# Determination of the Signal/Noise Ratio at the Input of Cable TV Amplifier Using Symmetrical N-Level Code 

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#### Abstract

The paper discusses the change in signal amplitude when interference is present during the transmission of a digital signal in the reverse channel of cable TV networks. We determine the level of noise and the signal/noise ratio as a function of the type of signals and the number of levels of the linear signal. This leads to the desired specifications of cable amplifiers, given a type of linear signal.


Keywords - N-level code, Signal-to-noise ratio, Amplifier, Bit error probability, Intersymbol interference.

## I. Introduction

Digital data transfer systems using the so-called "reverse channel" in coaxial CATV networks are being developed against the background of the existing analogue nets. This entails a flexible approach in the solution of the problems related to digital signal transfer.
The digital signal that is transmitted along the line is affected by noises, which bring about errors in certain symbols; the number of errors is expressed by the bit error rate [1].
Errors that appear during digital signals transmission are caused by noises and disturbances whose moment values exceed permissible boundaries, henceforth the appearance of many redundant pulses or the disappearance of those available. To avoid errors it is necessary to keep the proper signal-to-noise ratio at the input of the trunk line. When using multi-level signals errors could be generated whenever the moment value of noise exceeds the permissible level, and therefore, will have a sign opposite to that of the pulse itself. If the symbol " 0 " is accepted then the excess over the permissible level of noise is equally adverse for each polarity of this voltage.

Permissible noise voltage value $B_{0}$ should not exceed $0,5 B_{0}$ [6], and the probability of excess over that quantity will determine the bit error rate at the input of any amplifier.

[^0]
## II. EXPLANATION

Signal-to-noise ratio obtained in [1] is used to calculate the bit error rate and also allows to determine the properties which should be featured by given amplifier that is connected to the reverse channel of CATV networks at a preset signal $S_{p}(t)$. Signal-to-noise ratio according to [1] is determined by the following way:

$$
\begin{equation*}
b_{0}=\frac{B_{0}}{2 U_{S}}, \tag{1}
\end{equation*}
$$

where $B_{0} / 2$ is the permissible pulse amplitude and $U_{S}$ is the effective value of noise voltage.

For symmetrical N-level code [6]:

$$
\begin{equation*}
B_{0}=\frac{1}{(N-1)} \cdot S_{p}\left(t_{1}\right), \tag{2}
\end{equation*}
$$

where $t_{1}$ is the moment of decision making. If there are disturbances between symbols, the amplitude of the signal will vary as the result of the impact of neighboring symbols.

The digital signal used in digital systems to transmit data is called isochronous which means that its symbols appear at intervals with period of appearance $T_{T N}$. This signal can be rendered as [2, 6]

$$
\begin{equation*}
S(t)=\sum_{n=-\infty}^{\infty} S_{p}\left(t_{1}-n T_{T N}\right), \tag{3}
\end{equation*}
$$

where $n$ is the ordinal number of the symbol.
When reading the worst disturbances, caused whenever neighboring symbols obtain largest values, Eq. (3) will appear as:

$$
\begin{equation*}
B_{0}=\frac{S_{p}(t)}{N-1}-\sum_{n=-\infty}^{\infty} S_{p}\left(t_{1}-n T_{T N}\right), \tag{4}
\end{equation*}
$$

The disturbance power at the point of decision-making, i.e. at the amplifier output, will be:

$$
\begin{equation*}
U_{s}^{2}=\frac{1}{2 \pi} \int_{-\infty}^{\infty} p^{2}(\omega) n_{i n}(\omega) d \omega \tag{5}
\end{equation*}
$$

However, the transfer characteristic of an amplifier could be written in the following kind $[2,4]$ :

$$
\begin{equation*}
S_{p}(\omega)=S_{1}(\omega) \cdot T(\omega) \cdot P(\omega) \tag{6}
\end{equation*}
$$

from where:

$$
\begin{equation*}
P(\omega)=\frac{S_{p}(\omega)}{S_{1}(\omega) \cdot T(\omega)} . \tag{7}
\end{equation*}
$$

Hence taking into account relationships Eqs. (1), (4), (5) and (7) the most unfavorable value of the ratio between signal power and noise power can be expressed in the following way:

$$
\begin{equation*}
b_{0}^{2}=\frac{\left\{S_{p}(t)-(N-1) \sum_{n=-\infty}^{\infty} S(p)\left(t-n T_{T N}\right)\right\}^{2}}{(N-1)^{2} \frac{2}{\pi} \int_{-\infty}^{\infty} n_{i n}(\omega) \frac{S_{p}{ }^{2}(\omega)}{S_{1}{ }^{2}(\omega)} T^{-2}(\omega) d \omega} . \tag{8}
\end{equation*}
$$

It is evident from Eq. (8) that the signal $S_{p}(t)$ should be selected in such a way that will help reduce symbol disturbances to the minimum level. Furthermore the spectrum signal $S_{p}(\omega)$ should be as narrow as possible because, in this way, the influence of noises $n_{\text {in }}$ occurring at both inputs is reduced. At the same time, the input spectrum signal $S_{1}(t)$ $S_{1}(\omega)$ should be as large as possible in terms of noises $n_{i n}$.
For most cables and especially coaxial ones, the transition characteristic $T(\omega)$ cane be expressed as [1, 6]:

$$
\begin{equation*}
T(\omega)=e^{-\alpha} \sqrt{\frac{2 \omega}{\omega_{T}}} \tag{9}
\end{equation*}
$$

where $\alpha$ is the measured kilometer attenuation (in dB or Neper) for frequency that is numerically equal to the half value of the transfer rate of digital linear signal, and $\omega_{T}$ is clock frequency.

Setting Eq. (9) into Eq. (8) and assuming that the noise $n_{i n}(\omega)$ is frequency independent (this is valid for the thermal noise which is the main reason for the noise caused in the coaxial channel) then the expression Eq. (8) could be written as:

$$
\begin{equation*}
b_{0 N}^{2}=\frac{\left\{S_{p}\left(t_{1}\right)-(N-1) \sum S_{p}\left(t_{1}-n T_{\text {TN }}\right)\right\}^{2}}{(N-1)^{2} n_{i n} \frac{2}{\pi} \int_{-\infty}^{\infty} \frac{S_{p}^{2}(\omega)}{S_{1}^{2}(\omega)} e^{2 A \sqrt{\frac{2 \omega}{\omega_{t}}}} d \omega} \tag{10}
\end{equation*}
$$

By way of deduction it is possible to find the signal-tonoise ratio depending on $S_{p}(t)$ and $S_{1}(t)$ and the number of levels of the linear signal. Optimal decision is achieved for the signal $S(t)$ which is expressed by the Dirak function $h_{1} \delta(t)$ and signal $S(t)$ whereas for the signal $S_{p s}(t)$ adopting the kind shown below:

$$
\begin{equation*}
S_{p s}(t)=h p \frac{\sin \pi \frac{t}{T_{T N}}}{\pi \frac{t}{T_{T N}}}, \tag{11}
\end{equation*}
$$

whose spectrum is expressed by $[3,4]$ :

$$
S_{p S}(\omega)=\left\{\begin{array}{ll}
\frac{h p}{2} T_{T N} & \text { for } \omega \leq \frac{\omega_{T N}}{2}  \tag{12}\\
0 & \text { for } \omega>\frac{\omega_{T N}}{2}
\end{array} .\right.
$$

The function $S_{p s}(t)$ shows that the intersymbol transfer disturbances at the moments of decision $n T_{T N}$ are absent (at $t_{1}$ $=0$ ) and the frequency band is limited by the quantity $\omega_{T N} / 2$. By inputting the signals $S_{p s}(t), S_{p s}(\omega), S_{1} \delta(t)$, and also assuming that the mean value of the signal $h_{1} \delta(t)$ will be the same irrespective of the number of levels and equal to $h_{1 A V}$, we get the following simplified expression of Eq. (10):

$$
\begin{equation*}
b_{0 \text { ovs }}{ }^{2}=\frac{h_{1 A V} 2 A \sqrt{\frac{f_{T N}}{f_{T}}}}{4 n_{\text {in }} f_{T N}(N-1)^{2} e^{2 A \sqrt{\frac{f_{f_{T}}}{f_{T}}}}} . \tag{13}
\end{equation*}
$$

It is evident from [2,3] that with $N$-level linear code, the number of transferred states for a unit of time should be larger than the number of the possible states for a shared time in the transmitted binary code, i.e. to fulfill the inequality

$$
\begin{equation*}
\frac{N^{k}}{T} \geq \frac{2^{M}}{T} \tag{14}
\end{equation*}
$$

hence

$$
\begin{equation*}
K \log _{2} N \geq M \tag{15}
\end{equation*}
$$

where $K$ is the number of symbols expressed in N -level code which is used in the transfer of $M$ symbols of binary code for a time period $T$. With the transformation of the binary code into linear we should use the condition

$$
\begin{equation*}
T_{T} M=T_{T N} K \tag{16}
\end{equation*}
$$

where $T_{T}$ is the binary symbol duration, and $T_{T N}$ - the N -level symbol duration.

From Eq. (16) follows that the transmission rate of the linear signal $f_{T N}=1 / T_{T N}$ will include

$$
\begin{equation*}
f_{T N}=\frac{K}{M} f_{t} \tag{17}
\end{equation*}
$$

where $f_{T}=1 / T_{T}$ is the binary signal transmission rate. Setting Eq. (17) in Eq. (18) we get the condition, which determines the linear signal transmission rate

$$
\begin{equation*}
f_{T N} \geq \frac{f_{T}}{\log _{2} N} \tag{18}
\end{equation*}
$$

or

$$
\begin{equation*}
f_{T N}=\frac{f_{T}}{\log _{2} N}(1+r) \tag{19}
\end{equation*}
$$

where $r$ is the redundancy in linear signal
Taking into account Eqs. (17) and (19) and dividing the left part of Eq. (13) by $b_{0 \delta 2 S}^{2}$, and assuming that $r=0$, we get:

$$
\begin{equation*}
\frac{b_{0 \delta N S}}{b_{0 \delta 2 S}^{2}}=\frac{\sqrt{\log _{2} N} \cdot e^{2 A\left(1-\frac{1}{\sqrt{\log _{2} N}}\right)}}{(N-1)^{2}} . \tag{20}
\end{equation*}
$$



Fig. 1. Signal-to-noise ratio for multilevel code normalized in accordance with the quantity signal-to-noise for two-level signal.

This function has an extreme in relation to $N$, which depends on the quantity $A$ (Fig. 1). The error in the result bay the use of the function $S_{1}(t)$ instead of function $S_{1} \delta(t)$ (instead of using in praxis rectangular impulse with working fill $50 \%$ ), amounts to, for example, $1 d B$.

## III. CONCLUSION

In the presence of signal $S_{p}(t)$, which causes intersymbol transitional disturbances, the results will be considerably better than those shown in Fig. 1 The difference will be greater in proportion with the level of symbols belonging to the linear signal. Consequently, by the transmission of multilevel digital signal correction in preamplifying stage of the line amplifiers should be carried out very precisely. Very effective in this case is the requirement for step reverse feedback with the regenerator output at the point of decision making. This helps to suppress intersymbol disturbances.

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