Investigation and Frequency Characteristics of the High Order Voltage – Fed RLC Inverter for Induction Heating

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Abstract - **A voltage-fed RLC inverter with free - wheeling diodes for induction heating having a high order circuit is studied. The load is replaced by serially connected resistance and reactance. The inverter parameters are determined. The frequency controlling characteristics are obtained and confirmed. It is suggested to maintain resonance of the load.**

Keywords - **Voltage-fed inverter, RLC inverter, High-order circuit, Frequency characteristics, Resonance.**

I. INTRODUCTION

The voltage-fed RLC inverter with free - wheeling diodes (Fig. 1) applies widely for induction heating and melting due to the limited voltage across the power semiconductor devices even for large variations of the load. The circuit can operate either in oscillatory, critical or over damped mode depending on its parameters. In over damped, critical and oscillatory mode when the controlling frequency $\omega = 2\pi f$ is higher than the inherent resonant frequency ω_0 of the serial inverter circuit the inverter must be constructed with fully controllable devices (transistors or GTO thyristors) and free - wheeling diodes. Only in oscillatory mode when the inverter operates at a frequency lower than the inherent resonant frequency the power semiconductor devices (shown on the figure as switches) may be conventional thyristors making the power circuit cheaper. But thyristors require sufficient circuit turn-off time and have limitations concerning di/dt and dv/dt. The voltage-fed RLC inverter has not been analyzed in the practical case when the inverter circuit is of high order due to the influence of the load circuit [1,2]. The frequency characteristics of the main quantities of such an inverter when the controlling (output) frequency varies are also not known.

That's why this paper is aimed at performing an investigation of the high order power circuit of the voltage-fed RLC inverter with free – wheeling diodes for induction heating and melting and obtaining the frequency control characteristics of the main parameters. This will allow a method for control of the inverter operation to be suggested. Simulations and experiments should confirm the results.

II. ASSUMPTIONS

A bridge circuit is under study. Other configurations can be easily reduced to the bridge one. The power losses in the inverter are neglected. The commutation of the power semiconductor devices is instantaneous. The quality factor of the load circuit is sufficiently high that the voltage across the load has a close to the sine wave shape. The method of transitory values is applied.

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III. REPRESENTATION OF THE LOAD CIRCUIT

A parallel equivalent circuit containing the frequency dependent active resistance R_l and inductance L_l represents the inductor heater:

$$R_l = R_{lr}\left(\frac{\omega}{\omega_{lr}}\right)^{n_1} \tag{1}$$

$$L_l = L_{lr}\left(\frac{\omega}{\omega_{lr}}\right)^{n_2 - 1} \tag{2}$$

where ω_{lr} is the resonant frequency of the load circuit R_l, L_l, C_l

$$\omega_{lr} = \frac{1}{\sqrt{C_l L_{lr}}} \tag{3}$$

R_{lr}, L_{lr} are the inductor parameters for the resonant frequency. The values of n_1 and n_2 are given in [3].

If the quality factor of the load circuit R_l, L_l, C_l at resonance

$$Q_r = R_{lr}\sqrt{\frac{C_l}{L_{lr}}} \tag{4}$$

is high enough the load voltage is almost sine wave and the load can be represented by a serial equivalent circuit containing the parameters R_{eq}, X_{eq} (Fig. 2).

$$\frac{R_{eq}}{R_{lr}} = \frac{\left(\frac{\omega}{\omega_{lr}}\right)^{n_1}}{1 + Q_r^2\left(\frac{\omega}{\omega_{lr}}\right)^{(2n_1 - n_2 + 1)}\left(\frac{\omega}{\omega_{lr}} - \frac{\omega_{lr}}{\omega}\right)^2} \tag{5}$$

$$\frac{X_{eq}}{R_{lr}} = \frac{Q_r\left(\frac{\omega}{\omega_{lr}}\right)^{\left(2n_1 + \frac{1-n_2}{2}\right)}\left(\frac{\omega_{lr}}{\omega} - \frac{\omega}{\omega_{lr}}\right)}{1 + Q_r^2\left(\frac{\omega}{\omega_{lr}}\right)^{(2n_1 - n_2 + 1)}\left(\frac{\omega}{\omega_{lr}} - \frac{\omega_{lr}}{\omega}\right)^2} \tag{6}$$

Below the resonance of the load circuit the equivalent reactance X_{eq} is in fact a serial equivalent inductance L_{eq}. Therefore the serial inverter circuit consists of $R = R_{eq}$, $L = L_{eq} + L_S$, $C = C_S$. Above the resonance of the load the equivalent reactance X_{eq} is a serial equivalent capacitance C_{eq} and the serial inverter circuit consists of $R = R_{eq}$,

$L = L_S$, $C = \dfrac{C_{eq} C_S}{C_{eq} + C_S}$. Then the voltage across the inverter capacitor C is divided between the capacitors C_S and C_{eq} in accordance with the theory of electrical engineering [4].

$$\frac{L_{eq}}{L_{lr}} = \frac{Q_r^2 (\frac{\omega}{\omega_{lr}})^{(2n_1 + \frac{1-n_2}{2})} (\frac{\omega_{lr}^2}{\omega} - 1)}{1 + Q_r^2 (\frac{\omega}{\omega_{lr}})^{(2n_1 - n_2 + 1)} (\frac{\omega}{\omega_{lr}} - \frac{\omega_{lr}}{\omega})^2} \quad (7)$$

$$\frac{C_{eq}}{C_l} = \frac{1 + Q_r^2 (\frac{\omega}{\omega_{lr}})^{(2n_1 - n_2 + 1)} (\frac{\omega}{\omega_{lr}} - \frac{\omega_{lr}}{\omega})^2}{Q_r^2 (\frac{\omega}{\omega_{lr}})^{(2n_1 + \frac{1-n_2}{2})} (\frac{\omega^2}{\omega_{lr}^2} - 1)} \quad (8)$$

IV. MODE OF OPERATION

The mode of operation is determined from the diagram shown in Fig. 3 [5], where the load coefficient of the inverter is

$$B = \frac{\sqrt{1 + \omega^2 C^2 R_{eq}^2}}{\omega^2 LC} \quad (9)$$

and the load power factor of the inverter is

$$\cos\varphi = \frac{1}{\sqrt{1 + tg^2 \varphi}} = \frac{\omega C R_{eq}}{\sqrt{1 + \omega^2 C^2 R_{eq}^2}} \quad (10)$$

V. PARAMETERS OF THE INVERTER CIRCUIT

The fourth order inverter circuit is reduced to a second order RLC circuit. Its parameters are calculated as follows:

The damping coefficient is

$$\delta = \frac{R_{eq}}{2L} \quad (11)$$

The inherent frequency of the serial inverter circuit for:

- Oscillatory mode ($R_{eq} < 2\sqrt{L/C}$) is

$$\Omega = \omega_0 = \sqrt{\frac{1}{LC} - \delta^2} \quad (12)$$

- Over damped mode ($R_{eq} > 2\sqrt{L/C}$) is

$$\Omega = \sqrt{\delta^2 - \frac{1}{LC}} \quad (13)$$

- Critical mode ($R_{eq} = 2\sqrt{L/C}$) is

$$\Omega = \delta \quad (14)$$

The angle θ_2 ($\theta_2 > 0$) is

$$\theta_2 = \frac{\Omega}{\omega}\pi = \frac{\pi}{n} \quad (15)$$

The following functions and constants are introduced: $f_S(x)=\sin x$, $f_C(x)=\cos x$, c=+1 for oscillatory mode; $f_S(x)=shx$, $f_C(x)=chx$, c=-1 for over damped mode; $f_S(x)= x$, $f_C(x)=1=const$, c=0 for critical mode.

Then the a coefficient and the angle θ_1 ($\theta_1 > 0$) can be determined from

$$a = \frac{f_S(\theta_1)/f_C(\theta_1)}{1 - \frac{\delta}{\Omega} f_S(\theta_1)/f_C(\theta_1)} = \frac{f_S(\theta_2)}{e^{\frac{\delta}{\Omega}\theta_2} + f_C(\theta_2) - \frac{\delta}{\Omega} f_S(\theta_2)} \quad (16)$$

The generalized coefficient of hesitation is

$$K = \frac{1}{1 + \exp(-\frac{\delta}{\Omega}\theta)[(ca + \frac{\delta}{\Omega} + a\frac{\delta^2}{\Omega^2}) f_S(\theta_2) + f_C(\theta_2)]} \quad (17)$$

The peak and initial voltages across the inverter capacitor C are

$$V_{Cm} = [2(\frac{K}{K_1} - K) - 1]V_d \quad (18)$$

where K_1 is the value of K but for angle θ_1 instead of θ_2.

$$V_0 = (2K - 1)V_d \quad (19)$$

The average value of the inverter input current is

$$I_d = \frac{V_d}{\Omega L} \cdot \frac{1}{\theta_2} \cdot \frac{2(2K-1)}{\frac{\delta^2}{\Omega^2} + c} \quad (20)$$

The average value of the inverter input current between 0 and θ_1 is

$$I_1 = abs[\frac{V_d}{\Omega L} \cdot \frac{1}{\theta_2} \cdot \frac{2(2K_1 - 1)}{\frac{\delta^2}{\Omega^2} + c} \cdot \frac{K}{K_1}] \quad (21)$$

The average value of the inverter input current between θ_1 and θ_2 is

$$I_2 = abs[\frac{V_d}{\Omega L} \cdot \frac{1}{\theta_2} \cdot \frac{2}{\frac{\delta^2}{\Omega^2} + c} \cdot (\frac{K}{K_1} - 1)] \quad (22)$$

I_1 and I_2 are used for calculation of the average currents through the power semiconductor devices.

Initial inverter current for a half-period

$$I_0 = 2aK \cdot \frac{V_d}{\Omega L} \quad (23)$$

For $\omega \le \omega_0$ the thyristor circuit turn-off time is

$$t_{q.c.} = \frac{(\theta_2 - \theta_1)[rad]}{\omega_0} \quad (24)$$

The transitory value of the inverter current for a half-period is expressed by

$$i(\theta) = \frac{V_d}{\Omega L} .2K \exp(-\frac{\delta}{\Omega}\theta)[(1 + a\frac{\delta}{\Omega}) f_S(\theta) - a f_C(\theta)] \quad (25)$$

where $\theta = \Omega t$, t is time.

The transitory value of the inverter capacitor voltage C for a half-period is

$$v_C(\theta) = V_d \{1 - 2K \exp(-\frac{\delta}{\Omega}\theta)[(ca + \frac{\delta}{\Omega} + a\frac{\delta^2}{\Omega^2}) f_S(\theta) + f_C(\theta)]\} \quad (26)$$

VI. LOAD VOLTAGE DETERMINATION

The RMS of voltage across the load circuit R_l, L_l, C_l can be determined from its harmonic components as follows

$$V_l = \sqrt{\sum_{m=1,3,5,...} V_{l(m)}^2} \quad (27)$$

Each harmonic component of the load voltage is expressed by

$$V_{l(m)} = \frac{V_d}{\omega_0 L} \cdot \frac{c_{(m)}}{\sqrt{2}} \cdot \sqrt{R_{eq(m)}^2 + X_{eq(m)}^2} \quad (28)$$

where $R_{eq(m)}$ and $X_{eq(m)}$ are calculated from Eqs. (5) and (6), exchanging $\frac{\omega}{\omega_{lr}}$ with $\frac{m\omega}{\omega_{lr}}$.

The normalized peak value of the m – harmonic component of the inverter current is

$$c_{(m)} = \sqrt{a_{(m)}^2 + b_{(m)}^2} \quad (29)$$

The expressions for $a_{(m)}$ and $b_{(m)}$ are rather complicated functions of m, ω / Ω and δ / Ω. They can be derived from Eq. (25) in accordance with [4]. These expressions are known to the authors but for the sake of brevity are not given here.

VII. FREQUENCY CHARACTERISTICS

A MATLAB program processes all the mathematical information describing the steady state operation of the inverter together with its load circuit in the allowed frequency range. The frequency characteristics of the inverter are obtained and graphically displayed in Fig. 4 for the particular case corresponding to a practically implemented inverter with the following data: V_d=500 V; L_S=0.3 mH; C_S=4 μF; R_{lr}=4 Ω, f_{lr}=4000 Hz, $\cos\varphi_{lr}$=0.24254, n_1=0, n_2=1, f=3000 – 4800 Hz. The first set of graphics shows: five times the average input current of the inverter $5I_d$ [A] (solid line), the load voltage V_l [V] (dotted line), five times the circuit turn-off time of the thyristors $5t_{q.c.}$ [μS] (dashed line). The second set of graphics shows: one fifth of the peak serial capacitor voltage $V_{CSm}/5$ [V] (solid line), twice the RMS value of the inverter current $2I_S$ [A] (dotted line), five times phase angle of the load circuit $5\varphi_{eq} = 5 arctg(X_{eq} / R_{eq})$ [deg] (dashed line). The same MATLAB program can calculate frequency characteristics for different input data and modes of operation. These characteristics are original. Other parameters of the circuit can also be calculated and displayed. The calculation is more precise if the shape of the load voltage is closer to the sine-wave form. That is checked for each particular frequency.

Many conclusions about the inverter operation can be drawn from the frequency characteristics. In general the increase of the controlling frequency leads to increases of the input current (power), load voltage, peak serial capacitor voltage and RMS value of the inverter current, and to decreases of the circuit turn-off time of thyristors and phase angle of the load. But around the resonant frequency of the load the character of the most functions is opposite and the changes of the corresponding parameters are not so large. Therefore if the parameters of the load R_l, L_l vary during the induction heating process it is advisable to maintain a resonance of the load circuit R_l, L_l, C_l by exercising an influence on the inverter controlling frequency [6].

VIII. DIRECT SIMULATION OF THE STEADY STATE

The inverter steady state can be directly simulated by applying the method described in [7]. This method is experimentally confirmed. According to it the initial conditions of the circuit state variables for the steady state and continuous inverter current mode (most widely applied) can be calculated using a matrix exponential from:

$$[x(0+)] = ([Co] - \exp([A].T/2))^{-1}.$$
$$.([1] - \exp([A].T/2)).(-[A])^{-1}.[B].[V_d] \quad (30)$$

where $[A]$ and $[B]$ are the state matrixes of the circuit, $[V_d]$ is the vector of the supplying voltages (constant), $T/2 = 1/(2f)$ is the half-period, [1] is a unity matrix. $[Co]$ is a diagonal matrix of the same order as $[A]$, where the element Co_{ii}=-1 if it relates state variables in the beginning and in the end of the half period for the a.c. circuit of the inverter, and Co_{ii}=+1 if it relates state variables in the d.c. circuit of the inverter. For the exceptional discontinuous inverter current mode the calculation is more complicated but is also possible according to [7]. For the inverter under study the state variables vector is

$$[x] = \begin{bmatrix} u_{C_S} & i_{L_S} & u_{C_l} & i_{L_l} \end{bmatrix}^t \quad (31)$$

and the state matrixes are

$$[A] = \begin{bmatrix} 0 & 1/C_S & 0 & 0 \\ -1/L_S & 0 & -1/L_S & 0 \\ 0 & 1/C_l & -1/(R_l C_l) & -1/C_l \\ 0 & 0 & 1/L_l & 0 \end{bmatrix} \quad (32)$$

$$[B] = \begin{bmatrix} 0 & 1/L_S & 0 & 0 \end{bmatrix}^t \quad (33)$$

Co_{ii}=-1 for i=1;2;3;4.

After determining the initial conditions of the state variables for the steady state the state variables themselves are calculated as functions of time for one half-period from:

$$[x(t)] = \exp([A]t).[x(0+)] +$$
$$+ ([1] - \exp([A]t)).(-[A]^{-1}).[B].[V_d] \quad (34)$$

Taking into account Eqs. (30) - (34) a MATLAB program is created for direct simulation of the steady state processes in the inverter under study. The results from the MATLAB programs for calculating the frequency characteristics and for simulation of the steady state mode are compared. The comparison shows that both results are in good agreement. That confirms the correctness of the analysis, the frequency characteristics and the program for their calculation. The detailed simulation results for the power inverter are graphically displayed (Fig. 5) for the steady state and f=4000 Hz. The first set of diagrams shows: five times the inverter current i_{CS} [A] (solid line) and twice the load voltage v_l [V] (dotted line). The second set of diagrams shows: the serial capacitor voltage v_{CS} [V], twice the voltage across the first power electronic device v_{VS1} [V] (dashed line), twice the current through the load inductance i_{Ll} [A] (dotted line).

IX. A METHOD FOR CONTROL

As it was said when the load parameters R_l, L_l vary during the induction heating it is recommended to maintain resonance (φ_{eq}=0) of the load circuit R_l, L_l, C_l. This is done by following the phase angle between the load voltage and the inverter current, and influencing the controlling frequency. But the inverter current is distorted and has intervals of interruption in discontinuous inverter current mode. That is why it is advisable to follow the phase angle between the current through the load capacitor C_l – i_{Cl} and the voltage of the serial capacitor C_S - v_{CS}. For resonance these signals are in anti-phase but can be made to be in phase by proper connections of the windings of the measuring feedback transformers. The controlling system is fully described in [6].

X. CONCLUSION

An investigation of the high order power circuit of a voltage-fed RLC inverter with free – wheeling diodes for induction heating is performed. The original frequency control characteristics of the main parameters of the inverter circuit are calculated and graphically displayed. A MATLAB simulation program confirms the frequency characteristics. A method for control of the inverter operation is suggested.

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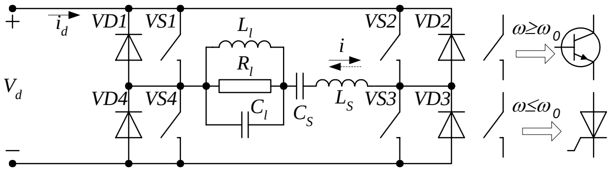

Fig. 1. Voltage – fed RLC inverter with free – wheeling diodes.

Fig. 2. Serial equivalent circuit of the load (L_l, R_l, C_l).

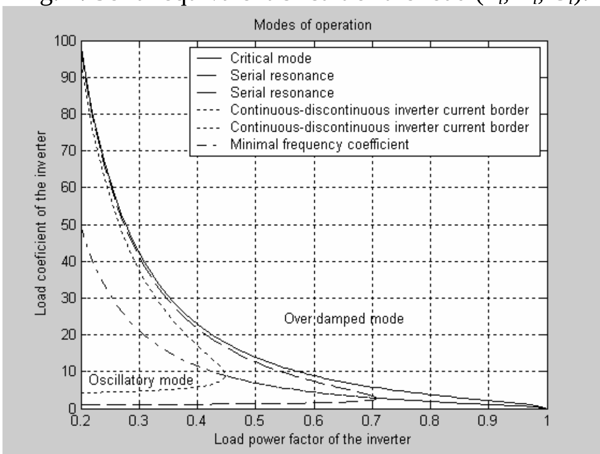

Fig. 3. The diagram of the inverter modes of operation.

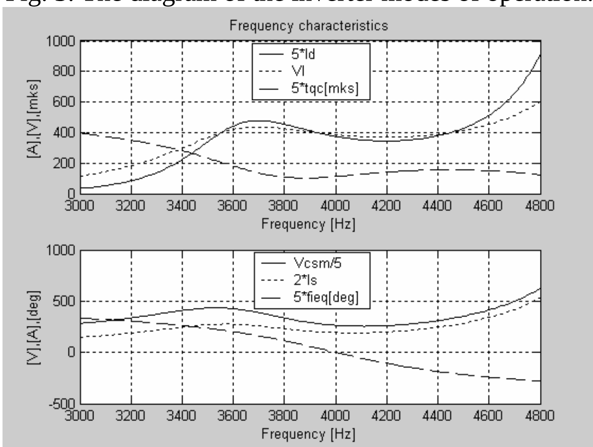

Fig. 4. Frequency characteristics of the inverter.

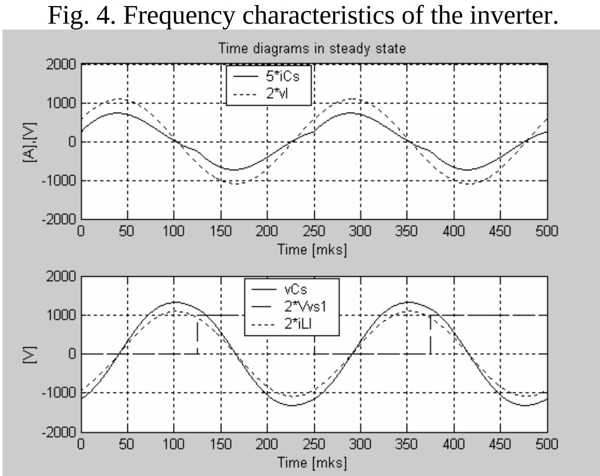

Fig.5. Detailed simulation results at f=4000 Hz.