Error Estimation of Adaptive 2D Interpolation of Images

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Abstract - **An analysis of 2D adaptive interpolation filter frequency characteristics for halftone image processing is presented. The adaptation is based on the local information of processed images and interpolation order is changed with zero or bi-linear. Experimental surfaces suggest that the effective use of local information contribute to minimize the mean-square error.**

Keywords - **digital signal processing, 2D image interpolation, local adaptation, image processing.**

I. Introduction

The basic methods for 2D interpolation of halftone images are separated in two groups: non-adaptive (zero, bi-linear or cubic interpolation) and adaptive interpolation [1], [2]. For non-adaptive methods with the increasing of interpolation order the sharp of brightens transitions are decreased. In other side with decreasing of interpolation order the artifacts ("false" contours) in the homogeneous areas are depicted [3], [4]. To reduce them a local adaptive method for 2D interpolation of images is developed [5]. Based on local characteristics of brightness, the interpolation order (zero or bi-linear) is changed and reduction of image distortions are achieved.

In this paper analysis of frequency characteristics of the developed adaptive 2D interpolation filter and estimation of mean-square errors for two kinds of interpolation are described.

II. Mathematical Description

The input m-level halftone image of size *M* x *N* and the output interpolated image of size *pM* x *qN* can be represented by the matrices:

$$\begin{aligned} &\mathbf{A}_{MxN} = \{\mathbf{a}(i,j)\ /\ i = \overline{0, M-1};\ j = \overline{0, N-1}\}, \\ &\mathbf{A}^{*}_{pMxqN} = \{\mathbf{a}^{*}(k,l)\ /\ k = \overline{0, pM-1};\ l = \overline{0, qN-1}\}, \end{aligned} \qquad (1)$$

where *p* and *q* are the interpolation coefficients in horizontal and vertical direction respectively [5].

The differences between each neighborhood pixels are given by the equations:

$$\begin{aligned} &\Delta_{2m+1} = \left| a(i+m, j) - a(i+m, j+1) \right|, \text{ for m} = 0,1; \\ &\Delta_{2n+2} = \left| a(i, j+n) - a(i+1, j+n) \right|, \text{ for n} = 0,1. \end{aligned} \qquad (2)$$

The four logical variables f_1, f_2, f_3 and f_4 are included, which depends from differences by the thresholds in horizontal (θ_m) and vertical (θ_n) directions as is shown in Eq. (3).

$$f_{2m+1} = \begin{cases} 1,\ if: \Delta_{2m+1} \geq \theta_m \\ 0,\ if: \Delta_{2m+1} < \theta_m \end{cases}; \quad f_{2n+2} = \begin{cases} 1,\ if: \Delta_{2n+1} \geq \theta_n \\ 0,\ if: \Delta_{2n+1} < \theta_n \end{cases}. \qquad (3)$$

Then each interpolated pixel can by presented as linear convolution of 4 basic elements:

$$a^{*}(k,l) = \sum_{m=0}^{1}\sum_{n=0}^{1} w_{m,n}(r,t)\, a(i+m, j+n), \qquad (4)$$

for $r = \overline{0,p}$; $t = \overline{0,q}$. Interpolation coefficients:

$$w_{m,n}(r,t) = F.Zr_{m,n}(r,t) + \overline{F}.Bl_{m,n}(r,t), \qquad (5)$$

are dependent from the logical function F, which give the type of the interpolation (zero or bi-linear):

$$F = f_1 f_3 \cup f_1 \overline{f_2} f_4 \cup f_2 f_3 \overline{f_4}.$$

The zero or bilinear interpolation coefficients are defined by the equations:

$$\begin{aligned} &Zr_{m,n}(r,t) = \frac{1}{4}\left[1 - (-1)^m \operatorname{sign}(2r-p)\right]\left[1 - (-1)^n \operatorname{sign}(2t-1)\right] \\ &Bl_{m,n}(r,t) = (-1)^{m+n}\left[1 - m - \frac{r}{p}\right]\left[1 - n - \frac{t}{q}\right] \end{aligned} . \qquad (6)$$

The dependence of functions F from the kind of brightness transitions and the coefficients f_1-f_4 as are given in [5] are shown on Table.1.

No.	f_1	f_2	f_3	f_4	F	Transitions
0	0	0	0	0	0	
1	0	0	0	1	0	
2	0	0	1	0	0	
3	0	0	1	1	0	
4	0	1	0	0	0	
5	0	1	0	1	1	
6	0	1	1	0	0	
7	0	1	1	1	1	
8	1	0	0	0	0	
9	1	0	0	1	0	
A	1	0	1	0	1	
B	1	0	1	1	1	
C	1	1	0	0	0	
D	1	1	0	1	1	
E	1	1	1	0	1	
F	1	1	1	1	1	

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The 2D interpolation process can by characterized by the generalized block scheme, shown on Fig.1.

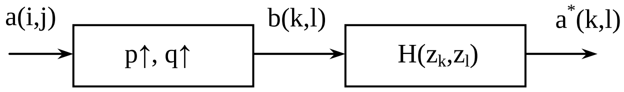

Fig.1 Generalized block scheme of the 2D interpolator

In the secondary sampling block the frequencies f_{sr} and f_{st} are increasing p and q times in the vertical and horizontal direction respectively. Consequently, the elements of the input image are added with zeros to receive b(k,l) from the equation:

$$b(k,l) = \begin{cases} a(k/p, l/q), \text{for } k = \overline{0,\pm(M-1)p}\ , l = \overline{0,\pm(N-1)q}, \\ 0 \text{ , for the other cases.} \end{cases} \quad (7)$$

The resulting image is processed by the 2D digital filter with z-transform function $H(z_k,z_l)$ and on our output the interpolated image $a^*(k,l)$ is received. From this the equation (4) can be presented as following:

$$a^*(k,l) = \sum_{m=0}^{1}\sum_{n=0}^{1} w_{m,n}(r,t).b(\left[\frac{k}{p}\right]+pm, \left[\frac{l}{q}\right]+qn), \quad (8)$$

where the operation $[x]$ is the largest integer not exceeding *x*.

Since the interpolation coefficients are repeating periodically, the distortion analysis can be accomplished for one block of the image. Then the amplitude and phase responses are described as following:

$$y(r,t) = \sum_{m=0}^{1}\sum_{n=0}^{1} w_{m,n}(r,t).x(pm,qn) \quad (9)$$

where x(pm,qn) are the basic elements in the current input block b(k,l) and y(r,t) are the interpolated elements in the output image block $a^*(k,l)$.

Zero interpolation characteristics.

After the 2D Z-transform of the equation (9) the result is:

$$H_{Zr}(z_r,z_t) = \frac{Y(z_r,z_t)}{X(z_r,z_t)} = \left[\frac{1-z_r^{-p}}{1-z_r^{-1}}\right]\left[\frac{1-z_t^{-q}}{1-z_t^{-1}}\right]. \quad (10)$$

After replacing of $z_r = e^{j\omega_r}$ and $z_t = e^{j\omega_t}$ in Eq.(10) the amplitude and phase responses are:

$$M_{Zr}(\omega_r,\omega_t) = \frac{\sin\left(\frac{p\omega_r}{2}\right)}{\sin\left(\frac{\omega_r}{2}\right)}\frac{\sin\left(\frac{q\omega_t}{2}\right)}{\sin\left(\frac{\omega_t}{2}\right)} = M_{Zr}(\omega_r)M_{Zr}(\omega_t)\ , \quad (11)$$

where $\omega_r = 2\pi f_r / f_{sr}$ and $\omega_t 2\pi f_t / f_{st}$. By analogy for the phase is received:

$$\Phi_{Zr}(\omega_r,\omega_t) = \frac{p-1}{2}\omega_r + \frac{q-1}{2}\omega_t = \Phi_{Zr}(\omega_r) + \Phi_{Zr}(\omega_t)\ . \quad (12)$$

Bi-linear interpolation characteristics.

By analogy after the 2D Z-transform of the equation Eq.(9) the results are:

$$H_{Bl}(z_r,z_t) = \frac{Y(z_r,z_t)}{X(z_r,z_t)} = \frac{1}{pq}\left[\frac{1-z_r^{-p}}{1-z_r^{-1}}\right]^2\left[\frac{1-z_t^{-q}}{1-z_t^{-1}}\right]^2 \quad (13)$$

$$M_{Bl}(\omega_r,\omega_t) = \frac{\sin^2\left(\frac{p\omega_r}{2}\right)}{\sin^2\left(\frac{\omega_r}{2}\right)}\frac{\sin^2\left(\frac{q\omega_t}{2}\right)}{\sin^2\left(\frac{\omega_t}{2}\right)} = M_{Bl}(\omega_r)M_{Bl}(\omega_t)\ , \quad (14)$$

$$\Phi_{Bl}(\omega_r,\omega_t) = (p-1)\omega_r + (q-1)\omega_t = \Phi_{Bl}(\omega_r) + \Phi_{Bl}(\omega_t)\ . \quad (15)$$

On the base of equations Eq.(11) and Eq.(14) the frequency responses for the different values of p and q are analyzed. The amplitude responses corresponding to these received from 2D low-frequency filters for the frequencies, multiple to π/p and π/q. From the equations Eq.(12) and Eq.(15) follow that the phase characteristics are the planes crossing to the beginning of coordinate system width slope, defined by the variables p and q.

III. Experimental Results

For the analyzes of interpolation distortions the mean-square error can be used as a criterion. The input images have the uniform spectral characteristics in the spaces $-\pi \le \omega_r \le \pi$ and $-\pi \le \omega_{\le} \le \pi$. Then the equation can be expressed as following [3]:

$$\overline{\varepsilon}^2 = \frac{1}{\pi^2}\int_0^\pi\int_0^\pi \left[1 - M_n(\omega_r,\omega_t)\right]^2 \partial\omega_r\partial\omega_t\ , \quad (16)$$

where:

$$M_n(\omega_r,\omega_t) = \frac{M(\omega_r,\omega_t)}{M(0,0)} = \frac{1}{pq}M(\omega_r,\omega_t)$$

is a normalized amplitude response of the adaptive 2D interpolator and

$$M(\omega_r,\omega_t) = \begin{cases} M_{Bl}(\omega_r,\omega_t)\ , \text{for } F=0\ ; \\ M_{Zr}(\omega_r,\omega_t)\ , \text{for } F=1\ . \end{cases}$$

After replacing equations Eq.(11) and Eq.(14) in Eq.(16) the following result is received:

$$\overline{\varepsilon^2}(p,q) = \begin{cases} 1 - \frac{1}{pq}\left[\frac{8\Im(p)\Im(q)}{\pi^2} - 1\right] \text{ for } F=1, \\ 1 - \frac{1}{pq}\left[2 - \frac{(2p^2+1)(2q^2+1)}{9p^2q^2}\right] \text{ for } F=0, \end{cases} \quad (17)$$

where $\Im(p)$ and $\Im(q)$ are integrals of the following kind:

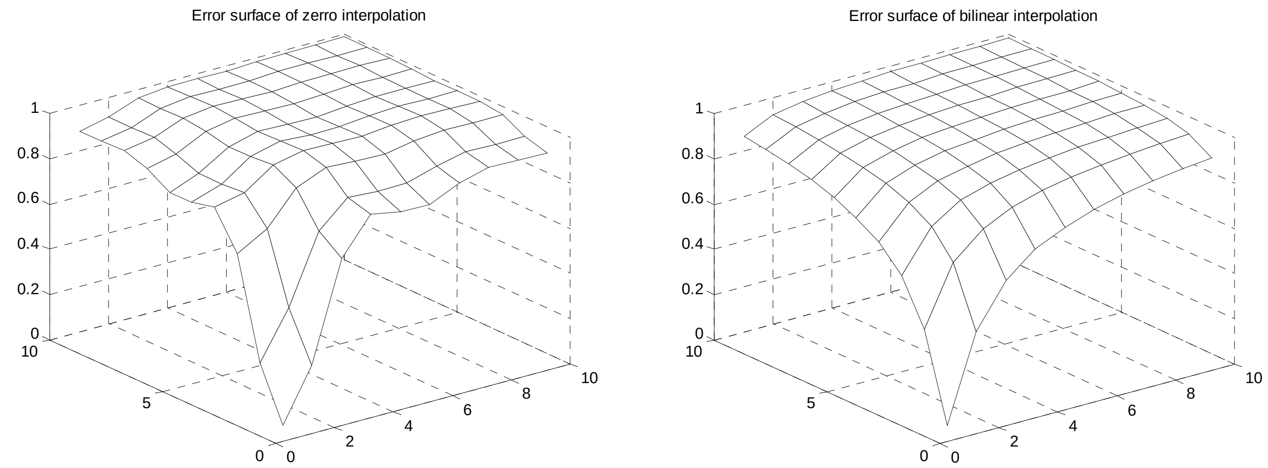

Fig.2. Error surfaces for zero and bi-linear interpolation

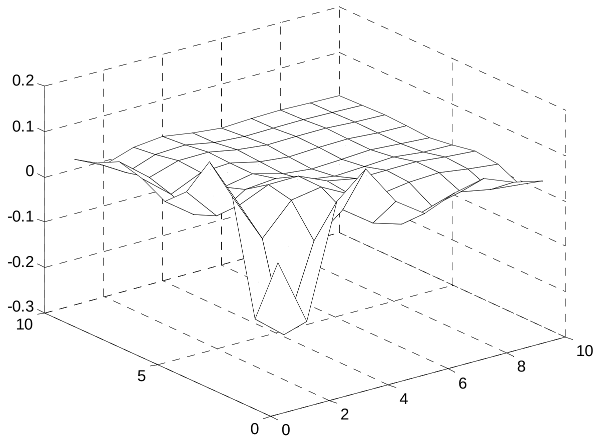

Fig.3. Difference error surface

$$\Im(u)=\int_0^{\pi/2}\frac{\sin(uv)}{\sin(v)}\partial v=\Im(u-2)+\frac{2\sin[(u-1)\pi/2]}{u-1}\,.$$

The error changing for the transition from zero to bi-linear interpolation, using equation Eq.(17), can be expressed as follows:

$$\Delta_{\overline{\varepsilon^2}}(p,q)\approx\frac{2}{pq}\left[\frac{4\Im(p)\Im(q)}{\pi^2}-1\right]\quad . \tag{19}$$

From equations Eq.(17) on Fig. 2 are presented the functions graph for mean-square errors, and on Fig.3 - their difference.

IV. CONCLUSION

On the base of error estimation for 2D adaptive interpolation on halftone images can be made the following conclusions:

- the digital interpolation filter doesn't include phase distortions in the processing images;
- to minimize mean-square error for p and q must be used variables in the space from 2 to 5;
- the maximal effective interpolation for the local characteristic of images can be achieved by using coefficients p,q=2-4.

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