Effect of Geometrical Aspect Ratio of a Rectangular Microstrip Element on Antenna Bandwidth

Nikola I. Dodov[1], Slavi R. Baev[2]

Abstract – **Analytical expressions based on the cavity model are used to evaluate some characteristics of the microstrip element. For the rectangular microstrip resonator these formulas provide clear information about the effect of the antenna geometrical dimensions on the bandwidth. When the dielectric substrate is very thin the accuracy of the formulas is sufficient which allows their use as approximate design expressions.**

Keywords – **antenna, microstrip resonator, bandwidth.**

I. INTRODUCTION

Microstrip antennas have a broad spectrum of applications in present-day communications due to their advantages like low volume and easy production technology. Main disadvantage of these antennas is the narrow bandwidth and many techniques for bandwidth enhancement have been developed. There are many publications [1-4] concerning the effect of the substrate parameters (dielectric permittivity, thickness) on the bandwidth, quality factor, radiation efficiency and other characteristics of the microstrip resonator.

The aim of the present publication is to analyze the relation between the bandwidth and the geometry of the microstrip resonator antenna. It has been found that some patch shapes have inherently low quality factor [1] – these are annular ring, square/rectangular ring, quarter-wave shorted patch and others. There is information about the strong dependence between the bandwidth and the geometrical aspect ratio (width to length ratio) of the rectangular patch [1], [5]. The analysis of this question is important because of the wide spread use of the rectangular microstrip element and the possibility for obtaining large bandwidth enhancement with simple variation of the antenna geometry.

In Section 2 of this publication are given basic expressions used in the analysis of microstrip resonator antennas. These analytical formulas are derived on the basis of the cavity model and include expressions for the resonant frequency, total quality factor, radiation efficiency and bandwidth. The formulas are given in form corresponding to the case of a rectangular microstrip patch and they provide insight into the effect of the geometrical aspect ratio on the antenna parameters. In Section 3 the above-mentioned expressions are used to obtain numerical results and a set of graphs for particular values of the dielectric permittivity, substrate thickness, resonant frequency and aspect ratio of the microstrip element.

II. THEORETICAL RELATIONS FOR THE RECTANGULAR MICROSTRIP PATCH

A rectangular patch is shown in Fig. 1. The designations are as follows: L – patch length, W – patch width, h – substrate thickness, ε_r – relative dielectric permittivity of the substrate material.

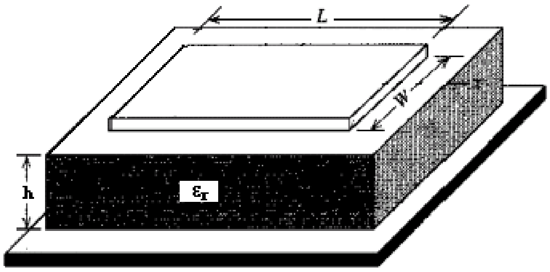

Fig. 1. Rectangular microstrip resonator

The analysis algorithm used for the purpose of this study is described next. All the expressions given in this section are derived for the dominant mode of a rectangular patch – TM_{010}, in which case the radiating edges are those with dimension W. Since the TM_{010} wave is used the most, the analysis shall be restricted only for this particular mode.

The effect of the aspect ratio on the resonant frequency can be evaluated with the expressions derived in [2]. From the resonance condition for the TM_{010} mode in the rectangular patch the following equation is obtained:

$$f_r = \frac{c}{2L\sqrt{\varepsilon_r}} \tag{1}$$

In this equation c is the speed of light in free space. When the resonant frequency f_r and the dielectric permittivity ε_r are given Eq. (1) can be used for determining the patch length. This formula does not account for the fringing effect, which leads to enlarged electrical area of the patch compared to the physical area. The including of the edge effects in Eq. (1) gives another value for the resonant frequency:

$$f_{rc} = \frac{c}{2L_e\sqrt{\varepsilon_{re}}} \tag{2}$$

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[1] Nikola I. Dodov is with the Department of Radiotechnics, Faculty of Communications and Communications Technologies, Technical University – Sofia, 8 ” Sv. Kliment Ohridski ” Blvd., Sofia 1000, Bulgaria, E-mail: ndodov@tu-sofia.bg

[2] Slavi R. Baev is with the Department of Radiotechnics, Faculty of Communications and Communications Technologies, Technical University – Sofia, 8 ”Sv. Kliment Ohridski ” Blvd., Sofia 1000, Bulgaria, E-mail: rosislav@yahoo.com

L_e is the effective length of the patch and ε_{re} is the effective dielectric permittivity of the substrate. Expressions for L_e and ε_{re} are given in [2]:

$$\varepsilon_{re} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + 12\frac{h}{W}\right)^{-\frac{1}{2}} \tag{3}$$

$$L_e = L + 2\Delta L \tag{4}$$

$$\Delta L = 0{,}412h\frac{(\varepsilon_{re} + 0{,}300)\left(\frac{W}{h} + 0{,}264\right)}{(\varepsilon_{re} - 0{,}258)\left(\frac{W}{h} + 0{,}813\right)} \tag{5}$$

ΔL is the extension of the length on each side of the patch due to the fringing effects.

It is observed that if the values of L, ε_r, and h are kept constant the variation of the ratio W/L shall alter the resonant frequency and more precisely the increasing of W/L shall lower f_{rc}.

To evaluate the shift of the frequency f_{rc} from the specified operating frequency f_r the so called fringe factor (length reduction factor) can be used:

$$n = \frac{f_{rc}}{f_r} \tag{6}$$

This factor n is of importance because it is used for length correction of the patch. When the aspect ratio W/L increases the fringing effect also increases and the resonant frequency f_{rc} lowers. To compensate for this shift of the frequency f_{rc} compared to the given resonant frequency f_r the length of the patch L must be reduced. The following equation gives the reduced patch length:

$$L_c = L_e n - 2\Delta L \tag{7}$$

This dimension L_c must be taken into account in the calculations concerning the antenna parameters.

Next the losses, the Q-factors associated with them and then the radiation efficiency of the patch are determined. There are four loss mechanisms to be considered in the microstrip resonator: radiation, loss due to surface wave propagation, conductor loss and dielectric loss. The total quality factor of the system Q_t is given by:

$$\frac{1}{Q_t} = \frac{1}{Q_{rad}} + \frac{1}{Q_{sw}} + \frac{1}{Q_c} + \frac{1}{Q_d} \tag{8}$$

For very thin substrates ($h/\lambda_0 << 1$) the losses due to the surface waves and Q_{sw} can be neglected. Formulas for the other Q-factors are given in [2], [4]:

$$Q_d = \frac{1}{\tan\delta} \tag{9}$$

$$Q_c = h\sqrt{\pi f_r \mu\sigma} \tag{10}$$

$$Q_{rad} = \frac{2\varpi_r \varepsilon_0 \varepsilon_r}{hG_{t/l}}K \tag{11}$$

$\tan\delta$ is the loss tangent of the substrate material, μ is the magnetic permittivity of the substrate, σ is the conductivity of the conductor used for the patch, ε_0 is the dielectric permittivity of free space, $G_{t/l}$ is the total conductance per unit length of the radiating aperture and:

$$K = \frac{\iint\limits_{area} |E|^2 dA}{\oint\limits_{perimeter} |E|^2 dl} \tag{12}$$

For a rectangular patch operating in the TM_{010} mode:

$$K = \frac{L_c}{4} \tag{13}$$

$$G_{t/l} = \frac{G_{rad}}{W} \tag{14}$$

G_{rad} is the radiation conductance of the equivalent slots. For thin substrates the expression for G_{rad} is [2]:

$$G_{rad} = \frac{I}{30\pi^3} \tag{15}$$

$$I = \int_0^{\pi}\int_0^{\pi}\left[\frac{\sin\left(\frac{k_0 W}{2}\cos\theta\right)}{\cos\theta}\right]^2 \sin^3\theta\cos^2\left(\frac{k_0 L_e}{2}\sin\theta\sin\varphi\right)d\theta d\varphi \tag{16}$$

The radiation efficiency of an antenna is defined as the power radiated over the input power or in terms of the Q-factors defined above:

$$e_r = \frac{Q_t}{Q_{rad}} \tag{17}$$

Eq. (16) allows evaluating the radiation efficiency as a function of the ratio W/L. In some publications [1], [3], [6] it is assumed that the radiation efficiency is almost independent of W/L. The results obtained in Section 3 prove that there is such dependency.

Next an expression for the bandwidth of a rectangular microstrip element is given. This equation clearly shows the dependence of the bandwidth from the substrate parameters, resonant frequency and geometrical aspect ratio. When the bandwidth is defined in terms of SWR (Standing Wave Ratio) and it is specified that SWR≤2 then the following expression for the bandwidth can be obtained [1], [6]:

$$BW = \frac{16}{3\sqrt{2}}\frac{p}{e_r}\frac{1}{\varepsilon_r}\frac{h}{\lambda_0}\frac{W}{L_c}q \tag{18}$$

$$p = 1 - \frac{0{,}16605}{20}(k_0 W)^2 + \frac{0{,}02283}{560}(k_0 W)^4 - 0{,}009142(k_0 L_c)^2 \tag{19}$$

$$q = 1 - \frac{1}{\varepsilon_r} + \frac{2}{5\varepsilon_r^2} \tag{20}$$

III. RESULTS OF THE PERFORMED SIMULATION ANALYSIS

The simulation analysis is performed according to the algorithm described in Section 2. Four substrate materials with different values of the dielectric permittivity ε_r are investigated: honeycomb (ε_r=1,07), duroid (ε_r=2,2), quartz (ε_r=3,8), alumina (ε_r=10). The first set of data is obtained for resonant frequency f_r=2 GHz and substrate thickness h=0,1588 cm. The initial value of the patch length is calculated by Eq. (1). The variation of the bandwidth *BW* and the radiation efficiency e_r is studied for a specified range of the aspect ratio (*W/L*=0,1÷3).

Fig. 2 shows the bandwidth *BW* as a function of *W/L* and some numerical values are given in Table I.

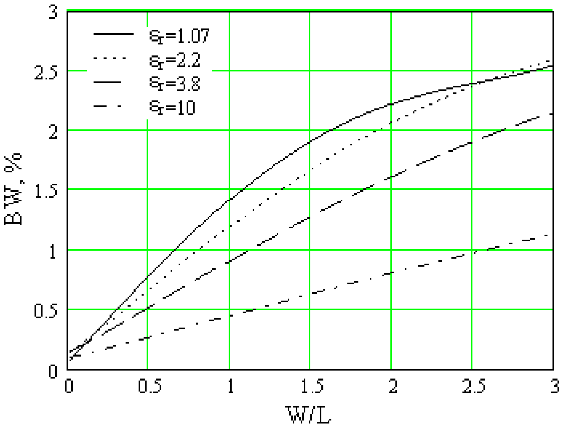

Fig. 2. Bandwidth as a function of the aspect ratio of the rectangular patch for four different substrates (h=0,1588 cm, f_r=2 GHz)

TABLE I
BANDWIDTH FOR DIFFERENT VALUES OF *W/L* AND ε_r

W/L	*BW,%*			
	ε_r=1,07	ε_r=2,2	ε_r=3,8	ε_r=10
0,5	0,782	0,666	0,519	0,267
1,0	1,420	1,195	0,908	0,448
1,5	1,908	1,669	1,277	0,629
2,0	2,220	2,065	1,613	0,805
2,5	2,391	2,370	1,905	0,973

As can be seen from Fig. 2 the bandwidth increases almost linearly with *W/L*. This is due to the larger radiating edges of the wider patch, which leads to increased radiated power and radiation conductance G_{rad} and decreased values of Q_{rad} and Q_t. The patch bandwidth decreases inversely with the increasing of ε_r. Increased value of ε_r leads to reduced fringing field (the electromagnetic field concentrates in the substrate) and radiated power, simultaneously the surface wave power increases. As a result the quality factor of the patch becomes higher thus reducing the bandwidth. Interesting fact is that for lower value of ε_r the relative bandwidth enhancement decreases, i.e., the useful effect from widening the patch is reduced.

Fig. 3 shows the radiation efficiency as a function of *W/L* for the four investigated dielectric substrates. Eq. (17) is used for the calculation of e_r. The materials with higher dielectric permittivity ε_r have lower radiation efficiency due to the reduced radiated power and increased surface waves as described above. This result is also confirmed by other publications [1], [3], [6], but the assumption in these papers that the radiation efficiency is almost independent of the aspect ratio *W/L* is true only for low values of the dielectric permitivity and large ratios *W/L* (approximately *W/L*≥1,5). The growing shape of the graphs is explained by the increased radiation in space waves when the patch is wider.

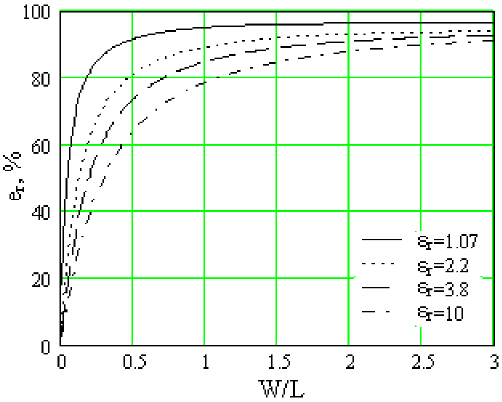

Fig. 3 Radiation efficiency as a function of the aspect ratio of the rectangular patch for four different substrates (h=0,1588 cm, f_r=2 GHz)

The next part of the study analyzes the effect of the aspect ratio *W/L* on the bandwidth *BW* of a rectangular microstrip resonator with dielectric permittivity of the substrate ε_r=2,2 and substrate thickness h=0,1588 cm for a range of resonant frequencies (f_r= 2, 4, 6, 8, 10 GHz). Fig. 4 illustrates the results.

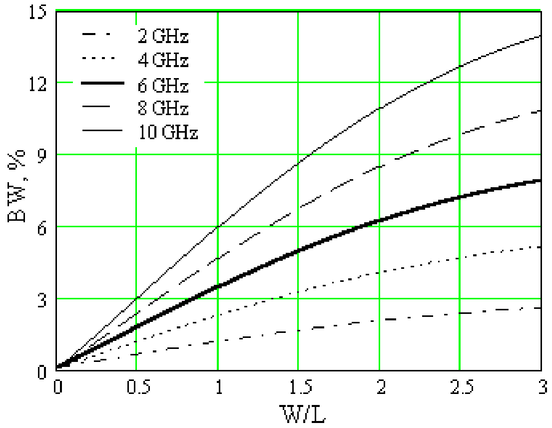

Fig. 4. Bandwidth as a function of the aspect ratio of the rectangular patch for five different frequencies (h=0,1588 cm, $\boldsymbol{\varepsilon_r}$=2,2)

As can be seen the bandwidth increases with f_r. The higher value of the resonant frequency increases the electrical thickness of the substrate ($h\sqrt{\varepsilon_r}/\lambda_0$), radiation conductance and radiated power. Similar results are reported in [1] and [3]. For low values of *W/L* the relation between *BW* and *W/L* is almost linear. The relative bandwidth enhancement increases with the frequency f_r. Table II contains values of *BW* for five ratios *W/L*.

TABLE II
BANDWIDTH FOR DIFFERENT VALUES OF *W/L* AND f_r

W/L	*BW,%*				
	f_r =2 GHz	f_r =4 GHz	f_r =6 GHz	f_r =8 GHz	f_r =10 GHz
0,5	0,666	1,204	1,778	2,382	3,018
1,0	1,196	2,290	3,450	4,679	5,981
1,5	1,671	3,264	4,958	6,757	8,671
2,0	2,068	4,077	6,219	8,500	10,933
2,5	2,373	4,704	7,193	9,851	12,692

IV. CONCLUSION

Theoretical formulas for the resonant frequency, radiation efficiency and bandwidth of the rectangular microstrip resonator are used to evaluate the variation of these parameters with the geometrical aspect ratio of the patch. The expressions in the present analysis can be used for obtaining quite accurate design data.

The results of the performed simulation clearly show that the increased patch width cause large bandwidth enhancement. This fact is explained by the increased radiating aperture and radiated power when the aspect ratio *W/L* is larger, which lowers the quality factor of the resonator. The bandwidth improvement of a wide rectangular patch (*W/L*=1,5) compared to a narrow patch (*W/L*=0,5) depends on the dielectric permittivity of the substrate and the resonant frequency and for the investigated values of these parameters it varies from 2,5 to 3 times. In other words the bandwidth can be changed to a great extent with a simple geometry variation. Interesting result is that the relative bandwidth enhancement becomes higher with the increase of f_r и ε_r, i.e., the useful effect of widening the patch is greater.

The main advantages of widening the rectangular patch are increased bandwidth, radiation efficiency, and directivity. This method for bandwidth improvement has also disadvantages, which must be taken into account for design purposes and which are not investigated in this study. These include larger patch size, generation of grating lobes in antenna arrays, effect on cross-polarization characteristics. In some papers [1], [4] it has been suggested that $W/L \leq 2$ for obtaining good aperture efficiency.

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