Fast Algorithm for Color Space K-L Image Transform

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Abstract - **The *Karhunen–Loeve (KL) Transform* is an optimal transform among all discrete transforms, which is used in image processing and computer vision for several tasks such as face and object recognition. Its computational demands and its batch calculation nature have limited its application.**
In this paper, we will propose a new fast algorithm for *Karhunen-Loeve Color Space Transform* (KLCST) which will facilitate the computational process and will allow reducing the data redundancy in the color space. This algorithm is faster in typical applications and is advantageous for real time image processing.

Keywords – **Fast Karhunen-Loeve Color Space Transform, reduction of data redundancy, feature extraction, color segmentation, image compression.**

I. Color Transforms: State of Art

The color space transform is critical for color feature extraction and data redundancy reduction. Many attempts have been made to model color perception by researchers of various fields: psychology, perception, computer vision, image retrieval, and graphics. Some of these resulted in well defined color spaces [1], [2]. The list of color spaces is almost endless. A few of the most important color spaces are:

- RGB – (Red, Green and Blue);
- YCbCr – (Luminance, Chrominance and Chrominance);
- HSV- (Hue, Saturation and Value);

In the present research, we adapt the W3C definition of color spaces: "A color space is a model for representing color numerically in terms of three or more coordinates." (http://www.w3.org/Graphics/Color/sRGB.html).

Each pixel of the image can be represented as a point in a 3D color space. Commonly used color space for image retrieval include RGB, YCbCr, HSV (or HSL, HSB). There is no agreement on which is the best.

RGB space is a widely used color space for image display. It is composed of three color components red, green, and blue. These components are called "additive primaries" since a color in RGB space is produced by adding them together.

YCbCr is a color space where Y represents the luminance of a color, while Cb and Cr represent the chromaticity of a color gray-vividness and brightness-darkness. One of the advantages of this transformation is that it reduces the psycho visual redundancy of an image.

In HSV (or HSL, or HSB) space is widely used in computer graphics and is a more intuitive way of describing color. The HSV depict the colors in a non linear space aiming closeness to the human perception. The three color components are *hue*, *saturation* (lightness) and *value* (*brightness*).

All these color transforms do not depend of the contents of an image except the Karhunen-Loeve transform. Karhumen-Loeve transform (KLT) is a unitary transform that diagonalizes the covariance or the correlation matrix of a discrete random sequence. The K-L transform has found many applications in traditional fields such as statistics and communication. In computer vision, it is used for a variety of tasks such as face recognition, object recognition, motion estimation, visual learning, and object tracking [3], [4], [5]. It is, however, used infrequently as it is dependent on the statistics of the sequence i.e. when the statistics change so also the KLT. Because of this signal dependence, generally it has no fast algorithm.

Our goal is propose a new fast algorithm for K-L optimal color transform which will facilitate the computational process and will allow the speed up of the process and the reducing of the data redundancy in the color space.

The first section of this paper introduces some of the most utilized color spaces and also presents a brief revue of the K-L transform. Afterwards in the second section we present the new algorithm for KLCST followed by the third section, where we present the evaluation techniques for this new algorithm. In section IV we will present the experimental results and a comparison between the performance of the KLCST and the introduced in section I color space transforms for reducing the data redundancy. We end this paper with plans for future research and the final conclusions.

II. Algorithm for Fast Karhunen-Loeve Color Space Transform

KLT is utilized as a tool to extract the significant inter-component information and eliminate the redundancy. KLT is linear transformation which uses the signal statistics to define a rotation of the original images in such a way that the new axes are orthogonal to each other and point the direction of decreasing order of the variances. The transformation components are totally uncorrelated. KLT is a transform depending upon the data to be transformed. Its matrix consists of the eigenvectors derived from the covariance matrix, so a KLT matrix is an orthogonal matrix. This transform is the optimal statistic transform and the most effective technique for data decorrelation.

In this paper all the images used are in RGB color space. The work is realized in two section containing twelve steps [5]:

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Section I – the direct transform algorithm:

Step 1. Calculate the mean values $\overline{R}, \overline{G}, \overline{B}$ for each color component of the S pixels, where S is the total number of pixels in the image; compare the calculated mean values with the corresponding threshold (T_{Green}, T_{Red}, T_{Blue}); eliminate a mean value if smaller than the threshold (this step will be explained in section III of the paper).

Step 2. Calculate the elements of the covariance matrix $[K_c]$:

$$k_{11} = k_1 = \frac{1}{S}\sum_{s=1}^{S}(R_s - \overline{R})^2 \quad (1)$$

$$k_{22} = k_2 = \frac{1}{S}\sum_{s=1}^{S}(G_s - \overline{G})^2 \quad (2)$$

$$k_{33} = k_3 = \frac{1}{S}\sum_{s=1}^{S}(B_s - \overline{B})^2 \quad (3)$$

$$k_{12} = k_{21} = k_4 = \frac{1}{S}\sum_{s=1}^{S}(R_s - \overline{R})(G_s - \overline{G}) \quad (4)$$

$$k_{13} = k_{31} = k_5 = \frac{1}{S}\sum_{s=1}^{S}(R_s - \overline{R})(B_s - \overline{B}) \quad (5)$$

$$k_{23} = k_{32} = k_6 = \frac{1}{S}\sum_{s=1}^{S}(G_s - \overline{G})(B_s - \overline{B}) \quad (6)$$

Step 3. Find the three coefficients a, b and c of the principal component analysis equation:

$$\lambda^3 + a\lambda^2 + b\lambda + c = 0 \quad (7)$$

$$a = -(k_1 + k_2 + k_3) \quad (8)$$

$$b = k_1k_2 + k_1k_3 + k_2k_3 - k_4^2 - k_5^2 - k_6^2 \quad (9)$$

$$c = k_1k_6^2 + k_2k_5^2 + k_3k_4^2 - k_1k_2k_3 - 2k_4k_5k_6 \quad (10)$$

Step 4. Calculate the following values:

$$\varphi = \arccos\left[-q/2\sqrt{(|p|/3)^3}\right] \quad (11)$$

$$q = 2(a/3)^3 - (ab)/3 + c \quad (12)$$

$$p = -(a^2/3) + b \quad (13)$$

Step 5. Using the Cardano solutions for cubic equation calculate the three eigenvalues:

$$\lambda_1 = 2\sqrt{\frac{|p|}{3}}\cos\left(\frac{\varphi}{3}\right) - \frac{a}{3} \quad (14)$$

$$\lambda_2 = -2\sqrt{\frac{|p|}{3}}\cos\left(\frac{\varphi - \pi}{3}\right) - \frac{a}{3} \quad (15)$$

$$\lambda_3 = -2\sqrt{\frac{|p|}{3}}\cos\left(\frac{\varphi + \pi}{3}\right) - \frac{a}{3} \quad (16)$$

for $\lambda_1 \geq \lambda_2 \geq \lambda_3$

Step 6. Calculate the following intermediate values for each λ_m, for m=1, 2, 3;

$$M_m = k_5(k_2 - \lambda_m) - k_4k_6 \quad (17)$$

$$N_m = k_6(k_1 - \lambda_m) - k_4k_5 \quad (18)$$

$$P_m = (k_1 - \lambda_m)(k_2 - \lambda_m) - k_4^2 \quad (19)$$

$$A_m = \pm\sqrt{P_m^2 + M_m^2 + N_m^2} \quad (20)$$

Step 7. Calculate the eigenvectors $\vec{\Phi}_m = [\Phi_{1m}, \Phi_{2m}, \Phi_{3m}]^t$ of $[K_c]$:

$$\Phi_{1m} = M_m / A_m \quad (21)$$

$$\Phi_{2m} = N_m / A_m \quad (22)$$

$$\Phi_{3m} = P_m / A_m \quad (23)$$

Step 8. Define the transformed K-L color vector $\vec{L}_s = [L_{1s}, L_{2s}, L_{3s}]^t$ for each pixel.

$$\begin{bmatrix} L_{1s} \\ L_{2s} \\ L_{3s} \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{21} & \Phi_{31} \\ \Phi_{12} & \Phi_{22} & \Phi_{32} \\ \Phi_{13} & \Phi_{23} & \Phi_{33} \end{bmatrix} \begin{bmatrix} R_s \\ G_s \\ B_s \end{bmatrix} \quad (24)$$

Section 2 – the inverse transform algorithm:

Step 9. Chess board decimation of the L_2 and L_3 components, the L_3 component is decimated twice.

Step 10. Interpolation of the L_2 and L_3 components followed by filtering.

Step 11. Compute the inverse color transform and return to the original color space RGB.

Step 12. Calculate the MSE (Mean Square Error) and SNR (Signal to Noise Ratio) between the original image and the recovered image [6].

The samples to make the matrix $[K_c]$ may be all pixels in each image or all the pixels of all the images in a group [7]. The three eigenvalues are useful to determinate the concentration of the highest energy or which component is the most important. The larger the eigenvalues is the, the more important its corresponding component is [8].

III. Evaluation of the New KLCST Algorithm

For the evaluation of the effectiveness of the new KLCST algorithm we use the following characteristics:

A. Computation complexity

We have succeeded to reduce the complexity of KLCST. The proposed algorithm requires O(15xMN) operations, M and N being the dimensions of each of the color components of an image. We could accelerate more the computational process by introducing three thresholds for each color component in Step 1 of the proposed algorithm:

$$T_{Green} > T_{Red} > T_{Blue} \qquad (25)$$

After calculating the mean values of each color component, we compare their values with the corresponding threshold, if smaller than its corresponding threshold the mean value is put to 0 and ignored in the following steps. This way the computation complexity drops considerably. The values of the thresholds are chosen depending of the desired MSE (Mean Squared Error) to be achieved. The descending order of the thresholds is chosen using the distribution of the MacAdam's color spheres [1].

B. Quality of KLCST

For the evaluation of the quality of the KLCST we use the standard MSE and SNR [1], [6].

IV. EXPERIMENTAL RESULTS

A. Color images – the database

In this paper, a database of 4 image groups is used. All of them are part of the database SCoPIe (http://www-mrim.imag.fr/projets/#scopie).

1. Deserts and beaches: Fig. 1 a) and b).
2. Forest: Fig. 1 c) and d).
3. Interiors: Fig. 1 e) and f)
4. Architectures: Fig. 1 g) and h).

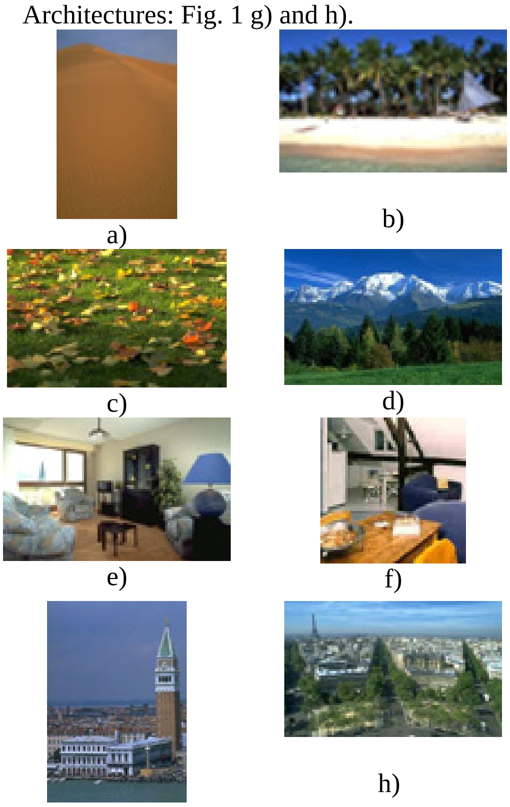


Fig. 1. Pictures in the database

B. Energy distribution

The goal of this experiment is to prove the effectiveness of the proposed algorithm:

- concentration of most of the energy in one component;
- decorrelation of the three components;
- minimization of the MSE;

We will also compare its performance with the performance of the YCbCr and HSV color transforms.

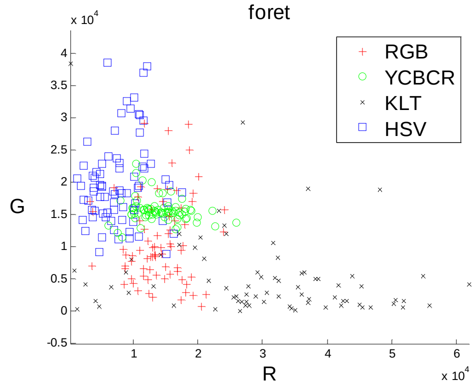
The K-L transform takes place in the RGB color space. The eigenvalues are sorted in the magnitude-descending order $\lambda_1 \geq \lambda_2 \geq \lambda_3$ and $\vec{\Phi}_1 = [\Phi_{11}, \Phi_{21}, \Phi_{31}]^t$ is the principal eigenvector. We calculate the relative power for each component of the transformed image. For better understanding the proportions between the energies concentrated in three transformed components, we introduce the value *eigPower*. This value represents the percentage of energies relative to the sum of all three.

We have obtained some very close results with individual image groups and even with some individual images. We have manually classified the images in four groups and in each group we observe that the principal eigenvector of each image is close to the principal eigenvector of the group. So instead of calculating a KLSCT for each image of each group we could calculate a single transform for all of the images in each group. We can observe the distribution of *eigPower* in the four groups on Table I:

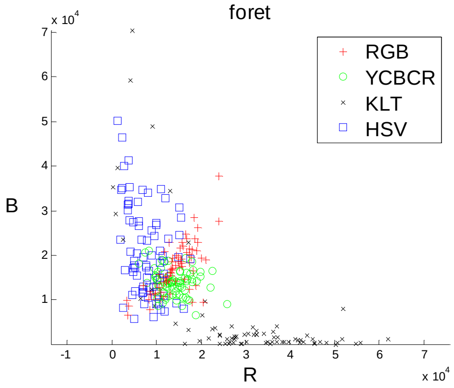
TABLE I
DISTRIBUTION OF EIGPOWER IN THE THREE COMPONENTS FOR EACH IMAGE GROUP

Group	Component 1	Component 2	Component 3
Desert/Beach	64.30	18.40	17.30
Forest	76.99	21.63	1.38
Indoor	48.66	35.51	15.82
City	58.53	23.66	17.81

We have performed the color transforms YCbCr and HSV using the same database of color images and calculated the relative power for each of the three components of the color transforms. Fig. 2 a) represents the relation between the relative energies between the firs and the second component. Fig. 2 b) represents the relation between the relative energies between the firs and the third component.



a)



b)

Fig. 2. Distribution of energies of the color bands

We can observe that the relationship between the energies of the first and the second component for the color transforms YCbCr and HSV is close but this is not the same for the K-L color transform. As opposed to the other color transforms the energies of the three bands are principally uncorrelated. The most energy is concentrated in the first component; the energies in the second and third component are almost non existent. From the proportional percentages between the *eigPower*, we can assume that the significance between the components in the K-L color space is roughly 4:2:1, this means that the first component can be left untouched, the second is twice as small as the original and the third is four times as small as the original. Using this resolution we can achieve a SNR within the limits of 28 to 33 dB when we restore the image to the original RGB color space. This also looks very nice and very useful in image applications. The percentages of the three relative powers give the information, which can be used in image retrieval and image compression during quantization.

V. Conclusions

The proposed algorithm for KLCST is elegant and simplified to the maximum. Its advantages can be formulated as following:

- the algorithm can be adapted to the statistical properties of each image, meaning the algorithm could be accelerated using the thresholds.
- the K-L color space transform is useful for many applications such as image retrieval, image coding, color segmentation, object recognition.

All these applications could be executed in real-time.

We note that farther improvement of the KLCST algorithm performance can be achieved if the principles of distributed arithmetic are used. Also it will be interesting to apply the integer numbers arithmetic to the Eq. (24) and to calculate the KLCST for each group of images.

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