Field Analysis of the Eccentric Archimedean Spiral Antenna by the Least Square Boundary Residual Method

Ana Jovanović, Svetozar Jovićević[1]

Abstract: **This paper demonstrates the applicability of the least square boundary method (LSBM) in the eccentric Archimedean spiral antenna analysis. It appeared that the boundary condition is highly fulfilled by a small number of eigenfunctions, which makes the obtained results very reliable. The radiation field is found in the far zone.**

Keywords: **Spiral antennas, the least-squares boundary residual method**

I. Introduction

Radiation problem is certainly one of the most complex problem in linear electromagnetic. For a long time in the antenna analysis the method of moments has been most frequently used. Although this method is still very much employed (see f.e. [1], [2], [3]) it seems that in the last decade Finite-difference time-domain (FDTD) has become more popular (some of the recent references f.e. [4], [5], [6]). Any attempt to find solution in a closed form by using eigenfunctions of the wave equation even for a relatively simple structure such as circular loop [7] or conic [8] antennas leads to a rather cumbersome procedure.

In this paper for field analysis of the eccentric Archimedean spiral antenna the Least-squares boundary Residual method is applied. Magnetic vector potential is expressed as a linear combination of the wave equation eigenfunctions, the unknown expansion coefficients follow from the minimization of the square error in the boundary condition fulfillment. This method, in original or modified form, has been found as simple and highly accurate in wide range of electromagnetic problems ([9],[10],[11],[12],[13],[14]).

II. Theoretical Analysis

There is an antenna's structure (figure 1) in xoy plane ($\theta = \frac{\pi}{2}$). Antenna is fed by a source of harmonic voltage with effective value U.

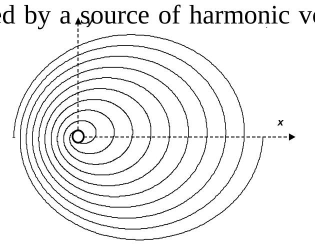


Fig. 1. Eccentric Archimedean spiral antenna

The antenna's structure such as Archimedean spiral has two arms described by the following equations in a spherical coordinate system:

$$r = a\varphi\sqrt{2K\cos\varphi + 1 + k^2} + b \quad 0 \le \varphi \le 2m\pi \quad \text{upper arm} \tag{1}$$

$$r' = a(\phi - \pi)\sqrt{-2K\cos\phi + 1 + k^2} + b \quad \pi \le \phi \le (2m+1)\pi \quad \text{lower arm}$$

where $a = \frac{\Delta}{2\pi}$ is a spiral constant, b is the initial point, k is eccentricity constant.

In a general case the field radiation, anywhere from the antenna does not belong to the TE and TM wave. All field components can be derived from the radial component of magnetic and electric vector potential.

$$A_r = \sum_m \sum_n C_{mn} B_n(kr) P_n^m(\cos\theta) e^{jm\varphi} \tag{2}$$

$$F_r = \sum_m \sum_n D_{mn} B_n(kr) P_n^m(\cos\theta) e^{jm\varphi}$$

where C_{mn} and D_{mn} are the unknown expansion coeficients, $B_n(kr)$ Bessel functions and $P_n^m(\cos\theta)$ Legandre polynoms.

Field components are given as [15]:

$$E_r = \frac{1}{j\omega\varepsilon}\left(\frac{\partial^2}{\partial r^2} + k^2\right)A_r \qquad H_r = \frac{1}{j\omega\mu}\left(\frac{\partial^2}{\partial r^2} + k^2\right)F_r$$

$$E_\theta = \frac{-1}{r\sin\theta}\frac{\partial F_r}{\partial\varphi} + \frac{1}{j\omega\varepsilon r}\frac{\partial^2 A_r}{\partial r\partial\theta} \qquad H_\theta = \frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\varphi} + \frac{1}{j\omega\mu r}\frac{\partial^2 F_r}{\partial r\partial\theta} \tag{2.a}$$

$$E_\varphi = \frac{1}{r}\frac{\partial F_r}{\partial\theta} + \frac{1}{j\omega\varepsilon r\sin\theta}\frac{\partial^2 A_r}{\partial r\partial\varphi} \qquad H_\varphi = -\frac{1}{r}\frac{\partial A_r}{\partial\theta} + \frac{1}{j\omega\mu r\sin\theta}\frac{\partial^2 F_r}{\partial r\partial\varphi}$$

The boundary condition is the annulation of the tangential electric field components on the antenna's arms surfaces. In this case tangential components on the antenna's arms surfaces have E_φ and E_r components. The boundary condition for upper and lower antenna's arms are given as:

$$\left.\begin{aligned} &E_{\varphi_t} + E_{r_t} = E_i \quad za \quad 0 \le r \le b \\ &E_{\varphi_t} + E_{r_t} = 0 \quad za \quad r \ge b \end{aligned}\right\} \text{upper arm} \tag{3.a}$$

$$\left.\begin{aligned} &E_{\varphi_t} + E_{r_t} = -E_i \quad za \quad 0 \le r' \le b \\ &E_{\varphi_t} + E_{r_t} = 0 \quad za \quad r' \ge b \end{aligned}\right\} \text{lower arm} \tag{3.b}$$

where :

[1] Authors are with the Faculty of Electronic Engineering, Cetinjski put bb, 81000 Podgorica, Serbia and Montenegro, E-mail: anaj@cg.ac.yu

$$E_{\varphi_t}=E_\varphi\cos\alpha,\quad E_{r_t}=E_r\sin\alpha \quad \text{upper arm,} \qquad E_{\varphi_t}=E_\varphi\cos\alpha',\quad E_{r_t}=E_r\sin\alpha' \quad \text{lower arm}$$

The angles between the tangent on the antenna lines and the field components are:

$$\alpha = arctg\frac{dr}{rd\varphi}\ ,\ \alpha' = arctg\frac{dr'}{r'd\phi}$$

Inserting the electrical field components (2.a), into the boundary condition (3.a,b), the boundary condition is given as:
for upper arm

$$\sum_{n=1}^{N}\sum_{m=0}^{n}\left(C_{nm}f_{nm}^{(1)}\left(r,\frac{\pi}{2},\varphi\right)+D_{nm}f_{nm}^{(2)}\left(r,\frac{\pi}{2},\varphi\right)\right)=\begin{cases}E_i & za\quad 0\le r\le b\\ 0 & za\quad r\ge b\end{cases} \tag{4.a}$$

where eigenfunctions are:

$$f_{nm}^{(1)}\left(r,\frac{\pi}{2},\varphi\right)=\left(\frac{n(n+1)}{yr^2}B_n(kr)\sin\alpha+\frac{jm}{yr}\frac{\partial B_n(kr)}{\partial r}\cos\alpha\right)P_n^m(\cos\theta)e^{jm\varphi}$$

$$f_{nm}^{(2)}\left(r,\frac{\pi}{2},\varphi\right)=\frac{B_n(kr)}{r}\frac{\partial P_n^m(\cos\theta)}{\partial\theta}\bigg|_{\theta=\frac{\pi}{2}}\cos\alpha\ e^{jm\varphi}$$

for lower arm

$$\sum_{n=1}^{N}\sum_{m=0}^{n}\left(C_{nm}f'^{(1)}_{nm}\left(r',\frac{\pi}{2},\phi\right)+D_{nm}f'^{(2)}_{nm}\left(r',\frac{\pi}{2},\phi\right)\right)=\begin{cases}-E_i & za\quad 0\le r'\le b\\ 0 & za\quad r'\ge b\end{cases} \tag{4.b}$$

where eigenfunctions are:

$$f'^{(1)}_{nm}\left(r',\frac{\pi}{2},\phi\right)=\left(\frac{n(n+1)}{yr'^2}B_n\left(kr'\right)\sin\alpha'+\frac{jm}{yr'}\frac{\partial B_n\left(kr'\right)}{\partial r'}\cos\alpha'\right)P_n^m(\cos\theta)e^{jm\phi}$$

$$f'^{(2)}_{nm}\left(r',\frac{\pi}{2},\phi\right)=\frac{B_n\left(kr'\right)}{r'}\frac{\partial P_n^m(\cos\theta)}{\partial\theta}\bigg|_{\theta=\frac{\pi}{2}}\cos\alpha'\ e^{jm\phi}$$

To simplify the systems (4.a,b), we shall transform them into the following expressions:
for upper arm

$$\sum_n\sum_m S_{nm}F_{nm}\left(r,\frac{\pi}{2},\varphi\right)=\begin{cases}E_i & za\quad 0\le r\le b\\ 0 & za\quad r\ge b\end{cases} \tag{5.a}$$

where are:

$$\left.\begin{aligned}F_{nm}\left(r,\frac{\pi}{2},\varphi\right)&=f_{nm}^{(1)}\left(r,\frac{\pi}{2},\varphi\right)\\ S_{nm}&=C_{nm}\end{aligned}\right\}za\quad n=1,2....N\quad i\quad m=0,.....,n$$

and

$$\left.\begin{aligned}F_{nm}\left(r,\frac{\pi}{2},\varphi\right)&=f_{n-N,m}^{(2)}\left(r,\frac{\pi}{2},\varphi\right)\\ S_{nm}&=D_{n-N,m}\end{aligned}\right\}\quad za\quad n=N+1,.....2N\quad i\quad m=0,...,n-N$$

for lower arm

$$\sum_n\sum_m S_{nm}F'_{nm}\left(r',\frac{\pi}{2},\phi\right)=\begin{cases}-E_i & za\quad 0\le r'\le b\\ 0 & za\quad r'\ge b\end{cases} \tag{5.b}$$

where :

$$\left.\begin{aligned}F'_{nm}\left(r',\frac{\pi}{2},\phi\right)&=f'^{(1)}_{nm}\left(r',\frac{\pi}{2},\phi\right)\\ S_{nm}&=C_{nm}\end{aligned}\right\}\quad za\quad n=1,2....N\quad i\quad m=0,.....,n$$

and

$$\left.\begin{aligned}F'_{nm}\left(r',\frac{\pi}{2},\phi\right)&=f'^{(2)}_{n-N,m}\left(r',\frac{\pi}{2},\phi\right)\\ S_{nm}&=D_{n-N,m}\end{aligned}\right\}\quad za\quad n=N+1,.....2N\quad i\quad m=0,...,n-N$$

We shall get system of algebraic equations for unknown coefficients S_{nm}, taking the F_{nm} as weighting functions:

$$\sum_n\sum_m S_{nm}a_{nm,n'm'}=b_{n'm'} \tag{6}$$

where :

$$a_{nm,n'm'}=\int_{gornjikrak}F_{nm}\ F^*_{n'm'}\,dl+\int_{donjikrak}F'_{nm}\ F'^*_{n'm'}\,dl'$$

$$b_{n'm'}=E_i\left(\int_0^b F^*_{n'm'}\,dl-\int_0^b F'^*_{n'm'}\,dl'\right)$$

The length elements of the upper and lower arms are:

$$dl=\sqrt{(rd\varphi)^2+dr^2}\ \text{ i }\ dl'=\sqrt{\left(r'd\phi\right)^2+dr'^2}$$

As the space includes the point $r\to 0$, $B_n(kr)$ are spherical Bessel functions. Because of the radiation condition, in the far zone the field potentials and consequently the field components are to be expressed by the Hanckel's functions. So the field potentials are given as:

$$A_r=\sum_m\sum_n\beta_{nm}^{(1)}H_n(kr)P_n^m(\cos\theta)e^{jm\varphi}$$
$$F_r=\sum_m\sum_n\beta_{nm}^{(2)}H_n(kr)P_n^m(\cos\theta)e^{jm\varphi} \tag{7}$$

where $\beta_{nm}^{(1)}$,$\beta_{nm}^{(2)}$ are the unknown explanation coefficients obtained by equalizing the expressions (2) and (7) on the sphere of radius $R=\max(r)$:

$$\beta_{nm}^{(1)}=C_{nm}\frac{B_n(kR)}{H_n(kR)}$$
$$\beta_{nm}^{(2)}=D_{nm}\frac{B_n(kR)}{H_n(kR)} \tag{8}$$

Knowing that in the far zone $(r\to\infty)$ Hanckel's functions become $j^{(n+1)}e^{-jkr}$**,**the field component $E_r\to 0$, and the only existing components are E_φ and E_θ .

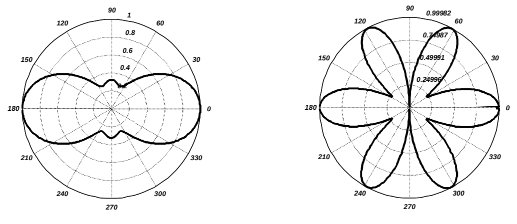
The radiation characteristics has a two components:

$$\hat{F}_{\varphi}(\theta,\varphi)=\left|\sum_{n=1}^{N}\sum_{m=0}^{n} j^{n+1} e^{jm\varphi}\left(\frac{km}{y\sin\theta}\beta_{nm}^{(1)}P_n^m(\cos\theta)+\beta_{nm}^{(2)}\frac{\partial P_n^m(\cos\theta)}{\partial\theta}\right)\right|$$

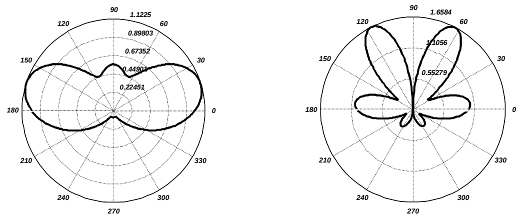
$$\hat{F}_{\theta}(\theta,\varphi)=\left|\sum_{n=1}^{N}\sum_{m=0}^{n} j^{n+1} e^{jm\varphi}\left(\frac{jk}{y}\beta_{nm}^{(1)}\frac{\partial P_n^m(\cos\theta)}{\partial\theta}-\frac{jm}{\sin\theta}\beta_{nm}^{(2)}P_n^m(\cos\theta)\right)\right|$$

III. Numerical Results

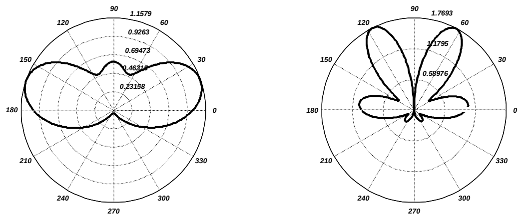
We analyzed eccentric Archimedean spiral antennas. On the figure 2.a,b radiation patterns of eccentric Archimedean spiral for two different spiral constant and two values of the eccentricity constants are given.



(a) $k=0$

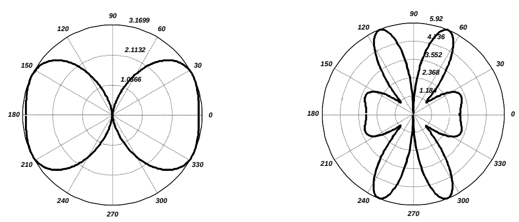


(b) $k=0.4$

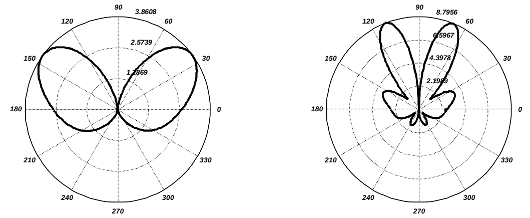


(c) $k=0.8$

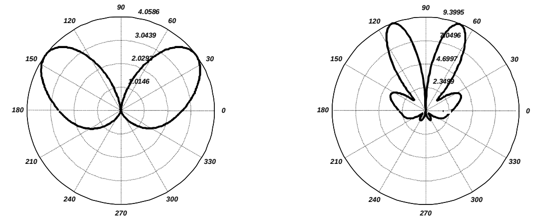
Fig. 2.a Radiation patterns of eccentricArchimedean spiral antenna. (left column E_{φ}, right column E_{θ} ; $\Delta=0.25\lambda$)



(a) $k=0$



(b) $k=0.4$



(c) $k=0.8$

Fig. 2.b Radiation patterns of eccentricArchimedean spiral antenna. (left column E_{φ}, right column E_{θ} ; $\Delta=0.5\lambda$)

Radiation patterns of E_{φ} and E_{θ} are given in left and right columns. The patterns are normalized to the case $k=0$, i.e. to the noneccentric antenna. The expected displacements of maximal radiations can easily be noticed and also that the greater eccentricity produces the grater displacement. The radiation characteristic for E_{φ} component in almost all cases has flater shape with small, in some cases even negligible side lobes. This is not so for E_{θ} component, which has very pronounced side lobes. All radiation patterns are plotted in the plane $\varphi=0^{o}$.

The maximal gains for three spiral eccentricities are given in Table I.

TABLE I.

spiral geometry	$k=0$	$k=0.4$	$k=0.8$
$\Delta=0.25\lambda$, $0\le\varphi\le 2\pi$	4.05 dB	5.22 dB	6.84 dB
$\Delta=0.5\lambda$, $0\le\varphi\le 2\pi$	3.99 dB	5.34 dB	7.18 dB

We can see that the larger eccentricity produces the greater gain in direction of maximal radiation.

Fig.3. gives the comparison of the results obtained by our theory and results of the theory discribed in [1].

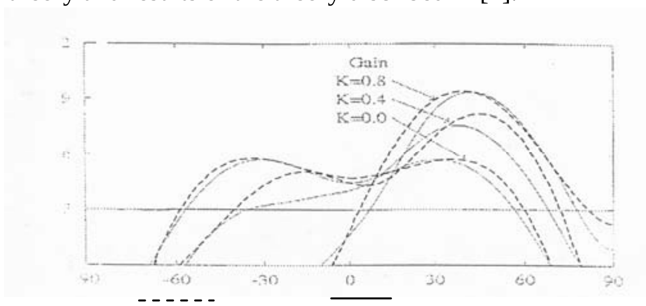


Fig. 3. Comparison of gains of the spiral with
$a=0.00144m/rad$, $90^{o}\le\varphi\le 1620^{o}$
(our results, results from [1])

A very good coincidence can be noticed for the case $k = 0$. The greatest deviation is for the spiral eccentricity $k = 0.4$

In all analyzed cases a very high accuracy in the boundary condition fulfillment is achieved with no more than twenty numbers of the series (2). As an example, the boundary condition error on upper and lower antenna arms is given on Fig. 4.

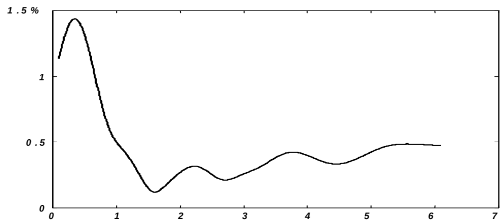


Fig. 4. The boundary condition error

IV. CONCLUSION

The applicability of the Least square boundary residual method to the analysis of eccentric Archimedean spiral has been demonstrated. The radiation patterns of eccentric spiral with two spiral constants and two eccentricities are obtained. Also, in all cases the directivities have been calculated. It was noticed that the enlargement of the spiral eccentricity produces higher gain in the direction of maximal radiation. The obtained gain patterns are compared with those given in Ref. [1]. A very good fulfillment of the boundary condition on the entire antenna was achieved with no more than twenty basic functions.

Therefore, we may conclude that in the analyzed cases the LSBR method turn to be accurate and relatively easy to be applied, which recommends it to even more complex radiation structures.

REFERENCES

[1] R.T.Gloutak, Jr. and N.G.Alexopoulos,”Two-Arm Eccentric Spiral Antenna”, IEEE Trans. on Antennas and Propag.,vol.45, no.4, april 1997.

[2] S.K.Khamas and G.G.Cook,”Moment-Method Analysis of Printed Wire Spirals Using Curved Piecewise Sinusoidal Subdomain Basis and Testing Functions”,IEEE Trans. on Antennas and Propag.,vol.45, no.6, jun 1997.

[3] H.Nakano, Y.Shinma and J.Yamauchi,”A Monofilar Spiral Antenna and Its Array Above a Ground Plane-Formation of a Circularly Polarized Tilted Fan Beam”, IEEE Trans. on Antennas and Propag.,vol.45, no.10, october 1997.

[4] O.M.Ramahi,”Near-and Far-Field Calculations in FDTD Simulations Using Kirchoff Surface Integral Representation”, IEEE Trans. on Antennas and Propag.,vol.45, no.5, may 1997.

[5] A.Shlivinski, E.Heyman and R.Kastner,”Antenna Characterization in the Time Domain”, IEEE Trans. on Antennas and Propag., vol.45, no.7, july 1997.

[6] O.M.Ramahi,” The Complementary Operators Method in FDTD Simulations“,IEEE Trans. on Antennas and Propag. magazine, vol.39, no. 6, december1997.

[7] L.W.Li, M.S.Leong, P.S.Kooi and T.S.Yeo,”Exact Solutions of Electromagnetic Fields in Both Near and Far Zones Radiated by Thin Circular-Loop Antennas: A General Representation”, IEEE Trans. on Antennas and Propag., vol.45, no.12, december 1997.

[8] S.N.Samadar and E.L. Mokole,” Biconical Antennas with Unequal Cone Angles”, IEEE Trans. on Antennas and Propag., vol.46, no.2, february 1998.

[9] LJ.Stanković, S.Jovićević,” Modified least squares method with application to diffraction and eigenvalue problems”, IEEE Procedings, vol.135, Pt.H, no.5, october 1988.

[10] LJ.Stanković, S.Jovićević,”A round-ridge waveguide”, Ann.Telecommun., 43, n^o 9-10, 1988.

[11] LJ.Stanković, S.Jovićević,” Boundary Condition Expansion of Basis Functions Method Implemented by Fast Fourier Transform Algorithms”, IEEE Trans. on Microwave Theory and Techniques, vol.38, no.3, march 1990.

[12] A.Jovanović, S.Jovićević”A general Solution of the Thin Circular Loop Radiation”, Electromagnetics, Vol. 23, no.1, january 2003.

[13] S.Jovićević, A.Jovanović,”The analysis of the biconical antenna by the Least-Squares Boundary Residual Method”, Int. J. Electron. Commun. (AEU) 57 (2003), no. 6, 415-419.

[14] A.Jovanović, S.Jovićević,”Analiza žičanih antena metodom najmanjih kvadrata”, XLII ETRAN sveska II, Vrnjačka Banja 1998.

[15] R.F.Harrihgton,Time-Harmonic Electromagnetic Fields, Mc Graw-Hill Book Company, New York 1961.

[16] L.W.Li, M.S.Leong, P.S.Kooi, T.S.Yeo,”Exact Solutions of Electromagnetic Fields in Both Near **and** Far Zones Radiated by Thin Circular-Loop Antennas: A General Representation”, IEEE Trans. Antennas and Propag., vol. 45, no. 12, dec.1997.