Influence of the Window Function to the Time-Frequency Characterization of Electrocardiogram

Sinisa S. Ilic[1], Vidosav Stojanovic[2], Aleksandar Zoric[3]

Abstract – **In this paper the study of influence of the window function to the Time – Frequency characterization of the ECG signal using STFT is presented. Our idea was to create real time STFT diagram of ECG signal in the existing PC based ECG device. In order to achieve STFT diagram with as many details as possible a lot of calculations in signal processing must be done. But, some calculations can take a time that is not enough for real time rendering of the STFT diagram. By proper selection of the window function and window length we can obtain good compromise for showing STFT diagram of ECG signal in real time.**

Keywords - **ECG, STFT diagram, window function, window length**

I. Introduction

The ECG signal represents an electrical stimulus to the heart muscles. Four chambers exist in the heart: Left and Right Atrium and Left and Right Ventricle. There are valves (Mitral and Tricuspid) between Atriums and Ventricles which enable the blood flow from Atriums to Ventricles. When the heart muscle contracts or beats (called systole) it pumps blood out of the heart. The heart contracts in two stages. In the first stage the Right and Left Atria contract at the same time, pumping blood to the Right and Left Ventricles. Then the Ventricles contract together to pump blood out of the heart. Then the heart muscle relaxes (called diastole) before the next heartbeat. This allows blood to fill up the heart again. Each step in the process described is stimulated by the electric signal generated in the Sinoatrial node which is located in the Right Atrium. Voltages which can be measured on the body skin are usually in the range ±1mV. Plotted changes of the voltages described create Electrocardiogram (ECG). Typical ECG signal, with denoted parts described above is shown in Figure 1.

P wave denotes the spread of electrical activity over the Atria and the beginning of its contraction, QRS complex denotes the spread of electrical activity over the ventricles and the beginning of its contraction and T wave denotes the recovery phase of the ventricles.

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It is obvious that changes in the electric stimulus result in the improper work of the heart. Doctors can find irregularity of the heart activity based on the waveform of the ECG signal in the time domain, but lot of information is also hidden in the frequency domain.

Our idea was to create “real time” computer ECG which would offer possibility to ECG specialist to monitor ECG signal in Time and Time – Frequency Domain. For some suspicious waveform of ECG signal in Time Domain, specialist could switch to Time – Frequency Domain monitoring in real time and look for spectral deviations. Of course, because of the refreshing rate of the ECG screen in the real time it is preferable to decrease sample rate and a number of samples in the window. So, the prime task was to select sample rate, window function and a number of samples in the window in order to render Time-Frequency diagram in real time, but also not to decrease quality of information consisted in the diagram.

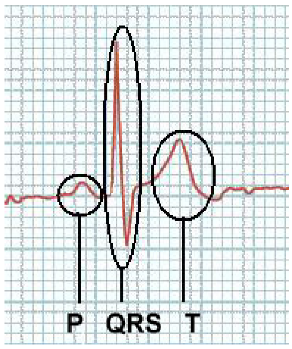

Fig. 1. Typical waveform of a single period of the electric stimulus of the heart

II. Background Of The Time-Frequency Analysis

We can take several periods (cycles) of the ECG waveform and calculate its spectrum, in order to obtain amplitudes and phases of the spectral components. This information is enough to determine which spectral components take part in the signal, what is the amplitude ratio between them and what is the bandwidth of the signal. But, this is not enough to indicate how a signal’s frequency contents evolve in time. This is very important for a range of heart diseases, i.e. deviation of the moments where some frequency components appear in relation to the beginning of the parts described (P, QRS, T) can show irregularity of the heart work.

There are several ways to obtain dynamic spectrum analysis over time. One of them, Short Time Fourier

Transform (STFT), uses functions that are concentrated in both time and frequency domains.

The short-time Fourier transform of a sequence x(n) is defined by

$$X_{STFT}(e^{j\theta}, n) = \sum_{m=-\infty}^{\infty} x(n-m)w(m)e^{-j\theta m} \quad (1)$$

where *w(m)* is suitable window sequence. Window function extracts finite-length portion of sequence *x(n)* such that spectral characteristic of the section extracted is approximately stationary over the window for practical purposes. The STFT is computed at the finite set of discrete values of θ. To be more precise, let the window be of length L defined in the range 0 ≤ m ≤ L-1. We sample $X_{STFT}(e^{j\theta}, n)$ at the N equally spaced digital frequencies $\theta_k = 2\pi k/N$ with N ≥ L as indicated in following equation

$$X_{STFT}(k, n) = \sum_{m=-\infty}^{\infty} x(n-m)w(m)e^{-j2\pi km/N} \quad (2)$$

for 0 ≤ k ≤ N-1. It follows from above equation, assuming $w(m) \neq 0$, that $X_{STFT}(k, n)$ is simple DFT of *x(n-m)w(m)*. Note that X_{STFT} is two-dimensional sequence and is periodic in k with a period N.

Finaly, the sampled STFT for a window defined in the region 0 ≤ m ≤ L-1 is given by

$$X_{STFT}(k, lR) = \sum_{m=-\infty}^{\infty} x(lR-m)w(m)e^{-j2\pi km/N} \quad (3)$$

where *l* and *k* are integers such that $-\infty < l < \infty$ and $0 \le k \le$ N-1.

Problem in STFT is that one cannot know what spectral components exist at what instances of times. What one can know are the time intervals in which certain band of frequencies exist, which is a resolution problem.

A simple way to characterize a signal simultaneously in time and in frequency is to consider its mean localizations and dispersions in each of these representations. This can be obtained [4], [6], [7] by considering $|x(t)|^2$ and $|X(\theta)|^2$ as probability distributions and looking at their mean values and standard deviations:

$$t_m = \frac{1}{E_x}\int_{-\infty}^{\infty} t|x(t)|^2 dt \quad (4)$$

$$\theta_m = \frac{1}{E_x}\int_{-\infty}^{\infty} \theta|X(\theta)|^2 d\theta \quad (5)$$

$$T^2 = \frac{4\pi}{E_x}\int_{-\infty}^{\infty} (t-t_m)^2|x(t)|^2 dt \quad (6)$$

$$B^2 = \frac{4\pi}{E_x}\int_{-\infty}^{\infty} (\theta-\theta_m)^2|X(\theta)|^2 d\theta \quad (7)$$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt < \infty \quad (8)$$

where t_m is "time center of the signal", θ_m is "frequency center of the signal", T is temporal width of the signal and B is spectral width of the signal. The product $T \cdot B$ shows the uncertainty area and should be greater or equal to 1 (it is equal to 1 only for Gauss window). We used three window function in this paper: Gauss, Hamming and Blackman. Equations for these windows are following:

$$gw(k+1) = e^{-\frac{1}{2}\left(\alpha\frac{k-N/2}{N/2}\right)^2} \quad (9)$$

$$hw(k+1) = 0.54 - 0.46\cdot\cos\left(2\pi\frac{k}{N-1}\right) \quad (10)$$

$$bw(k+1) = 0.42 - 0.5\cos\left(2\pi\frac{k}{N-1}\right) + 0.08\cos\left(4\pi\frac{k}{N-1}\right) \quad (11)$$

where $0 \le k < N$, and N is window length.

III. Optimisation Of STFT Parameters

ECG waveforms are recorded using the virtual 12-leads ECG device [1] with sample rate of 2880 Hz. According to the [2] valuable frequency components are up to 100 Hz, but it is for the spectrum of the whole signal. Some valuable frequency components may appear in time-frequency analyses which are greater than 100Hz. We have built a function in Matlab software, which has 3 input parameters: rate of reducing the sampling frequency (decimation), window function and window length.

A. Influence of the Window length

The window length is very important in two ways. By increasing the window length, resolution in frequency axis is better – spectral components are closer one to each other. But the time resolution is then poor because the same signal pattern will be included in a number of overlapping windows. It will result in impossibility to find out when the pattern starts in time. By decreasing the window length, resolution in time axis is better, but resolution in a frequency axis is worst. It means that spectral components along the frequency axis represent amplitude not of the frequency which exists in signal, but rather of the wider range of frequencies. For sampling frequencies between 72 and 288 Hz after decimation, and number of the samples in the window of 32 and 64, we can obtain that spectral component in the diagram represents ranges between 1.125 and 9 Hz. Also, according to the parameters selected, widths of the windows are between 0.111s to 0.889s. If we presume that basic frequency of the heart work (frequency of occurring PQRST components) is between 0.5s (120 bpm) to 2s (30 bpm), window width should be below 0.5s. It would ensure that window covers only or less than one single cycle of PQRST components.

B. Influence of the Window function

Each of the window functions has different influence on a signal it multiplies with. We have used Hamming, Gauss and Blackman window.

The criteria for selecting window function was spectrum of the functions and product T x B described above. Product T x B is calculated for discrete window functions and the values are: 0.996, 1.0041 and 1 for Hamming, Gauss and Blackman window functions respectively, but if there was no ECG signal.

In order to save calculations of an amplitudes of the window functions in time instants T, 2T, … nT which multiply appropriate samples in the time window of the ECG signal, it is enough to create 6 arrays: 3 arrays (for each window type) of 32 values (for 32 samples in window) and 3 arrays of 64 values.

C. Selection of the parameters

Our plan was to refresh the ECG signal in Time Domain with original sample rate, but for rendering the diagram in Time-Frequency Domain to select following parameters:

- Sample rate of 144 or 288 Hz,
- Hamming, Gauss, or Blackman Window,
- Number of the samples in the window: 32 or 64,
- Number of the samples between the overlapping windows: 4.

Sample rate of 144/288 Hz gives spectrum up to 72/144 Hz, which should be enough to show significant time-frequency components of ECG signal, and downsampling is then 10/20.

The relative sidelobe attenuation of the Hamming, Gauss and Blackman window functions are: -42.5dB, -312.5dB and 58.1dB respectively. Attenuation of the components in the Hamming window outside of the main lobe is around 42.5dB, which offer possibility to see higher frequency components in the diagram.

Product TxB is changing in time for the windowed ECG signal that can bee seen in the Figure 2. The maximum values of uncertainty are calculated by using Hamming and the minimum by using Gauss window function. But, as it can been seen later, decreasing of the product i.e. decreasing of the time-frequency uncertainty does not lead us to the better representation of time-frequency analysis of the signal.

The width of the window is 0.222s (for 32 samples) or 0.444s (for 64 samples) and between the overlapping windows is 0.027s which is enough to obtain time resolution for detailed monitoring of ECG signal.

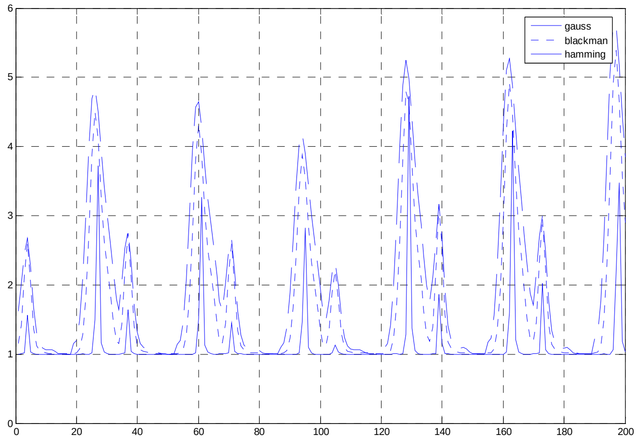

Fig. 2 – Product T x B for the windowed ECG signal

IV. Examples And Example

In the figures below, Time Domain waveform of 4th Cordial lead and it's 3D Frequency-Time diagrams are shown. Sample frequency (fs) is 144Hz, window length (w) is 0.44s and 0.22s, time resolution is Δt = 0.0278s, frequency resolution is Δf = 2.25Hz and 4.5Hz for number of the samples in the window of n=64 and n=32 samples. Figures 3 and 4 show diagrams using the Hamming and figures 5 and 6 using the Gauss windowing function with α = 10.

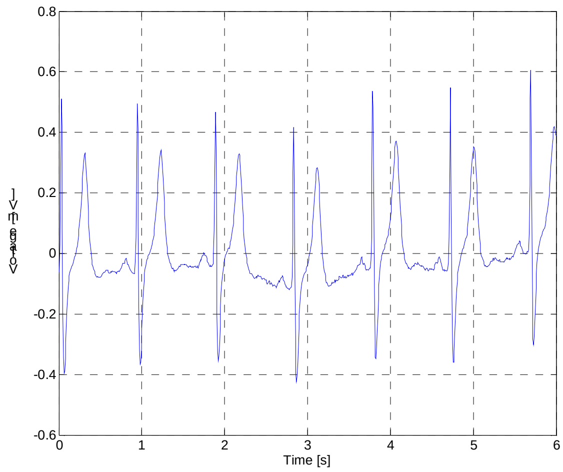

Fig. 3 – Time Domain waveform of the ECG signal has been analyzed

If we take a look on figures 4 and 5, we can see that better time resolution is obtained by using 32 samples in the window. Also, more details in higher frequencies can be seen in the diagrams which are obtained by using the Hamming window function.

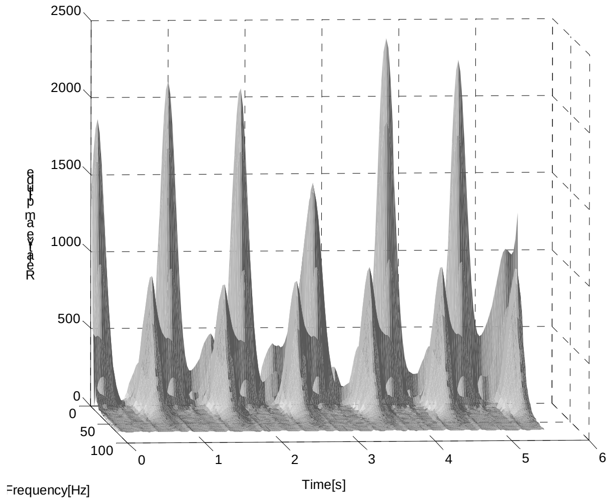

Figure 4 – 3D diagram using the Hamming window with w=0.44s, fs=144Hz, n=64 samples

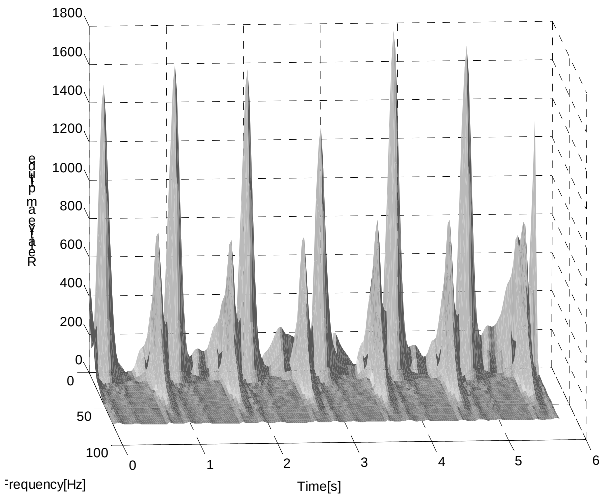

Fig. 5 – 3D diagram using the Hamming window with w=0.22s, fs=144Hz, n=32 samples

Considering processing time that computation of windowing function and FFT needs for 32 samples against 64 samples in the window, and quality of the diagrams presented for sample rate of 144Hz, we can say that selection of 32 samples in the window is good choice which guarantees satisfactory quality.

Also we can see that 3D spectrograms in figures 5 and 6 are similar although they represent processing using windows of different types and different number of samples in the window.

V. Conclusion

In this paper is presented study how window function impacts on the Time – Frequency characterization of the Semi – periodic biopotentials of the myocard by implementing the STFT. Although the initial idea was to select the window function that has the best resolution for the time-frequency diagram of the ECG signal according to minimization of the Heisenberg uncertainty principle, studies and examples showed that this is not the case for typical ECG signal.

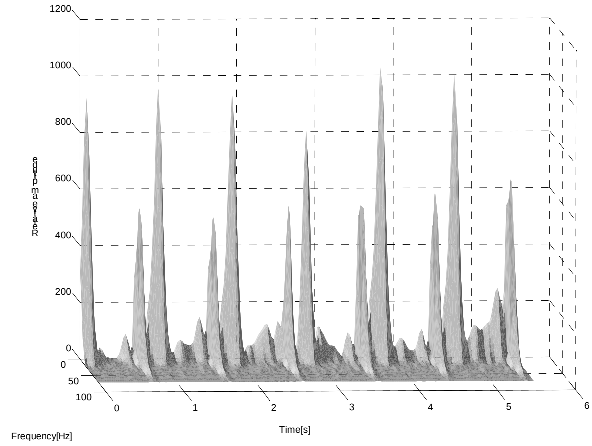

Fig. 6 – 3D diagram and the contour plot using the Gauss window with w=0.44s, fs=144Hz, n=64 samples

The aim of the study was to implement “real time” STFT analysis in the existing PC based Electrocardiogram software.

Results of comparing STFT of the real ECG signal of two types of window functions with different number of samples in the window show that better frequency resolution of STFT diagram is achieved by selection of the Hamming window function, while satisfactory time resolution is achieved by selection of 32 samples in the window.

Conclusion obtained represents good compromise of parameters selected in order to show STFT diagram in real time.

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