A New Adaptive Complex Narrowband Fourth-Order IIR Filter Section

Zlatka Nikolova[1], Georgi Iliev[2], Georgi Stoyanov[3]

Abstract – **In this paper a very low sensitivity fourth-order complex band-pass filter section with independent tuning of the central frequency and the bandwidth is developed. Then, a band-stop adaptive filter structure is formed around this section, using LMS algorithm to adapt the central frequency. The developed filter circuit is providing a low computational complexity and a very fast convergence.**

Keywords – **complex digital filters, sensitivity, adaptive digital filters.**

I. Introduction

Adaptive notch filters are indispensable for many digital signal processing (DSP) applications to separate, enhance or track narrow-band and sinusoidal signals mixed with a broadband noise. If such signal consists of real and imaginary parts, a complex coefficients adaptive filter must be implemented.

The complex adaptive systems are subjects of study in a great number of publications. In [1] a new complex adaptive notch filter for parameter estimation of complex sinusoids and chirp signals mixed with wideband noise is proposed. A complex multirate filter-bank-based adaptive notch filter is developed to solve the co-existing problem of CDMA and TDMA systems [2]. Reference [3] investigates digital complex adaptive filter algorithms to cancel the radio frequency interference in VDSL system. In [4] cascaded realization of complex adaptive infinite impulse response (IIR) notch filters are discussed.

In all this works, like in many others, the attention is focused mainly on the adaptive algorithms, their efficiency and methods of improvement. But the adaptive complex circuits used and their properties couldn't be neglected as they influence significantly the quality of the adaptive process. These circuits are, however, merely mentioned in [1]– [4] and they tend, generally to be quite complicated.

In this paper we develop a fourth-order complex band-pass (BP) filter section with independent tuning of the central frequency and bandwidth (BW). The new complex variable filter conducts very well in finite word-length environment and demonstrates very low coefficient sensitivity without influence of both – variable and adaptive processes parameters.

The paper is organized as follows. The fourth-order variable complex digital filter is derived and investigated in section 2. Section 3 defines the system model for adaptive complex narrowband filtering. In section 4, simulation results for elimination/enhancement of narrow-band complex signals are presented and discussed. Finally, section 5 concludes the paper.

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II. Complex digital filter circuit derivation

We shall try now to develop a fourth-order variable complex digital filter satisfying the following requirements. The filter must be of IIR type in order ensure higher selectivity with lower order transfer function. It should contain canonical number of multipliers and delays and it must be free of parasitic effects like delay-free loops. The last problem can be avoided after applying a proper design procedure, like the proposed in [5] new method of design of complex variable BP/BS filters with independently tuneable central frequency and BW. That method provides wider range of tuning of the BW and higher freedom of the central frequency tuning. The new filter should have low coefficient sensitivity in order to decrease the degradation of its magnitude characteristic after the coefficients are quantized. And we try to develop narrow-band variable BP filters because such filters are needed for most of the practical applications.

A proper real LP second-order circuit, such as to satisfy above-described requirements is the filter section shown in Fig. 1, good properties of which are certainly proved in [5], [6] after detailed investigations.

In order to make the LP filter BW-variable, we apply on $H(z)$ (2) the Constantinides LP to LP spectral transformation:

$$z^{-1} \to \frac{z^{-1}-\beta}{1-\beta z^{-1}} . \qquad (1)$$

Afterwards expand the coefficients functions of β into Taylor series taking the linear terms only in order to avoid delay-free loops, according to the well known procedure proposed in [7].

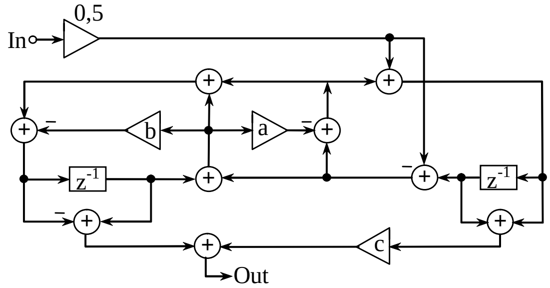

Fig.1 Real-coefficients second-order low-sensitivity LP filter section

The transfer function this section realizes is:

$$H(z)=\frac{(ac-b)-(a-2)}{2c}\cdot\frac{1+2\frac{(ac+b)+(a-2)}{(ac-b)-(a-2)}z^{-1}+z^{-2}}{1+(2a+b-2)z^{-1}+(1-b)z^{-2}}. \qquad (2)$$

Then we replace the multiplier coefficients *a*, *b* and *c* in the section (Fig. 1) by the composite multipliers ($\hat{a}$, $\hat{b}$ and $\hat{c}$) containing also β, after applying the LP to LP spectral transformation (1). Thereby, the transfer function $H_V(z)$ of the BW-variable LP filter is obtained. It is easy to show that $\hat{c}$ is independent of β, i.e. $\hat{c}=c$. Thus, we have two variable by β multipliers $\hat{a}=a+\beta.f$ and $\hat{b}=b+\beta.e$, where f and e are coefficients calculated by means of a and b as follows: $f=a(2a+b-4)$ and $e=b(2a+b-2)$. The structure of a composite multiplier is shown in Fig 2

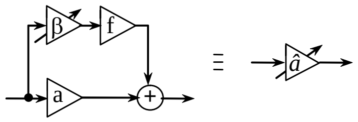

Fig.2 The composite multiplier obtained after Constantinides LP to LP spectral transformation and followed Taylor series truncation

The rotation transformation:

$$z=ze^{-j\theta} \text{ or } z^{-1}=z^{-1}(\cos\theta+j\sin\theta)\ , \qquad (3)$$

applied on $H_V(z)$ generates the BP complex coefficient transfer function $H_{CV}(z)$, which is variable with respect to both the BW and the central frequency. As complex function $H_{CV}(z)$ can be presented by two real coefficients fourth-order functions:

$$H_V(z)\xrightarrow{z=ze^{-j\theta}}H_{CV}(z)=H_R(z)+jH_I(z)\ , \qquad (4)$$

and realized by the complex structure in Fig.3.

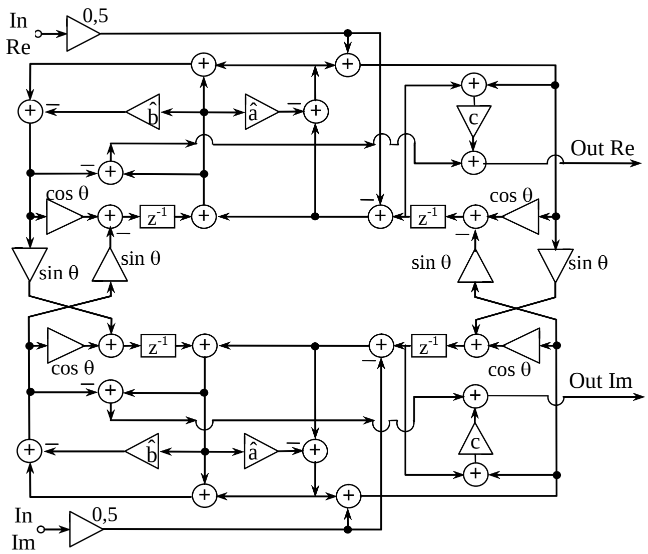

Fig.3 Complex-coefficient fourth-order low-sensitivity BP filter section

Having two inputs and two outputs (both couples real and imaginary), the structure is described by four real-coefficient BP transfer functions two by two equal with ±sign and related to $H_{CV}(z)$ as follows:

$$H_{RR}(z)=H_{II}(z)=H_R(z)=\frac{B_2-B_3}{2c}\cdot\frac{1+A\left(\frac{4B_3}{B_2-B_3}+B_1\right)z^{-1}+\left[\frac{2(B_2+B_3)(B_1-2)}{B_2-B_3}+(1-B_4)(A^2-C^2)\right]z^{-2}+A\left(\frac{2B_4(B_2+B_3)}{B_2-B_3}+B_1-2\right)z^{-3}+B_4z^{-4}}{1+2A(B_1-2)z^{-1}+\left[(B_1-2)^2+2B_4(A^2-C^2)\right]z^{-2}+2AB_4(B_1-2)z^{-3}+B_4^2z^{-4}} \qquad (5)$$

and

$$H_{RI}(z)=-H_{IR}(z)=H_I(z)=\frac{B_2-B_3}{2c}\cdot\frac{C\left(\frac{4B_2}{B_2-B_3}-B_1\right)z^{-1}+2AC(1-B_4)z^{-2}+C\left[(B_1-2)-\frac{2B_4(B_2+B_3)}{B_2-B_3}\right]z^{-3}}{1+2A(B_1-2)z^{-1}+\left[(B_1-2)^2+2B_4(A^2-C^2)\right]z^{-2}+2AB_4(B_1-2)z^{-3}+B_4^2z^{-4}} \qquad (6)$$

where

$$B_1=2\hat{a}+\hat{b};\ B_2=\hat{a}c-\hat{b};\ B_3=\hat{a}-2;\quad B_4=1-\hat{b};\quad A=\cos\theta;\quad C=\sin\theta\ . \qquad (7)$$

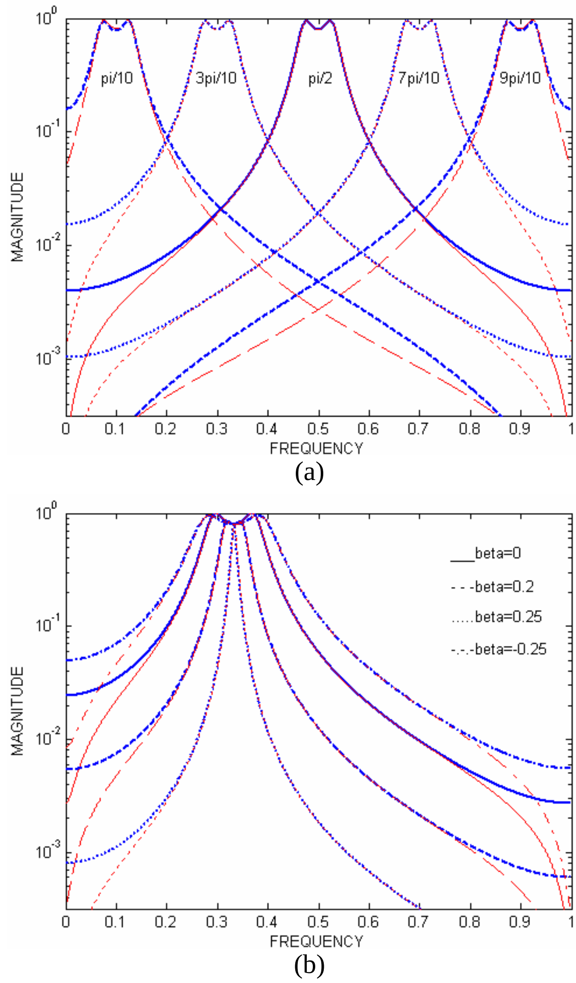

Fig.4 Magnitude responses of the variable complex BP fourth-order filter for (a) different values of θ; (b) different values β.

The BW of the obtained variable complex filter can be turned by changing of β, while the central frequency can be shifted by changing of θ.

In Fig.4a it is shown how the central frequency of the magnitude responses of $H_R(z)$ (5) (the bold lines) and $H_I(z)$ (6) is turned by trimming of θ, while $\beta=0.13$ is fixed. The tuning of the BW by changing β and fixed $\theta=\pi/3$ is without problems for rather wide range as it is shown in Fig.4b.

It is well known that all the properties (sensitivity including) of the real LP/HP prototype will be inherited by the complex BP/BS filter, obtained after the transformation (3) is applied. It was shown in [6] that both filters real and complex have very low coefficient sensitivity. The behavior of the complex section in a limited word-length environment is also investigated (Fig.5). Obviously the magnitude response isn't destroyed even for very short word-length.

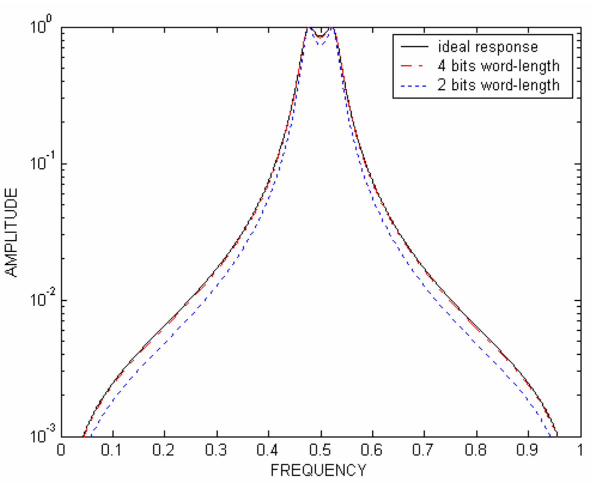

Fig.5 Magnitude responses of the variable complex BP fourth-order filter section for different word-length $(\theta=\pi/2;\beta=0.15)$.

III. ADAPTIVE COMPLEX NARROWBAND FILTERING

In Fig. 6 the block-diagram of the new adaptive filter is shown. In the following we consider the input/output relations for corresponding BP/BS filters - equations (8)-(11).

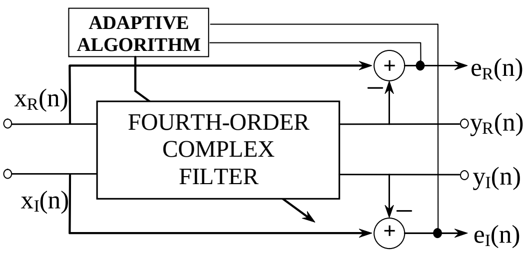

Fig. 6 Block-diagram of a versatile adaptive complex narrowband filter

For the BP filter we have the following real output:

$$y_R(n)=y_{R1}(n)+y_{R2}(n), \quad (8)$$

where $y_{R1}(n)$ is the part of the real output signal when the input signal is $x_R(n)$, while $y_{R2}(n)$ is obtained when $x_I(n)$ is the input signal. Similarly, for the imaginary output we get:

$$y_I(n)=y_{I1}(n)+y_{I2}(n). \quad (9)$$

For the band-stop filter we have - real output:

$$e_R(n)=x_R(n)-y_R(n), \quad (10)$$

and imaginary output

$$e_I(n)=x_I(n)-y_I(n). \quad (11)$$

The cost-function is the power of band-stop filter output signal:

$$[e(n)e^*(n)], \quad (12)$$

where

$$e(n)=e_R(n)+je_I(n). \quad (13)$$

We apply a Least Mean Squares (LMS) algorithm to update the filter coefficient responsible for the central frequency as follows:

$$\theta(n+1)=\theta(n)+\mu\mathrm{Re}[e(n)y'^*(n)], \quad (14)$$

μ is the step size controlling the speed of convergence, (*) denotes complex-conjugate, $y'(n)$ is the derivative of $y(n)=y_R(n)+jy_I(n)$ with respect to the coefficient subject of adaptation.

IV. EXPERIMENTS

We test our filter for elimination/enhancement of narrowband complex signals. Input signal is a mixture of white noise and complex (analytic) sinusoidal signal.

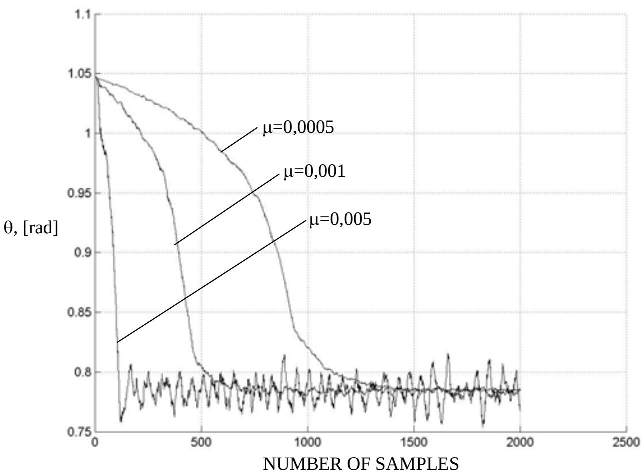

Fig. 7: Trajectories of filter coefficient θ for different step size μ

In Fig. 7 the learning curves for different values of step size μ are shown. It is seen that for $\mu>0.001$ the convergence is so fast that it takes less than 100 iterations to adapt.

In Fig. 8 the results for different filter bandwidth are presented. It is obvious that the filter is behaving quite conventionally. It should be mentioned that if some other (non low-sensitivity) LP-prototype was used, the coefficient β could not take values larger than 0.06-0.1 without destroying the magnitude shape. Thus we can always obtain a faster convergence because of the wider BW.

Finally, in Fig. 9 we show the behaviour of our filter for quite a wide range of sinusoidal frequencies. It is seen that in all the cases our filter converges to the proper frequency value.

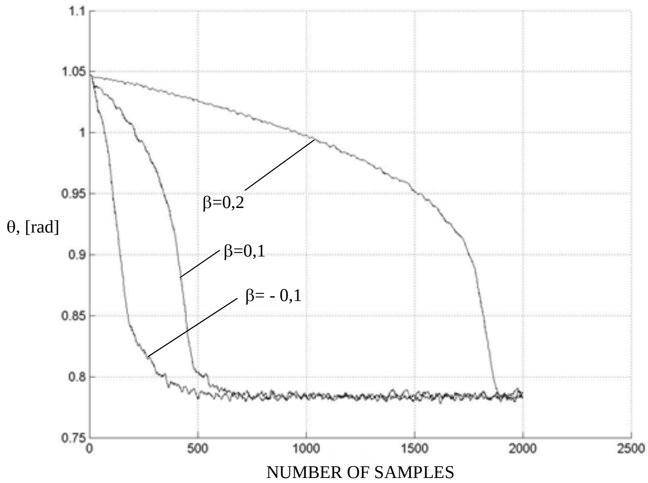

Fig. 8: Trajectories of filter coefficient θ for different BW

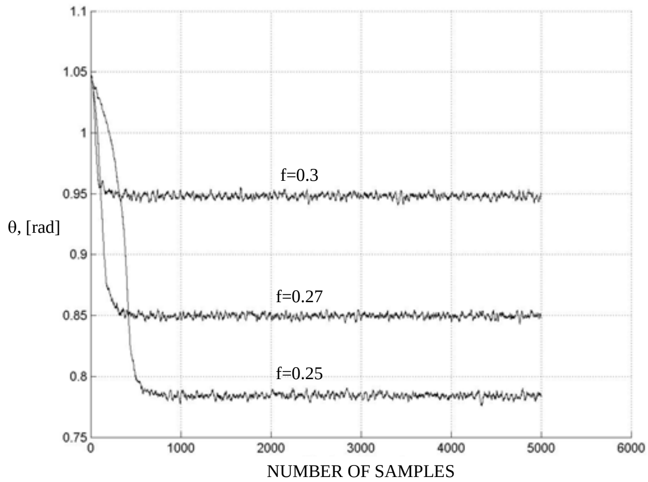

Fig. 9: Trajectories of filter coefficient θ for different central frequency

V. CONCLUSIONS

A very low-sensitivity variable complex fourth-order BP filter section with very precise and completely independent tuning of the BW and central frequency was designed in this work. The improved method of design permits the BW to be tuned in much wider frequency range compared to other well known methods and without any requirements to fix some of the cutoff frequencies. By changing the transformation factor θ, the central frequency is able to be tuned over the entire frequency band also adaptively by applying an LMS algorithm. The accuracy of tuning is very high and it is possible to use coefficients with shorter word-length decreasing thus the power consumptions for both, adaptive filtering and updating the coefficients.

The convergence of the adaptive algorithm for the developed variable complex filter is investigated experimentally and the efficiency of the adaptation is indisputable proved.

The main advantages of the proposed adaptive complex structure could be summarized as follows: very low coefficient sensitivity providing resistance against quantization effects, very high accuracy of tuning and adaptation – features very important for telecommunication applications. The structure is demonstrating a very fast convergence and has a low computational complexity.

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