = 0 \tag{3}$$

The roots of Eq. (3) are divided by their frequency ϖ, which means that it is accepted for a conditional unit: $(\varpi = 1)[(-\delta + j),(-\delta - j),(-\rho)]$. The real constants δ and ρ express the fading (reduction) for a unit of angle – radian.

According to the theorem for the residuum's the solution of Eq. (2) is a function of the roots of Eq. (3) and its general view is the following:

$$\begin{aligned} u(\theta) &= A_u \exp(-\delta\theta)\cos(\theta + \alpha) + B_u \exp(-\rho\theta) - 1 \\ i_1(\theta) &= A_{i1} \exp(-\delta\theta)\cos(\theta + \beta) + B_{i1} \exp(-\rho\theta) \\ i_2(\theta) &= A_{i2} \exp(-\delta\theta)\cos(\theta + \gamma) + B_{i2} \exp(-\rho\theta) \end{aligned} \tag{4}$$

The addends or the components can be grouped into two vectors:

$$\mathrm{a}(\theta) = \mathrm{rot}(\theta) + \mathrm{ex}(\theta) - 1 \tag{5}$$

The first vector is rotating and it is called **rot**-vector. The other one represents the movement on a straight line and it is called **ex**-vector. They represent the two types of free oscillations (FO): sinusoidal and aperiodic, which exist in the inverter.

By the increasing of the argument the vectors reduce their modules and they get near to the point of the steady solution (-1, 0, 0). Although they develop in the 3D phase space (PS) each of the vectors is with lower order. **Ex**-vector is a segment in the space and it is one-dimensional, **rot**-vector is a two-dimensional.

The structure of Eqs. (4) and (5) presumes that its components develop independently from each other.

III. Phase Model

The phase model represents the regime parameters – currents and voltages, with their dynamics and interconnection. It is situated in the 3D PS, marked by a coordinate system (u, i_1, i_2). Eq. (4) describes a phase trajectory (PT) when increasing the argument θ. PT is a space line put round rotational body, given in Fig. 2.

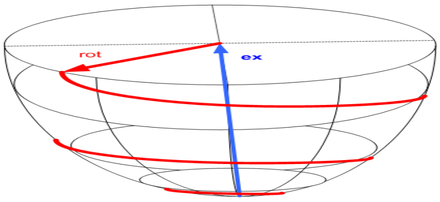

Fig. 2. Phase model

The body is flattened with less height corresponding to the **ex**-vector and with rounded vertex because of the bigger real root.

The vectors in the PS simultaneously express the regime (currents and voltages) and the energy which on the other hand is the main reason for the regime dynamics. The regime parameters (currents and voltages) are vectors and their belonging energy is a scalar. The quadrature of each vector in the PS defines a portion of energy [3].

Combining the regime and energy presentation defines the theorem for their mutual position:

If two vectors in the PS distribute the energy of a third vector – energy carrier, the two vectors are cathetuses of the energy-carrier. (6)

The three vectors make a right-angled triangle whose hypotenuse is the regime energy-carrier. Thus the quadrature of the hypotenuse, respectively the carried in energy, is distributed as the sum of the quadratures of the cathetuses.

The vectors (5) submit to the theorem (6), they are cathets and they divide the starting energy expressed by the hypotenuse – Figs. 2, 3 and 5.

If the inverter reaction is provoked in relation to its own aperiodic oscillation by substitution the current $i_2 = A\exp(-\rho\theta)$ and its derivative in Eq. (2), the coordinates of the **ex**-vector **ex**$[1,-\rho,k/(\sum{}^2\rho-\varepsilon)]$ are derived.

The **ex**-vector coordinates are defined only by the roots and the inverter circuit parameters. It defines an unchangeable straight line in the PS which is the “staunch” – the axis of the PM. It is extremely important that the **ex**-vector is a normal of the plane in which the **rot**-vector rotates.

IV. First Start

The first half period starts with the first activating of the switches S1 and S3 at zero starting conditions. The first half period has prevalent role as is in the development of the inverter, because on every next half period it reproduces itself on a higher energy level.

At the start the first equation of (4) is reduced to A+B=1. It expresses the distribution of the single x-axis segment between the two FO, presented by the vectors **rot**-vector and **ex**-vector. The single x-axis segment [(0, 0, 0)-(-1, 0, 0)] is the distance between the start point and the steady solution point of Eq. (4). It matches the voltage of the source E and it is the energy carrier which activates the free oscillations in the inverter.

There is a differential connection between the voltage u the primary current i_1, expressed by the first equation of Eqs. (1) and (2). This connection is applied in the second equation of (4) and for the start state of the ordinates it is derived: $\delta A - \rho B = 0$.

The two start equations are added in a system:

$$A+B=1 \qquad (4s)$$
$$\delta A - \rho B = 0,$$

from which the distributing constants of the voltage are defined:

$$A = \rho/(\delta+\rho)\,;\, B = \delta/(\delta+\rho) \qquad (7)$$

These are the two vectors voltages or their x-axes in the PS, depending only on the roots. The second equation in Eq. (4s) is about the primary current i_1:

$$i_{rot} = i_{ex} = (\delta\rho)/(\delta+\rho) \qquad (8)$$

It has one and the same start value and different directions at the two free oscillations. These currents are balancing and their sum expresses the zero starting condition of the start.

The projection of the **ex**-vector and the **rot**-vector in the main plane **u0i₁** are shown in Fig. 3. The projections of the two vectors form an obtuse angle faced towards the single x-axis segment. The x-axis segment, as a base of the projection (obtuse) triangle, is divided by its height on the mentioned dividing segments A and B. The proportion of their lengths is the same as that of the inverter roots: $A/B = \rho/\delta$.

The currents (8) are the height of the projection (obtuse) triangle. If its height is lengthened to the average geometric number of the dividing segments: $h= (AB)^{0,5}$, it will define the whole start triangle of the PS:

$$h = (\delta\rho)^{0,5}/(\delta+\rho) \qquad (9)$$

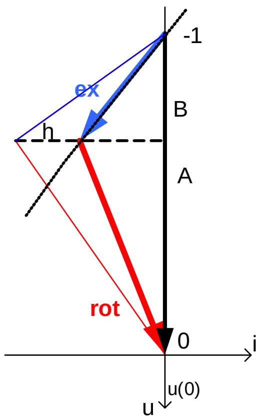

Fig. 3. Starting triangle

It is right-angled because it fulfils the condition of theorem (6) and it is in the main plane **u0i$_1$**, as it is shown in Fig. 3. The relation between the height of the start triangle and its projection is: $h/i=(\delta\rho)^{-0,5}$.

On the other hand the height of the start triangle in the PS is the vector sum of the two currents: $h^2=i_1^2+i_2^2$. The secondary current i_2 or the z-axis is:

$$i_2=(\delta\rho-(\delta\rho)^2)^{0,5}/(\delta+\rho)=i_1((1/\delta\rho)-1)^{0,5} \qquad (10)$$

The representing of the secondary current i_2 by the primary i_1 is possible in induction heating, where the secondary current develops in a massive and extremely heated part and it has no evident and accessible circuit for the ampere meters.

The dependence of the processes on the two currents vector sum: $h^2=i_1^2+i_2^2$ shows that the two currents influence on the FO as one resultant current. This leads to the conclusion that the separate free oscillations structure in their own elementary sub-spaces and that they are only included in the multi-dimensional PS.

The FO vectors in PS depend only on their roots:

$$\boldsymbol{ex}[\delta,-\delta\rho,-\delta\rho((1/\delta\rho)-1)^{0,5}]/(\delta+\rho) \qquad (11)$$

$$\boldsymbol{rot}[\rho,\delta\rho,\delta\rho((1/\delta\rho)-1)^{0,5}]/(\delta+\rho)$$

The vector coordinates x -axes (11) are influenced only by the distributing segments A and B. The rest of the coordinates are equal and balancing each other. They define the vectors in a way that except the regime values (voltage and currents) their adequate energy is also expressed.

The moduluses of (11) are the cathets of the single x-axis segment [(0,0,0),(-1,0,0)], which as a hypotenuse defines the start triangle:

$$|ex|=(\delta^2+\delta\rho)^{0,5}/(\delta+\rho)=(B)^{0,5} \qquad (12)$$

$$|rot|=(\rho^2+\delta\rho)^{0,5}/(\delta+\rho)=(A)^{0,5}$$

The moduluses (12) can be defined also by the distributing angle α - Fig.5, where:

$$tg\alpha=\delta/\rho; \quad |ex|=\sin\alpha; \quad |rot|=\cos\alpha \qquad (13)$$

An IM with a couplings coefficient k=0,6 and Q-factor=3 is taken as a numeric example, which roots are $\delta=0{,}08$ and $\rho=0{,}25$ [2]. The energy-carrier x-axis is divided between the **rot**-vector *A=0,76* and the **ex**-vector *B=0,24*. The dividing segments *A* and *B* express the distribution of the source voltage between the two inverter free oscillations. If *E=500 V*, then $A=\rho/(\delta+\rho)=25/33=76\%$ or 375 V activate the harmonic free oscillation, presented by the **rot**-vector, and the other 121 V activate the aperiodic free oscillation, presented in the PS by the **ex**-vector. It is just as if IM combines two autonomous inverters: serial inverter sourced with voltage E=379 V and aperiodic RC inverter, sourced with E=121 V. The primary current i_1 on the y-axis is 0,06, and the second i_2 on the z-axis is 0,42. The two currents in the two vectors are equal in length and with different signs:

ex [8,-2,-14]/(33) or [0.24,-0.06,-0.42];

rot [25,2,14]/(33) or [0.76,0.06, 0.42].

The divisor angle between **rot** and the x-axis in Eq. (13) is $\alpha=18°$; then the vector moduluses are: $|ex|=\sin\alpha=0{,}310$ and $|rot|=\cos\alpha=0.951$. The data is given in Table I.

TABLE I
VECTOR DATA

Distribution	**ex**	**rot**
Voltage %	24	76
Relative coordinates	8,-2,-14	25,2,14
Modulus %	31	95.1

V. Relative Coordinate System

A relative coordinate system (RCS) is selected, where FT is in canonic position as is shown in Fig. 5. The new RCS has been got from the old base coordinate system (BCS) by a rotation. The rotation orientates the z-axis of the new RCS towards its matching with the directress of the **ex**-vector, given in Fig. 4, where the starts of the two coordinate systems become common.

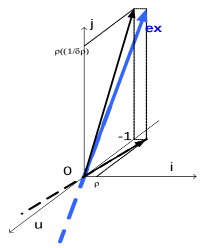

Fig. 4. **Ex**-vector in the PS

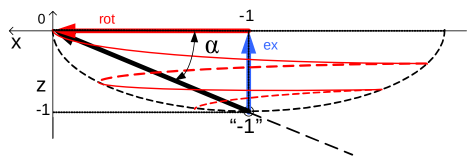

Fig. 5. Third main plane

The x-axis of the BCS is a half-line in the third projection plane (x0z) – Fig.5. The steady solution (-1) is on that half-line, where the center of the PT is, except the basic point (0,0,0). The cathets of the start triangle or the vectors of the FO are oriented towards the x-axis and the z-axis.

The vectors of the FO are connected by the energy-carrier – the hypotenuse, but they develop in their own independent spaces of the RCS. The **ex**-vector is only on the z-axis and the rest coordinate axes are for the **rot**-vector. It develops only in the first main plane **x0y.**

Magnifying scales $1/\cos\alpha$ for the x-axis and $1/\sin\alpha$ for the y-axis are introduced, so the steady point "**-**1" in BCS is reproduced also in "**-**1" on the x-axis and on the z-axis.

VI. Dynamics of the First Phase Vector

The development of the processes is expressed by the PV dynamics (Eq. 5), which was decomposed to the vectors of the two FO and it was structured in the RCS. It begins its development during the first half-period, starting from zero starting conditions. This "primary" development deserves special attention, because in every next half-period it reproduces itself on a higher level.

The aperiodic FO vector ***- ex*** θ**,** forms the z-axis in RCS:

$$z(\theta) = -1 + \exp(-\rho\theta) \tag{14}$$

The periodic FO vector, rotating $\boldsymbol{r}(\theta)$ (**rot-**vector), develops in the main plane **x0y** of the RCS. The vector $\boldsymbol{r}(\theta)$, decomposed by the x-axis and the y-axis vectors in a matrix, assumes the following:

$$\vec{r}(\theta) = \begin{bmatrix} r_x \\ r_y \end{bmatrix} = \begin{bmatrix} \exp(-\delta\theta)\cos\theta \\ \exp(-\delta\theta)\sin\theta \end{bmatrix} \tag{15}$$

The first main plane **x0y** is express as complex, where Eq. (15) assumes the complex view:

$$\dot{r}(\theta) = \exp((-\delta + j)\theta) \tag{16}$$

Eq. (5) changes, expressed as a matrix of coordinate vectors:

$$\vec{F}_1\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \begin{bmatrix} \mathrm{Re}(\dot{r}(\theta)) \\ \mathrm{Im}(\dot{r}(\theta)) \\ z(\theta) \end{bmatrix} \tag{17}$$

The periodic vector complex equation (16) allows reducing Eq. (17):

$$\vec{F}_1(\dot{r}, z) = -\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \dot{r}(\theta) \\ z(\theta) \end{pmatrix} \tag{18}$$

The x-axis and the y-axis are expressed by the real and the imaginary parts of the complex operator $\dot{r}(\theta)$ (Eq. 16), and the z-axis by the scalar $z(\theta)$ (Eq. 14).

The final point of Eq. (!8) is a combinations of the final points of Eqs. (16) and (14) at $\theta = \theta_s$:

$$F_1(\theta_s) = \begin{pmatrix} -1 + \dot{q} \\ -1 + q \end{pmatrix} \tag{19}$$

where the complex *q'=exp(p')* is the final value of Eq. (16) $p' = (-\delta + j)\theta_s$.

The real *q=exp(p)* is the final value of Eq. (14) $p = -\rho\theta_s$.

The start point (0,0,0) and the final point (19) define the PT segment, passed by the primary PV for the half-period interval $(T_t / 2 = \theta_s / \omega))$.

The primary PV dynamics finishes with so called commutation. The timer activates the inverse bridge switches at the final point and it initiates a source direction change, represented by a change of the signs in Eq. (19).

The second half-period starts at point:

$$S_2 = \begin{pmatrix} 1 - \dot{q} \\ 1 - q \end{pmatrix} \tag{20}$$

VII. Components of the IM (Sub invertors)

The upper operations have been made simultaneously, but they have been separated in two independent "containers".

The development of the periodic FO or **rot-**vector is exactly the serial inverter development [3], [4]. The aperiodic FO simultaneous development, represented by the **ex-**vector, is a one-dimensional elementary inverter development [5].

These are the two sub-inverters – Fig. 6, to which IM is decomposed. They develop synchronous, but independently from each other. Their common connection is only the injected energy at the commutation transition, which they distribute each other. The quadratic forms of this distribution have been proved.

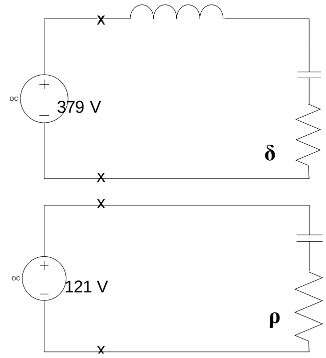

Fig. 6. Sub-inverters

The regime distribution of the source voltage has been demonstrated in the example and it has been shown in Fig. 7.

The serial sub-inverter has got a stationary mode, defined by the commutation point: S_s=2,085(26,5^0)=1,866+j0,93; scale M_a=0,5CE^2=0,144[J]; power P=14,42 kW at 10 kHz [4]. The aperiodic sub-inverter has got a stationary mode, defined by the point S_a=0,63/1,37=0,462, that assigns it power P=4,56 kW.

VIII. Conclusions

1. The theorem of the residuum's can be applied on the power inverters.
2. The complex inverters can be decomposed on components – sub-inverters from first and second order.
3. The inverter phase model is effective in the three-dimensional space too.
4. The free oscillation processes develop in independent relative sub-spaces embraced by the three-dimensional phase space.

References

[1] M. H. Rashid, "Power Electronics, Circuits, Devices and Applications", Pearson Prentice-Hall International, Inc., 2004.

[2] Х. П. Хинов, "Характеристично уравнение на инвертор с индуктивна връзка", *UNITECH'04 Conference Proceedings*, 1-222, Gabrovo, Bulgaria, 2004.

[3] H. P. Hinov, V. V. Rankovska, "Analysis of an Asymmetric Inverter in Phase Space", *ICEST 2004 Conference Proceedings*, pp. 403-406, Bitola, Macedonia, 2004.

[4] Х. П. Хинов, "Фазов анализ на симетричен инвертор". Известия на ТУ Габрово /под печат/.

[5] Ch. P. Chinow, "Geometrische Modellierung elektromagnetischer Prosesse in einfachen Wechselrichtern", 4-WK, pp. 19-21, Mittweida, DDR, 1976.