

# Short Cycle Inverter

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**Abstract** - The paper determines the dependence between inverter frequency and cycle. It formulates the stationary mode based on the symmetry at the bridge circuit. The phase model has been built and the inverter characteristics, recuperation and control have been proved. A relation has been formulated between the inverter mode and energy. A map has been drawn out, covering the mode and power of all short cycle inverters. The following conclusions have been drawn: The stationary mode in the case of bridge inverters can be formulated as an effect of functional symmetry; The transistor is only activated after the current in the diode on the same bridge arm has been reduced to zero; The diode is switched on when the transistor on the adjacent arm of the bridge is deactivated (switched off); The inverter is only controlled by the trailing edges of the gate pulses; The diodes of short cycle inverters perform a typical recuperative process; Soft commutation is practically impossible; The inverter power depends only on the voltage of the start commutation point.

**Keywords** - Inverter, Phase model.

## I. INTRODUCTION

The series inverter /SI/ is a source of high-frequency energy and a fundamental object of research in power electronics [1]. The inverter forms its operation cycle within the timer half-period  $T_t/2$ . The object of the present article is the SI which generates frequency higher than its own, as a result of which it develops a shortened operation cycle.

The aim is to formulate the stationary mode, which, in addition to voltages and currents, should cover the inverter power.

The phase analysis method is used for this purpose, and its basic postulates are applied to the SI [2]. The applications there refer to inverters which operate at a frequency lower than their own and develop a lengthened operation cycle ( $\theta_s = \pi + \alpha$ ).

The present article is an expansion of [2] into a mode area, where the object has different properties.

A series bridge inverter with reverse diodes /SI/ is shown in Fig. 1. Normally the switches are transistors controlled by a timer, which determines the generated frequency. The timer closes the switches along one of the bridge diagonals and opens the switches along the other diagonal at intervals  $T_t/2$  – half-periods that determine the generated frequency. This

causes a periodical change in the direction of source E relative to the resonant CLR circuit and free oscillations, which are the object of the present article.

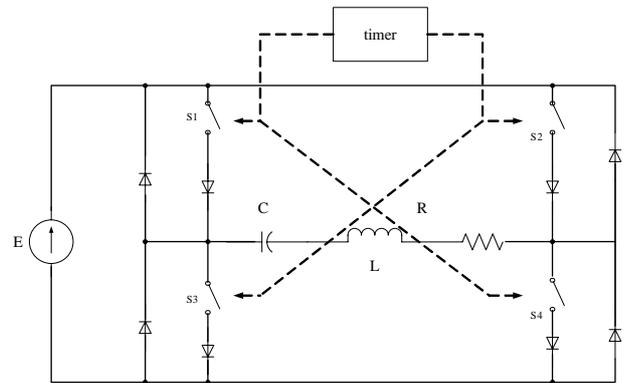


Fig.1. Series inverter circuit

## II. EQUATION AND PARAMETERS

The differential equation of the inverter is:

$$L(dI/dt) + RI + (1/C)\int I dt = E$$

The roots of the characteristic equation determine:

- Natural frequency:  $\omega = [(LC)^{-1} - (R/2L)^2]^{0.5}$
- Attenuation:  $\varepsilon = (R/2L)/\omega = 1/2Q$

A priority of the method applied is the replacement of the current time by a current angle called “phase”:  $\theta = \omega t$ .

Just like the clock represents time by means of an angle (thirty degrees mean an hour), in the same way the phase angle represents the current time in the inverter.

The phase is a limited argument:  $0 \leq \theta \leq \theta_s$ . The maximum phase, called cycle:  $\theta_s = \pi (T_t/T_0)$ , is proportional to the timer period  $T_t$ , relative to the own period  $T_0 = 2\pi/\omega$ . The cycle  $\theta_s$  is the angular path traveled by the “inverter clock hand” over the timer interval  $T_t/2$  or  $\theta_s < \pi$ , ( $\theta_s = \pi - \alpha$ ).

The four circuit parameters (C, L, R and period  $T_t$ ) have been reduced to just the two dimensionless quantities – attenuation  $\varepsilon$  and cycle  $\theta_s$ . These are the degrees of freedom or inverter coordinates. They are combined in a single complex parameter:

$$p = (-\varepsilon + j) \theta_s \quad (1)$$

The solutions to the differential equation are represented in dimensionless quantities:

- Capacitor voltage:

$$u(\theta) = -1 + u_s \exp(-\varepsilon\theta) \cos \theta$$

where:  $u = U/E$ ,  $u_s$  arbitrary start voltage.

- Current across inductance:

$$i(\theta) = i_s \exp(-\varepsilon\theta) \sin \theta$$

where:  $i = I(L/C)^{0.5}/E$ ,  $i_s$  arbitrary start current.

The two mode quantities are represented together by a single complex quantity, the real part of which is the voltage, and the imaginary one is the current [3].

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$$F(\theta) = -1 + (1 + S_n) \exp((- \varepsilon + j) \theta) \quad (2)$$

The start significations  $u_s$  and  $i_s$  correspond to a zero phase  $\theta = 0$  and refer to a random number (n) of the half-period  $F(0) = S_n$ . The start complex point is

$$S_n = (u_s + j i_s)$$

Both the combination of voltage and current, and their equivalent complex form Eq. (2) define the phase vector /PV/. It reduces its module when the phase  $\theta$  increases and tends to reach the “stable” point  $u(\infty) = -1$ ,  $i(\infty) = 0$ .

When in the complex plane, the PV rotates and, although its hand gets shorter, it represents the “inverter clock”. The peak of the PV describes a spiral, which starts from the starts point  $S_n$  and tends to the stable point  $(-1, j0)$ .

The spiral segment in inverters has a cycle of about  $180^\circ$ ,  $\theta_s = \pi \pm \alpha$ , which, substituted in Eq. (2) determines the final point of the half-period:

$$F(\theta_s) = -1 + (1 + S_n) \exp(p) \quad (3)$$

When in the final point Eq. (3), the timer activates the inverse switches of the bridge and causes the so-called commutation in the inverter. The commutation causes a change in the direction of the source, which can be expressed by a change in the signs of Eq. (3), therefore the next half-period starts from the point:

$$S_{n+1} = 1 - (1 + S_n) \exp(p) \quad (4)$$

### III. STATIONARY MODE

In the stationary mode each of the following half-period reproduces the preceding one, which only requires the reproduction of the start point:

$$S_{n+1} = S_n = S$$

The reproduction principle applied in Eq. (4) determines the start commutation point describing the inverter stationary mode:

$$S = (1 - \exp(p)) / (1 + \exp(p)) \quad (5)$$

The point and the mode depend only on the two inverter coordinates, attenuation  $\varepsilon$  and phase  $\theta_s$ . The start point can also be represented in a hyperbolic form:

$$S = \text{th}(-p/2) \quad (6)$$

The interpretation of the general dependences under the condition of a short cycle  $\theta_s = \pi - \alpha$  specifies (1):

$$p = -\exp(-\varepsilon(\pi - \alpha)) \exp(-j \alpha) \quad (1.1)$$

The substitution of Eq. (1.1) in Eq. (5) or Eq. (6) determines the position of S, similar to p, in the fourth quadrant, in the case of negative voltage and negative current.

The start of the phase trajectory is related to the commutation in the inverter or the replacement of the bridge arms. The above-mentioned negative current before the start commutation point can only be the current of the inverse transistor, and immediately after the commutation the negative current can only be transferred into the reverse diode. Hence the commutation in a short cycle inverter is connected to the deactivation (switching off) of the transistor and the transfer of current in the diode of the bridge adjacent arm. The commutation is an effect of the trailing edge of the gate pulse.

The relationships proved above form a complete mathematical model of the inverter stationary mode.

### IV. PHASE MODEL

The geometric interpretation of the mathematical dependences is the phase model /PM/, which visualizes the dynamics and the relationships in the inverter.

The phase trajectory /PT/ of odd-numbered half-periods is described by the vector Eq. (2), which is centered on the abscissa in point -1. The vector starts in point S and, rotating counter-clockwise, completes the upper spiral segment in point -S.

The PT or the spiral of even-numbered half-periods is an inverse repetition of the preceding one:  $F(\theta_s + \theta) = -F(\theta)$ . The spiral of even-numbered half-periods intersects the first closed curve, broken in commutation, in points S and -S.

The commutation is reflected by the PM as a change in the vector center.

Fig. 2 presents an example referring to inverters with Q-factor = 2.5 and cycle  $\theta_s = 2.5$  ( $143^\circ$ ), which corresponds to frequency relation  $T_t/T_0 = 0.8$ .

The ordinate of the rotating vector represents the current and its function can be traced from the right-hand intersection point with the abscissa. The transistor current there is zero and increases in the first quadrant. After its maximum  $i_m = 2.15$ , with phase  $\theta_{im} = 80^\circ$ , the current is completed in point -S with coordinates: voltage  $u_s = -1.65$  and current  $i_s = 1.85$ . The transistorized segment has a duration of  $\theta_t = 110^\circ$ . The transistor is switched off in commutation point -S and the current is taken over by the diode in the bridge inverse wing.

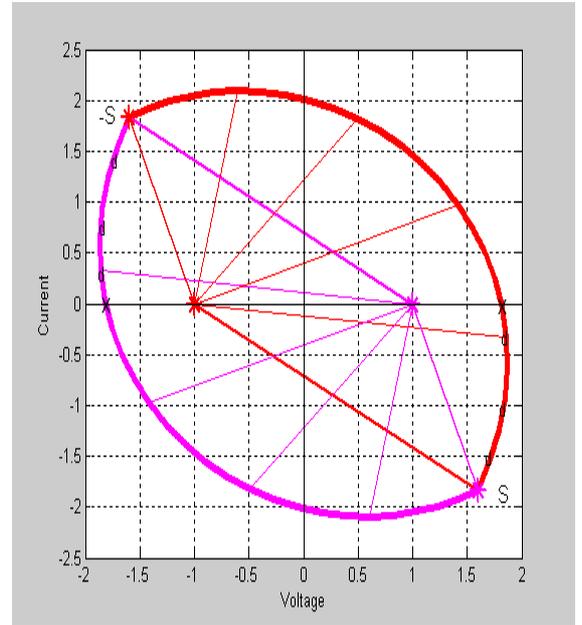


Fig.2. Phase model

The diode segment is positioned in the second quadrant and continues  $\theta_a = 33^\circ$  ( $\theta_s = \theta_t + \theta_a$ ). The diode segment  $\theta_a$  is determined relative to the inverse center +1.

The inverse diode is switched off when the current is reduced to zero.

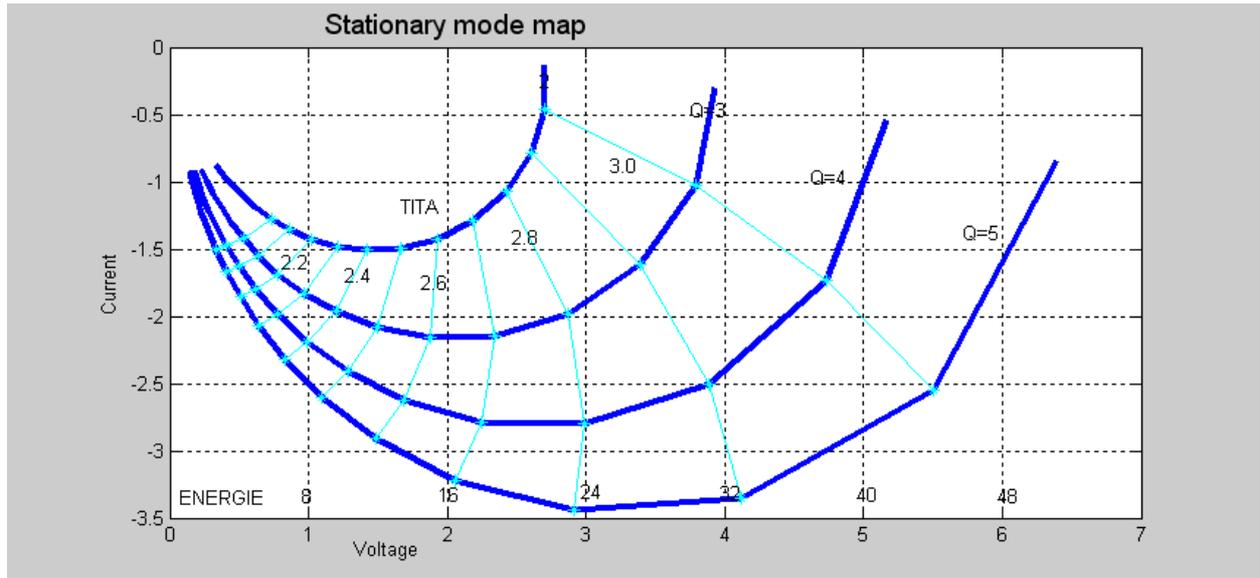


Fig.3 Stationary mode map

The diode segment reduces the current to zero, with a steep drop to the abscissa, accompanied by an insignificant rise in voltage. It can be concluded that the inverse diode performs recuperation, transferring the reactor energy mainly to the source, a process typical for the object.

The connection to the inverse transistor is performed when the current is zero and the development of the odd-numbered half-period repeats the processes described above.

This connection does not change the conductivity of the bridge arm; like in the case of a regular conductor the current has changed its direction.

The soft commutation of the transistor requires the leading edge of its gate pulse to be passed in advance to an arbitrary point in the previous diode interval. The exact choice of the moment of the gate leading edge is insignificant, because the voltage is zero and the transistor has to wait until the moment the diode is switched off. This gives ground for the following conclusions:

- The transistor is only activated after the current in the diode on the same bridge arm is reduced to zero.
- The diode is switched on when a transistor on the bridge adjacent arm is deactivated (switched off).
- Short cycle inverters are only controlled by the trailing edges of the gate pulses.
- The gate pulses must have greater duration than that of the transistor current.

The duration of the gate pulses is limited between the transistor interval and the cycle:  $\theta_t < \theta_g < \theta_s$ . Although the gate pulse can last as long as cycle  $\theta_s$ , the stationary mode automatically shortens the transistor interval to  $\theta_t$ , at the expense of the recuperation diode interval  $\theta_d$ .

The rhomboid with vertexes in the start commutation points S, -S and in the centers (1,0), (-1,0), has the angles of the inverter cycle  $\theta_s = \pi - \alpha$ . More precisely,  $\theta_s$  is the angle at the centers, while  $\alpha$  is the angle at S. As the cycle  $\theta_s$  increases, its complainer  $\alpha$  decreases and the rhomboid tends to a position on the abscissa. This process is described in the following parts of the article.

## V. STATIONARY MODE MAP

The start point S on the spiral identifies the entire stationary mode. In addition to its function for constructing the PM, it illustrates the inverter power. S is the radius of circles, which are used to determine the maximum voltage, maximum current, and a number of other parameters.

The set formed by all start points is the mode map, which illustrates the properties of all inverters of this type – Fig. 3.

Inverters in power electronics have the typical significations:  $2 \leq Q \leq 5$  ( $Q\text{-factor} = (L/C)^{0.5}/R$ ) and cycle  $1.5 \leq \theta_s \leq 3.1$ , which, when applied to Eq. (5) or Eq. (6) determine the start point S.

The mode map is a net of two arc families. The lengthened arcs have a fixed Q-factor and are the frequency characteristics of the inverter. The approximation of cycle  $\theta_s$  to  $\pi$  ( $\alpha > 0$ ) increases the power in the second abscissa scale. It is essential to maintain the distance to this limit  $\theta_s \neq \pi$  ( $T_t < T_0$ ), otherwise there is a risk of short circuit across the transistors. The vertical arcs have a fixed cycle  $\theta_s$ . These arcs are the inverter load characteristics, since the varying Q represents the changes in the load.

The mode amplitude is typical for both the great values of Q, and the great values of  $\theta_s$ , where the timer frequency approaches the inverter natural frequency. The map can be used for design and optimization of the series inverter, as well as for other applications.

## VI. ENERGY AND POWER

Power is a priority in power electronics, since the efficiency and productivity of various devices is based on it. Conventional methods reach the power required by means of time-consuming procedures connected with working out equations, solutions, stylization of the solutions, such as average or quadratic mean current, etc. These procedures are

additionally complicated with inverters due to their discrete-switch principle of operation.

For direct determination of the inverter power we use the quadratic relationship between mode levels and energy levels in the PM – Fig. 2. If the capacitor of an arbitrary inverter is charged up to the source voltage E, then its energy is:

$$Ma = 0.5CE^2 \text{ [J]} \quad (7)$$

Then the abscissa unit - (1,0) in Fig.2, will present simultaneously the voltage E of the capacitor and the energy portion Eq. (7) in it.

The same is proved for the ordinate unit (0,1) of the PM, which presents the base portion of energy Eq. (7) simultaneously with the current.

The base portion of energy Eq. (7) is the energy scale of the inverter.

In addition to the mode significance (voltage and current) of its coordinates (u, i), any point of the PM, also presents the sum of their coordinates energies.

$$A = (u^2+i^2)*Ma \text{ [J]} \quad (8)$$

The mode quadratures between the parentheses in Eq. (8) are the quadratures of the radius vector, which summarizes the remarkable relation between the mode and the energy:

**Any vector in the PM represents energy that is proportional to its module quadratures.**

The energy which is realized in resistance R of the inverter is illustrated by the difference between the modules of the start vector and the final vector. These vectors intersect in start point S – Fig.2. They are the equal sides of an isosceles triangle, the base of which is the abscissa segment between spiral centers –1 and +1.

The quadratures of the difference in the sides is easy to determine using the coordinates of the start point S ( $u_s, i_s$ ):

$$a/2 = [(u_s+1)^2 + i_s^2] - [(u_s-1)^2 + i_s^2]$$

The equality shows the energy realized for a half-period. Although it presents a quadratures, its relatively simple transformation eliminates the coordinate quadratures. The energy for a full period is:

$$a = 8 uS \quad (9)$$

The active energy in the inverter depends only on the start point voltage. This allows the mode map in Fig. 3 to be supplemented with a second abscissa scale, along which the active energy can be reported directly.

The inverter power is the product of the multiplication of Eq. (9) by the scale Eq. (7) and by the frequency -  $f = 1/T_i$ :

$$P = f \cdot a \cdot Ma \text{ [W]} \quad (10)$$

## VII. CONCLUSIONS

The stationary mode in bridge inverters can be formulated as an effect of the functional symmetry.

The transistor is only activated after the current in the diode on the same bridge arm is reduced to zero.

The diode is switched on when a transistor on the bridge adjacent arm is deactivated (switched off).

The inverter is only controlled by the trailing edges of the gate pulses.

The diodes perform a recuperative process.

Soft commutation is practically impossible.

The inverter power depends only on the voltage at the start point.

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