Determination of the BER Characteristics with M-QAM Signal by Weibull Noise Distribution in Hybrid Cable TV Networks

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Abstract – **The paper discusses the influence of noise in hybrid cable TV networks with M-QAM signal. Non-linear distortions and thermal noise are unknown factors in CATV systems. This requires the determination of CSO and CTB levels, particularly with the use of digital signals. We use the properties of Weibull distribution to model the statistical properties of the "composite" distortions, leading to the BER characteristics of the M-QAM signal.**

Keywords – **BER, M-QAM, Weibull noise distribution, CSO, CTB.**

I. Introduction

The use of hybrid coaxial networks in CATV is a key technology in expanding the transferred services and in improving their performance quality [2, 4, 5]. Taking in consideration the use of M-array quadrature amplitude modulation (M-ary QAM) for transmitting data along with conventional amplitude modulation (AM) there should be some due attention paid to certain features, which occur with such modulations only. As is known the main line is built up by optic cable whose branches are coaxial lines [2, 3]. The optical part of the line causes the rise of thermal noise, nonlinear distortions and impulse noises. On the other hand the coaxial part of the network becomes the source of thermal noises, nonlinear distortions and low frequency noises.

Of all disturbances occurring in TV channels nonlinear distortions are best studied because they are active in both optical and coaxial part of the network.

Nonlinear products of the kind $f_m \pm f_n$ are called Second Order Intermodulation Distortion (IMD2) whereas those of the kind $f_m \pm f_n \pm f_p$ are called Third Order Intermodulation Distortion (IMD3). These distortions, coming from different channel signals, could come across some channel and by plying one over another cause additional phase distortions, which are known as "composite" distortions. In the existing CATV networks AM signal mixed with IMD2 is called Composite Second Order (CSO), and is in the area of *±1,25MHz* around the carrier of AM signal. Signal, which is mixed with IMD3, is called Composite Tipple Beat and is usually within the range of *±2,5MHz* around the carrier of AM signal. Generally, CSO cause most of the problems in the optic part of the net whilst CTB creates problems in the coaxial part. [5].

Nonlinear distortions as well as thermal noises are unknown factors in the Cable TV systems, which is why it is so important to determine the levels of nonlinear distortions, which would guarantee decent quality of digital service. The latter is closely related with the determination of the mean value of BER characteristic of the digital channel in presence of M-ary QAM signal. If the function of the random density of the probability distribution $P(u,v)$ is used it will be possible to estimate the BER characteristic of M-ary QAM signal.

Investigation of CSO and CTB disturbances during digital data transfer proves that they are closer in semblance to noise rather than frequency divided harmonics and, also, their level differences are considerable [6].

In this case the noise can be regarded as such with limited frequency band and lending itself to statistical description. Normally "composite" distortions are regarded as noise which is distributed by the Gauss law [2] and not taking into account the function $P(u,v)$ or the theoretical analysis of BER characteristic of M-ary QAM signals.

This article presents an optional use of statistical model of Weibull distribution whereby it is very convenient to model the mathematical features of "composite" distortions.

II. Explanation

Weibull distribution of a random function is rendered by the formula [7]

$$P(x) = C.\alpha.x^{\alpha-1}.e^{-cx^{\alpha}} , \qquad (1)$$

where C and α are determining parameters.

It is evident that a Weibull distribution of the random quantity $P(x)$ has the same form as that of a Reylei distribution which in fact is an amplitude distribution by Gaussian law. Accordingly, the Weibull distribution can be applied to all types of "composite" distortions by carefully selecting the level of asymmetry α no matter whether they have the same behavior or not.

A randomly selected Weibull variable could be rendered in a complex form by the following way

$$r = u + jv,$$

where u and v are the components "in phase" and "quadrature" of the variable. In this case the normal density of

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distribution of the probability *P(u,v)* could be rendered by the expression

$$P(u,v)=\frac{1}{2\pi\sigma^2}\cdot\frac{\alpha}{2}\left(u^2+v^2\right)^{\frac{\alpha}{2}-1}\cdot e^{-\frac{1}{2\sigma^2}\left(u^2+v^2\right)^{\frac{\alpha}{2}}}. \qquad (2)$$

In Eq. (2) σ^2 is the signal dispersion. In this particular case it is the variable, which is connected to the power of the Weibull variable *r*.

By inserting in Eq. (2) the parameter

$$b=\sqrt[a]{2\sigma^2}\ , \qquad (3)$$

then the relationship between a, b and energy *E* cane be expressed through the Γ-function

$$E\left(u^2+v^2\right)=\frac{2b^2}{a}\Gamma\left(\frac{2}{a}\right). \qquad (4)$$

The left part of Eq. (4) cane be substituted for the alteration interval N_0 in case *u* and *v* have identical spectrum density of distribution:

$$E\left(u^2+v^2\right)=2N_0\,. \qquad (5)$$

By substituting Eqs (3), (4) and (5) in Eq.(2) we get:

$$P(u,v)=\frac{a}{2}\cdot\frac{\left(u^2+v^2\right)^{\frac{a}{2}-1}}{n^{\frac{a}{2}}}\cdot\exp\left[\left(-\frac{1}{n^{\frac{a}{2}}}\right)\cdot\left(u^2+v^2\right)^{\frac{a}{2}}\right], \quad (6)$$

where $n=\pi\left[\frac{aN_0}{\Gamma\left(\frac{2}{a}\right)}\right]^{\frac{a}{2}}$.

Summarizing the expression for normal density of probability distribution we get:

$$P(u)=P(v)=\int_{d}^{+\infty}\int_{-\infty}^{+\infty}P(u,v)dudv=P_q\ . \qquad (7)$$

It is well known that BER characteristic for M-QAM signal using the Gray code is expressed by [8]

$$P_e=\frac{2}{\log_2(M)}\left(\sqrt{M}-1\right)P_q\,, \qquad (8)$$

where P_q is the probability for the undesired amplitude of the output signal to go beyond the appropriate level *q* of the coherent detector.

Then, by substituting Eqs. (6) and (7) in Eq. (8) we get Eq. (9) for the signal P_e:

$$P_e=\frac{2}{\log_2(M)}\left(\sqrt{M}-1\right)\int_{d}^{+\infty}\int_{-\infty}^{+\infty}P(u,v)dudv \qquad (9)$$

i.e.

$$P_e=\frac{2}{\log_2 M}\left(\sqrt{M}-1\right)\times$$
$$\times\int_{d}^{+\infty}\int_{-\infty}^{+\infty}\frac{a}{2}\frac{\left(u^2+v^2\right)^{\frac{a}{2}-1}}{n^{\frac{a}{2}}}\exp\left[\left(-\frac{1}{n^{\frac{a}{2}}}\right)\left(u^2+v^2\right)^{\frac{a}{2}}\right]. \quad (10)$$

Since *d* is the minimum distance between two adjacent symbols it can be expressed by Eq. (8) by means of the formula:

$$d=\sqrt{\frac{3T_S}{M-1}P_S}\ , \qquad (11)$$

where P_S is the average power of M-QAM signal and *Ts* is the time needed for the appearance of a symbol.

If in Eq. (6) it is assumed that $k=\frac{aN_0}{\Gamma\left(\frac{2}{a}\right)}$, then from Eqs. (4) and (11) it follows:

$$\frac{d}{\sqrt{kN_0}}=\sqrt{\frac{3T_SP_S}{k(M-1)N_0}}=\sqrt{\frac{3\Gamma\left(\frac{2}{a}\right)\delta}{a(M-1)}}\ , \qquad (12)$$

where $\delta=\frac{T_SP_S}{N_0}$.

In order to compare the results of both theoretical calculations and practical measurements, BER of 16-QAM signal for “composite” distortions were measured.

Figs. 1 and 2 show the results of BER measurement of 16-QAM signals for CSO and CTB and also the theoretical characteristics obtained in Eq. (10), which were compared with the BER characteristic of noise.

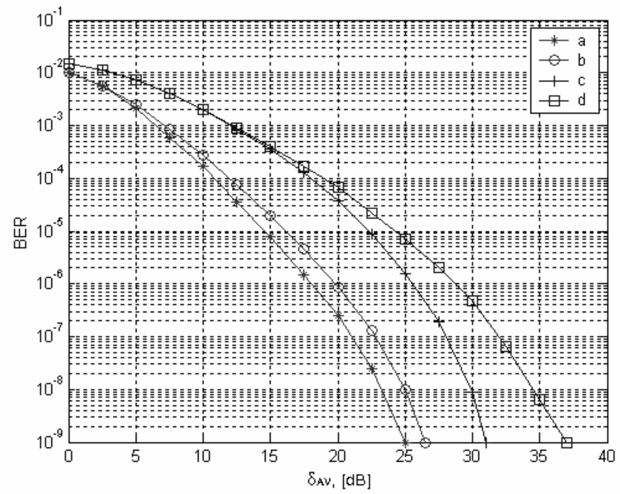

Fig. 1. BER as s function of δ_{AV} for 16-QAM signal in presence of CSO and random noise
a – theoretical results in presence of CSO
b – experimental results in presence of CSO
c – theoretical results in presence of random noise
d – experimental results in presence of random noise

From Fig. 1 it is evident that with values of error P_b for BER less than 10^{-3} the difference of δ_A with available noise and with CSO is about 2 dB. This result allows for the obtained properties of amplitude distribution of CSO and larger amplitudes with lower probabilities of occurrence than those of the noise itself.

The experimental results for BER of 16-QAM in presence of CTB are shown in Fig. 2

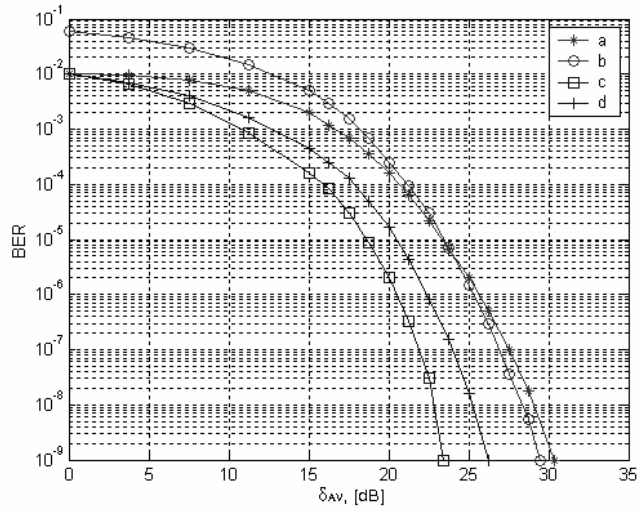

Fig. 2. BER as a function of δ_{AV} for 16-QAM signal in presence of CTB and random noise

a – theoretical results in presence of CTB
b – experimental results in presence of CTB
c – theoretical results in presence of random noise
d – experimental results in presence of random noise

It is evident that the results obtained for BER of CTB are very close to those of the noise for $P_b > 10^{-4}$, but after that they part smoothly. The connection between theoretical values and measured quantities is found in the fact that BER of the measured values are lower than the theoretical ones for $P_b > 10^{-5}$ and coincide for P_b in the area of 10^{-5} whereas for values $P_b < 10^{-5}$ it can be said that they fully coincide. For BER at $P_b < 10^{-5}$ the δ_{AV} for CTB have a *3dB* difference as compared with noise.

Accordingly, this model proves to be effective for BER at lower values.

III. CONCLUSION

The measured results for amplitude distribution show that the statistical properties of "composite" distortions are different from those of Gaussian process, i.e. Weibull distribution can be applied in calculating "composite" distortions. These results also show that the level of asymmetry of CSO is smoother than that of the noise unlike the asymmetry of CTB whose fluctuations of amplitude are much more uneven than those of the noise.

Also, the results obtained for BER show that the influence of CTB on these characteristics is stronger than noise. This is why it is important to give special attention to the CTB level control rather than to the level of both noise and CSO. The small difference between theoretical and experimental results in BER analysis by means of Weibull distribution indicates that such type of analysis is appropriate with CATV system design.

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