Low Sensitivity Design of IIR Filters Obtained as a Tapped Cascaded Interconnection of Identical Allpass Subfilters

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Abstract – **In this paper an improved method of design of low sensitivity multiplierless IIR digital filter is proposed. First a tapped cascaded interconnection of identical allpass subfilters is designed and then first- and second-order allpass sections with minimized sensitivities are introduced, which is improving considerably the filter behavior in a limited wordlength environment. The method is verified experimentally.**

Keywords – **IIR digital filters, allpass filters, low sensitivity, multiplierless**

I. Introduction

The number of the structures for realization of recursive digital filters, developed in the last 40 years, appeared to be very high. Most of these structures, however, are having only a textbook value, because when realized in a limited wordlength environment, they behave badly and their magnitude characteristics can not meet even some simple specifications. One of the best known realizations is build as a parallel connection of allpass structures [1]. Even though having extremely low sensitivity in the passband (PB) this realization creates problems in the stopband (SB) and in order to keep the SB attenuation within given limits, it is necessary to work with very high wordlength. A very interesting way to solve this problem was developed in [2], where some additional tap coefficients have been introduced in the parallel structure and then it was decomposed to a cascade of identical substructures. Another way to improve the SB behavior of the parallel allpass structure was advanced in [3] and it was based on the sensitivity minimization of the allpass sections used.

The main aim of this paper is to apply this approach to the structure from [2]. And if it works, to investigate the possibilities to obtain a multiplierless realizations without using some complicated optimization procedures, in order to avoid the degradation of the shape of the filter magnitude, as it was done in [4].

II. Design Method Description

In this section we shall introduce briefly the method of Saramaki and Renfors [2]. It starts with a tapped parallel allpass structure, as shown in Fig. 1a, where $A(z)$ and $B(z)$ are all pass filters, usually realized as cascades of first- and second-order allpass sections.

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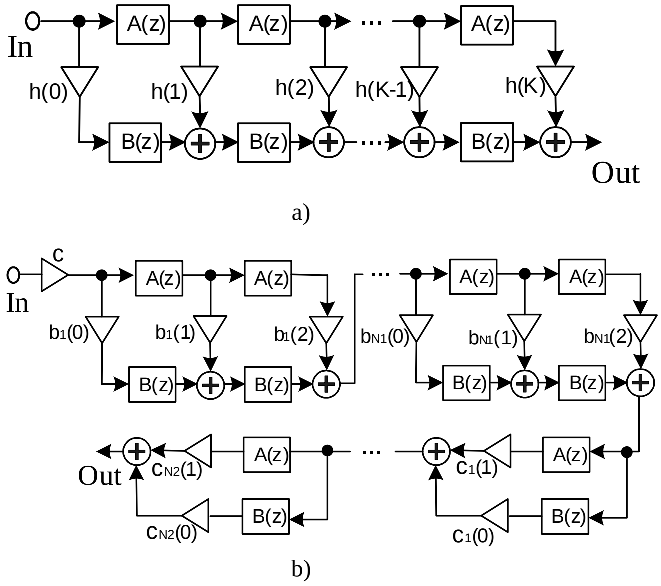


Fig. 1 Initial tapped cascaded interconnection of identical allpass subfilters structure (a); final structure (b)

The overall transfer function of this structure is [2]:

$$H(z) = \sum_{n=0}^{K} h(n)[A(z)]^n [B(z)]^{K-n} \qquad (1)$$

The specifications for the overall filter are formulated as

$$\begin{aligned} 1-\delta_p \le |H(e^{j\omega})| \le 1+\delta_p \quad & for\ \omega \in X_p \\ |H(e^{j\omega})| \le \delta_s \quad & for\ \omega \in X_s, \end{aligned} \qquad (2)$$

where the PB regions X_p and SB regions X_s may consist of several bands.

The design of the overall filter $H(z)$ can be separated into two parts – the design of a prototype nonlinear phase FIR filter $\tilde{H}(z)$, containing the additional tap coefficients $h(n)$ and the design of an elliptic filter $F(z)$, determining the two allpass sections.

$F(z)$ is an IIR filter implemented as a parallel connection of two allpass filters. The amplitude response of the elliptic filter

$$F(z) = \frac{1}{2}[A(z) + B(z)] \qquad (3)$$

is given by

$$|F(e^{j\omega})| = |\cos\{[\varphi_B(\omega) - \varphi_A(\omega)]/2\}| \qquad (4)$$

where $\varphi_B(\omega)$ and $\varphi_A(\omega)$ are the phase responses of the allpass filters $A(z)$ and $B(z)$.

The frequency response of the overall filter can be obtained from the frequency response of the prototype filter as it is described in [2]. The frequency transformation

$$\Omega = \varphi_B(\omega) - \varphi_A(\omega), \tag{5}$$

converts PB and SB performance of a prototype nonlinear phase FIR into that of overall filter:

$$\tilde{H}(e^{j\Omega}) = \sum_{n=0}^{K} h(n) e^{-jn\Omega}. \tag{6}$$

Both lowpass nonlinear phase FIR filter and allpass subfilters meet following specifications:

$$\begin{aligned} 1-\delta_p \le |\tilde{H}(e^{j\Omega})| \le 1+\delta_p \quad & for\ 0 \le \Omega \le \Omega_p \\ |\tilde{H}(e^{j\Omega})| \le \delta_s \quad & for\ \Omega_s \le \Omega \le \pi \end{aligned} \tag{7}$$

$$\begin{aligned} 1-\hat{\delta}_p \le |F(e^{j\omega})| \le 1 \quad & for\ \omega \in I_p \\ |F(e^{j\omega})| \le \hat{\delta}_s \quad & for\ \omega \in I_s, \end{aligned} \tag{8}$$

where

$$\hat{\delta}_p = 1-\cos(\Omega_p/2), \quad \hat{\delta}_s = \cos(\Omega_s/2). \tag{9}$$

The optimal allpass sections can be found by designing a minimum odd order lowpass elliptic filter which meets the requirements of Eq. (8). This filter is implemented as a sum of two allpass sections $A(z)$ and $B(z)$ whose orders differ by one. The Herrmann – Schüssler technique (see [2][5] for details) is used for designing the nonlinear phase FIR filter meeting Eq. (7).

III. ALLPASS FILTER STRUCTURES

It is well known that digital filters realized as a parallel connection of two allpass structures have an extremely low passband sensitivity and low roundoff noise. But in order to have only real coefficients, the transfer function (3) must be only of odd-order, when it is lowpass or highpass. The allpass branches $A(z)$ and $B(z)$ are usually realized as a cascade connection of first and second order sections. In [2][5] these sections are realized using the well known wave lattice structures (Fig. 2a and Fig. 3a), often called Gray-Markel (GM)- sections [6]. The transfer functions of these sections are:

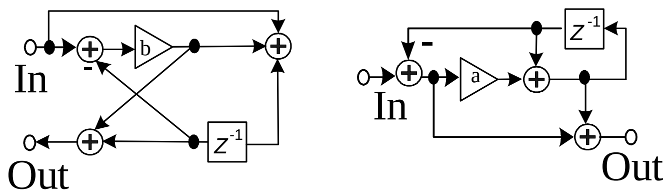
$$H_{GM1}(z) = \frac{b+z^{-1}}{1+bz^{-1}} \tag{10}$$

$$H_{GM2} = \frac{-a_1 - a_2(1-a_1)z^{-1} + z^{-2}}{1 - a_2(1-a_1)z^{-1} - a_1 z^{-2}} \tag{11}$$

The GM-sections are really famous and posses indisputable merits, but our observations show that they behave badly when realizing poles near z=1. It was shown in [3] that much better for realization of such poles are the section ST1 [7] (Fig. 2b) and LS2 [3] (Fig. 3b). The transfer functions of these sections are given by:

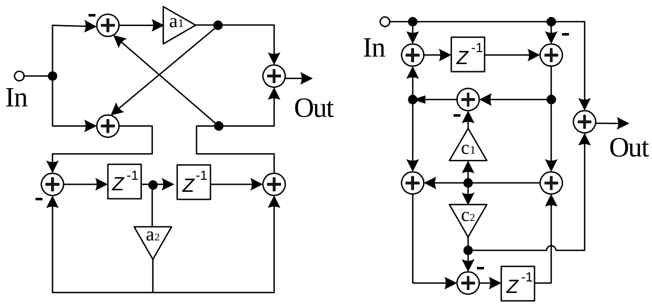
$$H_{ST1}(z) = \frac{-(1-a)+z^{-1}}{1-(1-a)z^{-1}} \tag{12}$$

$$H_{LS2} = \frac{1-c_2+(-2+2c_1+c_2)z^{-1}+z^{-2}}{1+(-2+2c_1+c_2)z^{-1}+z^{-2}} \tag{13}$$



a) GM1 b) ST1

Fig. 2 First-order allpass sections



a) GM2 b) LS2

Fig. 3 Second-order allpass sections

We have investigated the sensitivities of the GM1, GM2, ST1 and LS2 for the transfer function coefficient values as given in Table II corresponding to poles very near z=1 (a narrow-band LP filter). The results for the worst-case sensitivities vs. frequency are given in Figs. 4 and 5. It is clearly seen that the low-sensitivity sections ST1 and LS2 are having many times lower sensitivities for all frequencies, but the difference is especially strong at the maximal points. It is expected that after using the low-sensitivity sections, it will be possible generally to shorten the filter coefficients wordlength and, in our case, to introduce handy multiplierless representations of these coefficients without causing some unacceptable degradation of the magnitude shape.

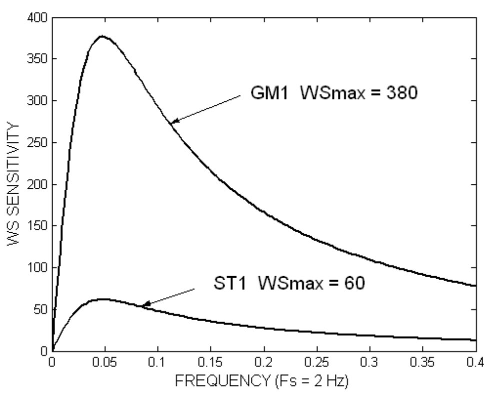


Fig. 4 Worst-case sensitivities of the first-order allpass sections with coefficients a and b as given in Table II

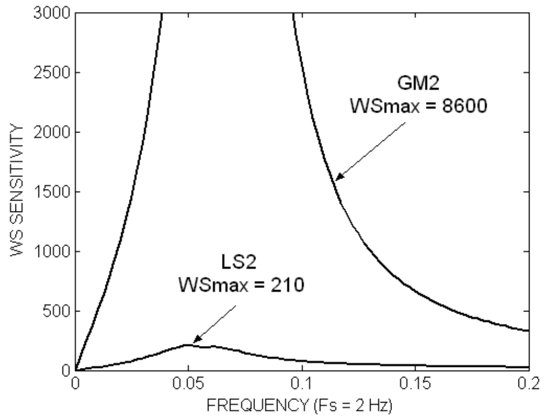


Fig. 5 Worst-case sensitivities of the second-order allpass sections with coefficients a_i and c_i as given in Table II

IV. EXPERIMENTS AND COMPARISONS

In this experiment a lowpass IIR filter with PB edge $\omega_p = 0.05\pi\ rad/s$ and SB edge $\omega_s = 0.12\pi\ rad/s$ is considered. The maximum ripple in PB is $\delta_p = 0.0009$ and in SB – $\delta_s = 0.000009$. For elliptic filter realized as a parallel connection of two allpass sections, the filter order 9 is estimated.

If the number of subfilters K in structure shown in Fig. 1 is 4, then the estimated order of an elliptic subfilter $F(z)$ is 3. Therefore, the order of $A(z)$ is two, and the order of $B(z)$ is one.

The magnitude of the prototype FIR filter obtained using the method described in [5], is shown in Fig. 6. The best extraripple prototype filter solution meeting the specifications in Eq. 7 has PB and SB frequencies $\Omega_p = 0.14\pi\ rad/s$ and $\Omega_s = 0.9835\pi\ rad/s$. In this case, the required ripples for the elliptic subfilter are $\hat{\delta}_p = 0.024$ and $\hat{\delta}_s = 0.0258$. The prototype filter transfer function $\tilde{H}(z)$ can be factored into one second order section and two first order sections (Fig. 1b). Table I presents received tap coefficients.

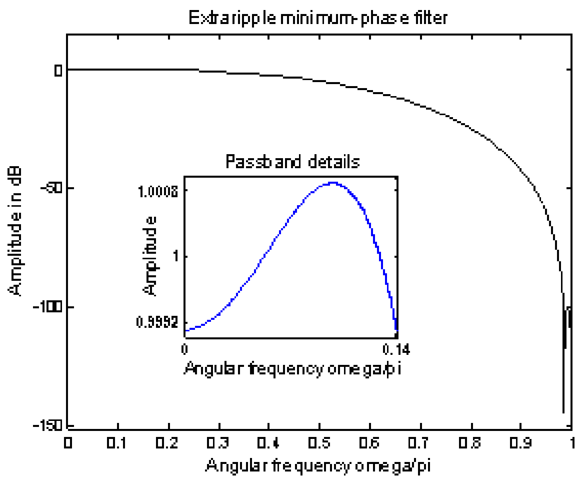


Fig. 6 The prototype FIR filter

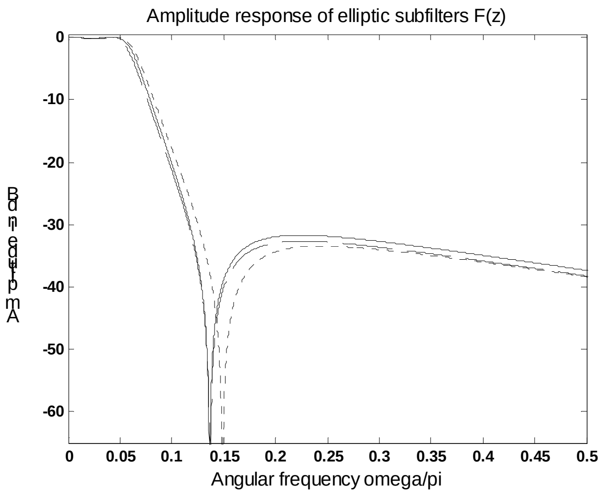
TABLE I
QUANTIZED TAP COEFFICIENTS

First order sections			
	$c_k(0)$	$c_k(1)$	
First section	$2^{-3}+2^{-4}+2^{-8}$	$2^{-3}+2^{-4}+2^{-8}$	
Second section	1	$-2^{-2}-2^{-4}-2^{-5}-2^{-8}$	
Second order sections			
First section	$b_k(0)$	$b_k(1)$	$b_k(2)$
	1	$2^{1}-2^{-9}$	1

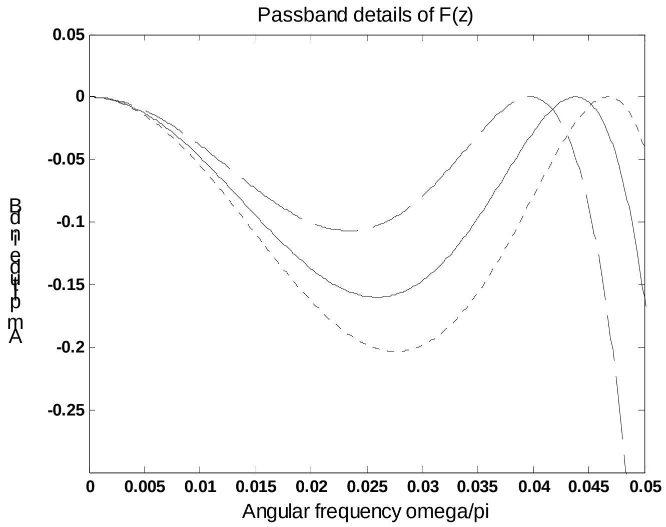
In this paper, different low-sensitivity structures are used for implementation of elliptic subfilter $F(z)$. First, the low-sensitivity sections LS2 and ST1 are used for design of the allpass filters $A(z)$ and $B(z)$. Then, for comparison, the filters $A(z)$ and $B(z)$ are designed with GM-sections. Fig. 7 gives the amplitude responses of the elliptic subfilter $F(z)$ realized with LS2 and ST1 ($F_{LS}(z)$), and by wave lattice sections ($F_{GM}(z)$). Because of the extremely low sensitivities of the sections LS2 and ST1 to the multiplier coefficients variation, the elliptic subfilter $F_{LS}(z)$ realization magnitude does not differ from the ideal after the coefficients are quantized with 8 bits. Fig. 8 gives the optimal amplitude response of the overall filter and PB and SB details. It is clear from this figure that filter implemented with subfilters $F_{LS}(z)$ meets filter specification in Eq. 2 without general multipliers better than filter realized with subfilters $F_{GM}(z)$. All the coefficient values in Table II are representable as a sum of powers of two.

TABLE II
QUANTIZED COEFFICIENTS FOR ALLPASS SUBFILTERS

	Low sensitivity		Wave lattice
a	$2^{-3}+2^{-6}$	b	$2^{0}-2^{-3}-2^{-6}$
c_1	2^{-6}	a_1	$-2^{0}+2^{-3}-2^{-7}$
c_2	$2^{-3}-2^{-6}$	a_2	$2^{0}-2^{-5}+2^{-7}$

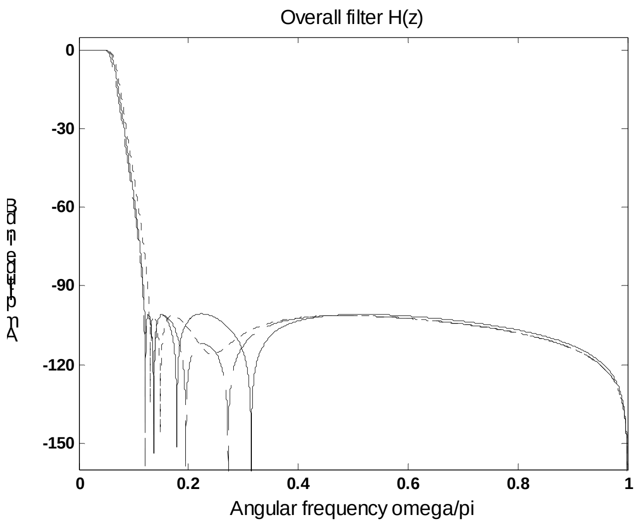


a)



b)

Fig. 7 The amplitude response - solid line; $F_{LS}(z)$ - dashed line and and $F_{GM}(z)$ – dotted line a) overall filter $F(z)$ b) passband details



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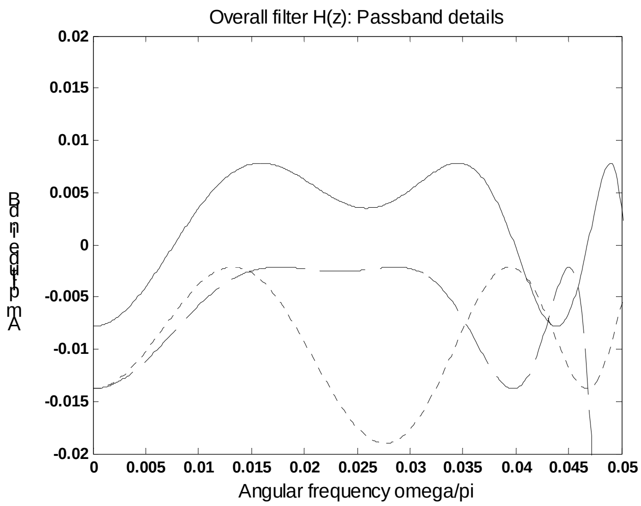

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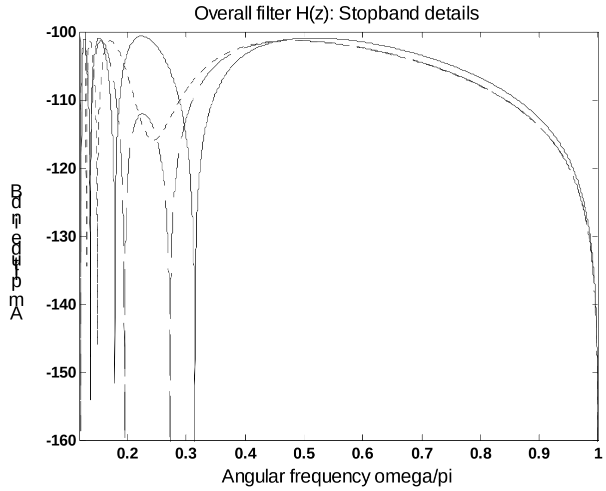

c)

Fig. 8 The amplitude response: a) the overall original IIR filter - solid line; quantized filter with $F_{LS}(z)$ subfilters - dashed line and quantized filter with $F_{GM}(z)$ subfilters – dotted line b). Passband details c). Stopband details

V. Conclusion

A substantial improvement of the method of Saramaki and Renfors for design of recursive digital filters was achieved in this paper by introduction of an additional step in the design procedure. It was shown that the sensitivity of the stopband attenuation can be considerably reduced if the allpass sections used in the subfilters are with minimized sensitivities for the stopband frequency range. As a result, it becomes possible to reduce the coefficient wordlength or to simplify the multiplierless representation of these coefficients without destroying the magnitude shape. Shorter coefficients means, on the other hand, lower power consumption, which is very important for portable telecommunication equipment realizations.

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