Active Inverse Hausdorff Filters

Peter St. Apostolov[1], Michael Bl. Momchedjikov[2]

Abstract - **This paper presents a method for synthesis of two kinds of active inverse filters with Hausdorff-type transfer characteristic. The filter frequency characteristics are determined and a comparison with Chebyshev inverse filters is given.**

Keywords - **Approximation, Polynomial, Synthesis, Active inverse filter, Hausdorff, Chebyshev, Frequency characteristic.**

I. INTRODUCTION

Hausdorff filters are implemented by "shifted" Delta-function approximation $\chi(x)$ with Hausdorff polynomial (Fig. 1):

$$P_n(x) = \varepsilon T_n\left(\frac{2x}{2-\alpha\varepsilon}\right) = \varepsilon T_n\left(\frac{x}{1-\alpha\varepsilon/2}\right), \qquad (1)$$

where ε is the Hausdorff space, T_n is Chebyshev polynomial of first order and n degree, α is a parameter. The $\alpha\varepsilon$ product determines precision estimation of approximated "shifted" Delta-function in point 1 where it has first-order indeterminacy or infinite steepness.

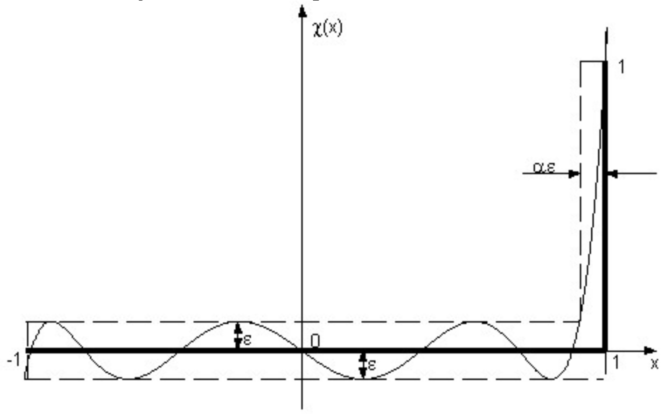

Fig. 1

The inverse Hausdorff filters are of two types: A type and B type [2]. They have characteristics similar to Chebyshev inverse filters. Magnitude responses of third-order Hausdorff inverse filters compared to Chebyshev inverse filter are shown on Fig. 2. The three characteristics have identical pass-bands. For infinite attenuation frequencies the following equations are valid:

$$f_A = f_C/(1-\alpha\varepsilon/2);\ f_B = f_C(1-\alpha\varepsilon/2). \qquad (2)$$

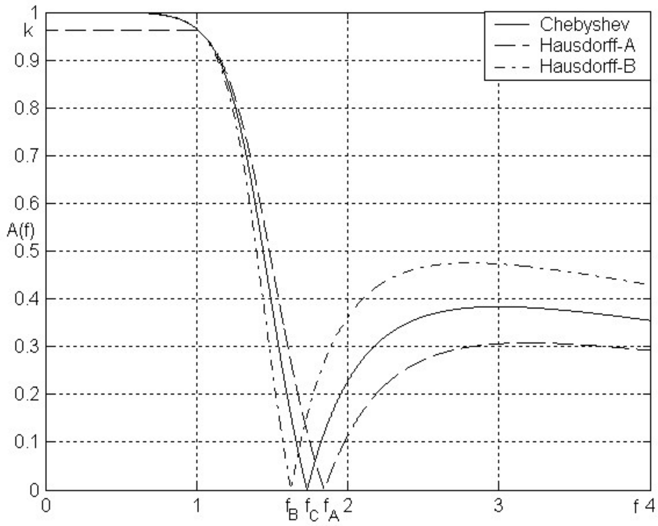

Fig. 2

II. SYNTHESIS IMPLEMENTATION

Let's take an example to illustrate the filter synthesis:
An active inverse Hausdorff filter is to be calculated - of A type, forth order $n=4$, cut-off frequency $f_c = 1\text{KHz}$; stop frequency $f_s = 2.135\text{kHz}$, limit frequency characteristic drop $DA = 2\text{dB}$, pass-band gain $A_0 = 1$, selective component values $R = 10\text{k}\Omega$ and $C = 10\text{nF}$. The filter will consist of two cascade two-order amplifier stages - Boctor circuit [3], [4].
The limit frequency characteristic drop is calculated:

$$k = \sqrt{10^{0.1DA}-1} = 0.7648. \qquad (3)$$

The attenuation of inverse Chebyshev filter of the same stop frequency's input data is defined [1]:

$$DS_{ch} = 10\log\left[k^2\text{ch}^2\left(n\text{Ach}\frac{f_s}{f_c}\right)\right] = 40\text{dB}. \qquad (4)$$

From the established value the Hausdorff space and the product $\alpha\varepsilon$ could be found [2]:

$$\varepsilon = \frac{1}{\sqrt{10^{0.1DS_{ch}}-1}} = 0.01; \qquad (5)$$

$$\alpha\varepsilon = 2\frac{\text{ch}\left[\frac{1}{n}\text{Ach}\left(\frac{1}{\varepsilon}\right)\right]-1}{\text{ch}\left[\frac{1}{n}\text{Ach}\left(\frac{1}{\varepsilon}\right)\right]+1} = 0.6725. \qquad (6)$$

The inverse Hausdorff filter stop-band minimal attenuation and equivalent ripple ε_1 is defined:

[1]Peter St. Apostolov, Ms. Sc., Assoc. Fellow, IST-Sofia, E-mail: p_apostolov@abv.bg
[2]Michael Bl. Momchedjikov, Prof. D-r, Eng, TU – Sofia, FKTT, Radiocommunication; E-mail: momche_tu@abv.bg

$$DSH_{\min} = 10\log\left\{k^2 \mathrm{ch}^2\left[n\mathrm{Ach}\frac{f_s}{f_c\left(1-\alpha\varepsilon/2\right)}\right]+1\right\} = 55.4531\mathrm{dB} \quad (7)$$

$$\varepsilon_1 = \frac{1}{\sqrt{10^{0.1DSH_{\min}}-1}} = 0.0017\cdot \quad (8)$$

The active inverse Hausdorff filter stop frequency attenuation is equal to:

$$DSH = -10\log\frac{\varepsilon_1^2 \mathrm{ch}^2\left(n\mathrm{Ach}\frac{1}{1\text{-}\alpha\varepsilon/2}\right)}{1+\varepsilon_1^2\mathrm{ch}^2\left(n\mathrm{Ach}\frac{1}{1\text{-}\alpha\varepsilon/2}\right)} = 27.8341\mathrm{dB}\cdot \quad (9)$$

The filter poles and zeros are defined:

$$\sigma_i = -\sin\left(\frac{2i-1}{n}\frac{\pi}{2}\right)\mathrm{sh}\left[\frac{1}{n}\mathrm{Ash}\left(\frac{1}{\varepsilon_1}\right)\right]; \quad (10)$$

$$\Omega_i = \cos\left(\frac{2i-1}{n}\frac{\pi}{2}\right)\mathrm{ch}\left[\frac{1}{n}\mathrm{Ash}\left(\frac{1}{\varepsilon_1}\right)\right]; \quad (11)$$

$$\omega_{Pi} = \frac{1}{\left(1-\frac{\alpha\varepsilon}{2}\right)\left(\sigma_i + j\Omega_i\right)} = \begin{pmatrix} -0.1216 - j0,3110 \\ -0,3186 - j0.1399 \\ -0,3186 + j0.1399 \\ -0.1216 + j0,3110 \end{pmatrix}; \quad (12)$$

$$\omega_{Zi} = \frac{j}{\left(1-\frac{\alpha\varepsilon}{2}\right)\cos\left(\frac{2i-1}{n}\frac{\pi}{2}\right)} = \begin{pmatrix} j1.0824 \\ -j1.0824 \\ j2.6131 \\ -j2.6131 \end{pmatrix}; i = 1,...,n\ . \quad (13)$$

The filter amplifier stages indexes are calculated [4]:

$$\Omega_{Pi} = \left|\omega_{Pi}\right| = \begin{pmatrix} 0,334 \\ 0,348 \end{pmatrix}; \quad (14)$$

$$\Omega_{Zi} = \left|\omega_{Zi}\right| = \begin{pmatrix} 1.0824 \\ 2.6131 \end{pmatrix},\ i = 1,...,\mathrm{int}\left(\frac{n+1}{2}\right)\cdot \quad (15)$$

The coefficient is defined:

$$k_f = \mathrm{ch}\frac{\mathrm{Ach}\frac{\sqrt{\frac{1}{10^{0.1DA}-1}}}{\varepsilon_1}}{n} = 3.2166\cdot \quad (16)$$

Denominated pole-zero frequencies can be found from:

$$\omega_{Pi} = 2\pi f_c \Omega_{Pi} k_f = \begin{pmatrix} 21876 \\ 52813 \end{pmatrix}\mathrm{Hz}; \quad (17)$$

$$\omega_{Zi} = 2\pi f_c \Omega_{Zi} k_f = \begin{pmatrix} 21876 \\ 52813 \end{pmatrix}\mathrm{Hz}\cdot \quad (18)$$

Selective values 10kΩ and 1nF are assigned to R_{7i} and C_{8i} to define circuit minimal capacitance:

$$C_{1\min i} = \frac{C_{8i}A_0^2\left[Q_{Pi}^2\left(\omega_{Zi}^2-\omega_{Pi}^2\right)+\omega_{Pi}^2\right]^2}{\omega_{Zi}^2 Q_{Pi}^2\left(A_0-1\right)+\omega_{Zi}^2\omega_{Pi}^2 Q_{Pi}^2\left(A_0^2-A_0\right)+\omega_{Pi}^2 A_0\left(\omega_Z^2-A_0\omega_{Pi}^2\right)} = \begin{pmatrix} 37.713 \\ 5.5399 \end{pmatrix}\mathrm{nF}\ . \quad (19)$$

Standard values are accepted:

$$C_{1i} = \begin{pmatrix} 47 \\ 10 \end{pmatrix}\mathrm{nF}\cdot \quad (20)$$

Discriminants are calculated:

$$D_i = C_{1i}^2\omega_{Zi}^4 - 4C_{1i}C_{8i}A_0^2\omega_{Pi}^2\left(\omega_{Pi}^2+Q_{Pi}^2\omega_{Zi}^2\right) = \begin{pmatrix} 497.7723 \\ 776.2355 \end{pmatrix}. \quad (21)$$

Then the remain circuit elements are determined:

$$R_{4i} = \frac{R_{7i}\left(\omega_{Zi}^2 - A_0\omega_{Pi}^2\right)}{\omega_{Pi}^2 A_0} = \begin{pmatrix} 95.048 \\ 553.98 \end{pmatrix}\mathrm{k\Omega}; \quad (22)$$

$$R_{2i} = \frac{C_{1i}\omega_{Zi}^2 - \sqrt{D_i}}{2C_{1i}C_{8i}A_0 Q_{Pi}\omega_{Pi}^3} = \begin{pmatrix} 4.5668 \\ 8.2311 \end{pmatrix}\mathrm{k\Omega}; \quad (23)$$

$$R_{3i} = \frac{1}{C_{1i}C_{8i}R_{2i}\omega_{Pi}^2} = \begin{pmatrix} 102.27 \\ 245.65 \end{pmatrix}\mathrm{k\Omega}; \quad (24)$$

$$R_{5i} = \frac{-Q_{Pi}R_{2i}}{Q_{Pi} + R_{2i}^2 C_{1i}C_{8i}Q_{Pi}\omega_{Pi}^2 - R_{2i}C_{1i}\omega_{Pi}} = \begin{pmatrix} 454.36 \\ 310.01 \end{pmatrix}\mathrm{k\Omega}; \quad (25)$$

$$R_{6i} = \frac{R_{7i}Q_{pi}}{C_{8i}\omega_{Pi}\left(R_{2i}R_{4i}C_{1i}Q_{Pi}\omega_{pi} - R_{7i}\right)} = \begin{pmatrix} 11.360 \\ 4.7029 \end{pmatrix}\mathrm{k\Omega}\cdot \quad (26)$$

The active inverse Hausdorff A-type filter electrical schematics is shown on Fig. 3

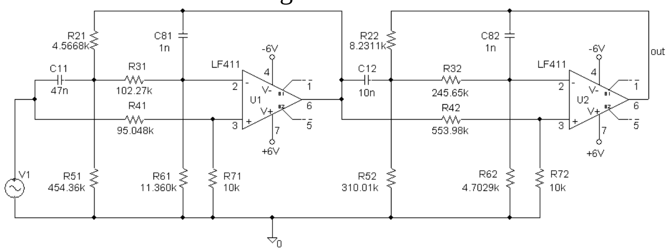

Fig. 3

The filter magnitude response is shown on Fig. 4. The marked values coincide with the values calculated in formulas (7) and (9).

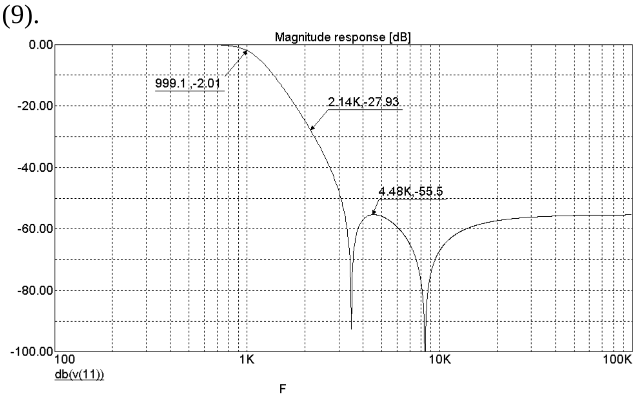

Fig. 4

An active inverse Hausdorff B-type filter [2] can be calculated using the described procedure. The changes are the following:

Formula (7) becomes:

$$DSH_{\min B} = 10\log\left\{k^2\mathrm{ch}^2\left[n\mathrm{Ach}\frac{f_s\left(1-\alpha\varepsilon/2\right)}{f_c}\right]+1\right\}\cdot \quad (27)$$

Formula (9) becomes:

$$DSH_B = -10\log\frac{\varepsilon_1^2\mathrm{ch}^2\left[n\mathrm{Ach}\left(1\text{-}\alpha\varepsilon/2\right)\right]}{1+\varepsilon_1^2\mathrm{ch}^2\left[n\mathrm{Ach}\left(1\text{-}\alpha\varepsilon/2\right)\right]}\cdot \quad (28)$$

Formula (12) becomes:

$$\omega_{Pi} = \frac{1-\frac{\alpha\varepsilon}{2}}{\sigma_i + j\Omega_i}\cdot \quad (29)$$

Formula (13) becomes:

$$\omega_{Zi} = \frac{j\left(1 - \frac{\alpha\varepsilon}{2}\right)}{\cos\left(\frac{2i-1}{n}\frac{\pi}{2}\right)}. \quad (30)$$

An active inverse Hausdorff B-type filter electrical schematics with the same input data is shown on Fig. 5.

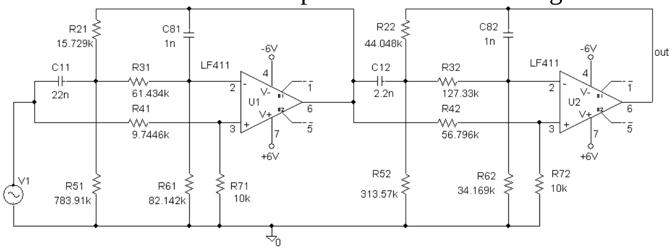

Fig. 5

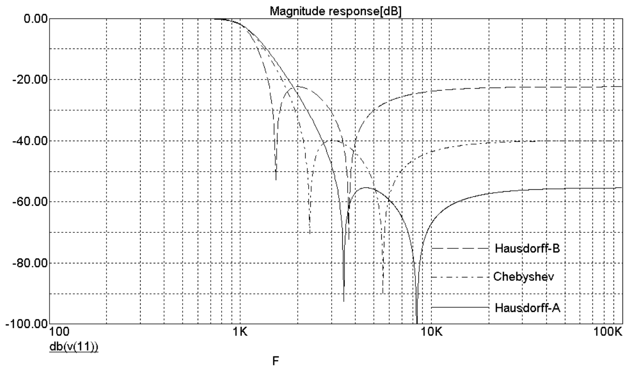

Fig. 6

Active inverse Hausdorff A and B - type frequency characteristics compared to the same input data Chebyshev filter frequency characteristics are shown on Fig. 6, 7 and 8.

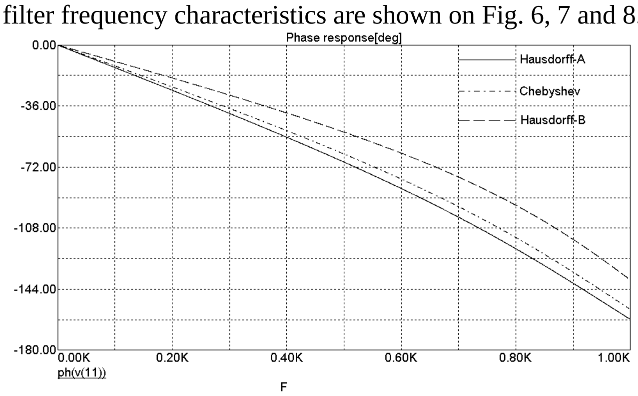

Fig. 7

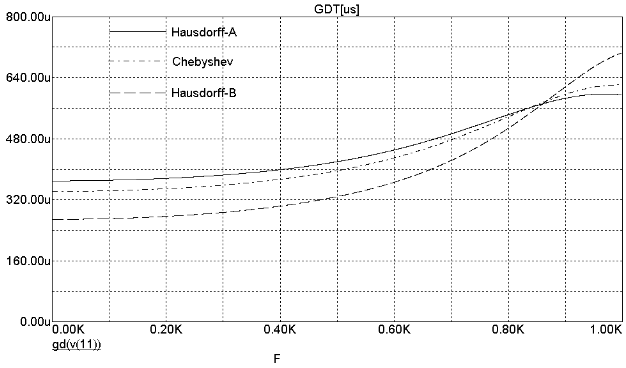

Fig. 8

III. CONCLUSION

On the figures we can see that the two types of inverse Hausdorff filters are complimentary of one another. Compared to Chebyshev's, the first type shortcomings are second type advantages. This leads to greater opportunities in filter design. The frequency characteristics specificity is defined from Hausdorff space with a parameter α. This means that the inverse Hausdorff filters are unique and without analogous using other type of approximation.

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