# Performance Analysis of Diversity Systems in Land Mobile Satellite Channels

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*Abstract* - We present the analysis of LMS channel performances in the presence of multiple interferers employing various diversity receivers, including SC, MRC, and OC. The dependence of the BER of the average SNR/bit per branch is used as the measure of performances.

*Keywords* – **BPSK signaling, SC, MRC, OC, Rayleigh fading, log-normal shadowing, diversity combining, cochannel interference.** 

## I. INTRODUCTION

Land Mobile Satellite (LMS) systems are becoming important part of today digital wireless systems. They offer a great number of applications and can serve many users over a wide area with low cost. It is very important to develop an accurate statistical model for the LMS channel between satellite and mobile user because the overall performances of LMS system strongly depends on it. In this paper we describe one single model of LMS channel in the presence of multiple cochannel interference and additive white Gaussian noise (AWGN). In a single model, the channel is characterized by a single statistical distribution, while a mixture model refers to a combination (weighted summation) of several statistical distributions. Single models are valid for stationary conditions, where the channel statistics remain approximately constant over the time interval of interest in a small area.

Namely, digital signaling is often followed by presence of fading and shadowing. Fading is the term used to describe the rapid fluctuations in the amplitude of the received radio signal over a short time interval caused due to the interference between two or more versions of the transmitted signals which arrive at the receiver at slightly different times [1]. The resultant received signal can vary widely in amplitude and phase, depending on various factors such as the intensity, relative propagation time of the waves, bandwidth of the transmitted signal etc... In mobile environments transmitted signal can be also affected by effect of shadowing which results in the long- term attenuation of received signal due to specific propagation environment (vegetation, buildings). The shadow fading can be classified on the line-of-sight (LOS) shadow fading and multiplicative shadow fading. LOS shadow fading affects only LOS component of received desired signal and attenuate it, while multiplicative fading affects both the LOS and scattered component of received desired signal attenuating the total power of desired signal.

In this paper we consider LMS channel with LOS shadowing modeled with Nakagami distribution, while fading is modeled with Rice distribution with shadow effect on LOS component. This model is modification of Loo's model widely used in literature where shadowing is modeled by lognormal distribution. Also, multiple interfering signals affected the desired multipath signal is modeled with Rayleigh distribution. Some simulation results are given in order to show improvement attained by implementing various types of diversity receivers including Selection Combining (SC), Maximal Ratio Combining (MRC) and Optimum Combining (OC).

### II. SYSTEM MODEL

Consider BPSK signaling in LMS channel with AWGN and additional cochannel interference and corresponding diversity receiver. Let L be the number of diversity branches which is assumed to be independent and M is the number of independent interference signals. The received signal in matrix notation may be written as [2]

$$\mathbf{r} = \mathbf{u}_s \cdot x_s + \sum_{j=1}^M \mathbf{u}_j \cdot x_j + \mathbf{\eta}$$
(1)

where

 $\mathbf{u}_{s} = \left[\alpha_{s,1}(t), \alpha_{s,2}(t), ..., \alpha_{s,L}(t)\right]^{T}$  - *L*-dimensional vector of fading amplitudes of desired signal.

 $\mathbf{u}_{j} = \left[\alpha_{j,1}(t), \alpha_{j,2}(t), ..., \alpha_{j,L}(t)\right]^{T}$  - *L*-dimensional vector of fading amplitudes of *j*th interfering signal.

 $x_s$  - desired signal equals  $\sqrt{E_b}$  or  $-\sqrt{E_b}$  with a priori probability <sup>1</sup>/<sub>2</sub>.

 $x_j$  - *j*th interfering signal equals  $\sqrt{E_b}$  or  $-\sqrt{E_b}$  with *a* priori probability 1/2.

 $\mathbf{\eta} = \left[\eta_1(t), \eta_2(t), ..., \eta_L(t)\right]^T - L\text{-dimensional vector of additive complex Gaussian noises.}$ 

 $E_b$  - energy per bit.

LMS channel model we consider [1] models fading amplitudes of desired signal with lowpass-equivalent complex envelope of the stationary narrowband shadowed Rice singled model which can be written as

$$\alpha_{s,l}(t) = |A_l(t) \cdot \exp[j\alpha_l(t)] + S_l(t) \cdot \exp[j\zeta_{l0}]|, \ l = 1,...L \ (2)$$

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where  $\alpha_l(t)$  is the stationary random phase process with uniform distribution over  $[0,2\pi)$ , while  $\zeta_{l0}$  is the deterministic phase of the LOS component. The independent stationary random processes  $A_l(t)$  and  $S_l(t)$ , which are also independent of  $\alpha_l(t)$ , are the amplitudes of the scatter and the LOS components, following Rayleigh and Nakagami distributions, respectively

$$p_A(a) = \frac{a}{b_s} \exp\left(\frac{-a^2}{2b_s}\right), \quad a \ge 0,$$
(3)

$$p_{s}(s) = \frac{2m^{m}}{\Gamma(m)\Omega^{m}} s^{2m-1} \exp\left(\frac{-ms^{2}}{\Omega}\right), \quad s > 0 \quad (4)$$

where we except subscript l denoting the diversity branches. If we consider Loo's model, the LOS component is modelled by lognormal distribution

$$p_s(s) = \frac{1}{\sqrt{2\pi\sigma_s s}} \exp\left(-\frac{(20\log s - \mu)^2}{2\sigma_s^2}\right)$$
(5)

where  $\mu[dB] = E[20\log s]$  i  $\sigma_s[dB] = Var[20\log s]$ . The corrensponding relation between sets of parameters for Nakagami na lognormal distribution is as follows [3]:

$$\mu[dB] = 10 * \left( \log\left[\frac{\Omega}{m}\right] + \frac{\Psi(m)}{\ln 10} \right)$$
(6)

$$\sigma_s[dB] = \frac{10}{\ln 10} \sqrt{\Psi'(m)} \tag{7}$$

where  $\Psi(\cdot)$  is psi function and  $\Psi'(m)$  is its first order derivativ.

As we consider model where changes of fading amplitudes are sufficiently slow during one bit interval, we assumes that they are constant during one bit interval.

Quantity  $E[A^2] = 2 \cdot b_s$  denotes the average power of the scatter component,  $\Omega_s = E[S^2]$  denotes the average power of the LOS component,  $\Gamma(m)$  is the gamma function, and m is Nakagami parameter (case m = 1 corresponds Rayleigh distribution), which determines the magnitude of obstruction of of the LOS. Smaller values of m correspond to urban areas, moderate values correspond to suburban and rural areas, while bigger values of this parameter correspond to open areas with no obstruction of the LOS.

After some algebraic manipulations, using (3) and (4) we obtain fading envelope pdf of desired signal, which is

$$p_{\alpha_{s}}(\alpha_{s}) = \left(\frac{2b_{s}m}{2b_{s}m + \Omega_{s}}\right)^{m} \frac{\alpha_{s}}{b_{0}} \exp\left(-\frac{\alpha_{s}^{2}}{2b_{s}}\right)$$

$$\cdot {}_{1}F_{1}\left(m, 1, \frac{\Omega_{s}\alpha_{s}^{2}}{2b_{s}(2b_{s}m + \Omega_{s})}\right), \quad \alpha_{s} \ge 0$$
(8)

where  $_{1}F_{1}(\cdot,\cdot,\cdot)$  is the confluent hypergeometric function. For m = 0, (8) simplifies to the Rayleigh pdf, while for  $m = \infty$ , it simplifies to the Rice pdf.

In order to derive system performances in sense of dependency of the average Bit Error Rate (BER) of the average signal-to-noise ratio (SNR), we must concern the second moment of fading anyelopes pdf, which is

$$E[\alpha_s^2] = \Omega_s + 2b_s.$$
<sup>(9)</sup>

If we define instant SNR/bit as  $\gamma_b = \alpha_s^2 \frac{E_b}{N_0}$ , where

 $N_0(W/Hz)$  denotes one-sided spectral power density of additive white Gaussian noise, then average SNR/bit can be written as

$$\bar{\gamma}_b = E[\alpha^2] \frac{E_b}{N_0} = (\Omega_s + 2b_s) \frac{E_b}{N_0} = 2b_s (K_s + 1) \frac{E_b}{N_0} \quad \text{where}$$

 $K_s = \frac{\Omega_s}{2b_s}$  denotes Rice factor of desired signal fading

anvelope ..

Also, in this paper, we assume interference model where fading amplitudes of interfering signal is modeled as [2]

$$\alpha_{j,l} = a_{j,l} \cdot \cos(\phi_{j,l}), \quad j = 1,...,M, \quad l = 1,...,L \quad (10)$$

where  $\phi_{j,l}$  denotes random phase of *j*th interfering signal in *l*th diversity branch, assumed to be uniformly distributed over the interval  $[0,2\pi)$ , and  $a_{j,l}$  denotes fading anyelope of *j*th interfering signal in *l*th diversity branch that has Rayleigh pdf

$$p_{a_j}(a_j) = \frac{a_j}{b_j} \exp\left(\frac{-a_j^2}{2b_j}\right), \quad a_j \ge 0.$$
 (11)

where average power of interfering signal fading anyelope is  $E[a_i] = 2 \cdot b_i$ 

In this analysis we assume that fading anyelope of interfering signals are identically distributed with equal average power so  $b_i = b_i$ , for j = 1, ..., M.

Channel type	$b_s$	т	$\Omega_s$	$K_s[dB]$
Channel 1 (light shadowing)	0.158	19.4	1.29	6.1
Channel 2 (heavy shadowing)	0.063	0.739	8.97x10 <sup>-4</sup>	-21.48
Channel 3 (average shadowing)	0.126	10.1	0.835	5.2

 TABLE 1

 Channel parameters for three different scenarios, taken

 FROM [2]

### **III.** THE SYSTEM PERFORMANCES

We analyze performances of proposed model of LMS channel with multiple cochannel interferers involving the three sets of parameters estimated from experimental measurements, which is taken from [3]. These parameters is given in Table 1. Channel 1 corresponds to light shadowing scenario which is associated with bigger values of parameters m and  $K_s$ . Channel 2 is used for description of heavy shadowing scenario corresponding with smaller values of m and  $K_s$ , and channel 3 is associated with average shadowing conditions. Percentage of time received desired signal level is greater than apscisa is given on Fig.1. This quantity is analytically derived using (8) as

$$P(\alpha_{s} > \alpha_{st}) = \int_{\alpha_{st}}^{\infty} p_{\alpha_{s}}(\alpha_{s}) d\alpha_{s} .$$
 (12)

Results derived by computer simulation of these quantity is also depicted in Fig. 1, and they agree with analytical results. We also presents some results derived by simulation of described LMS channel in the presence of multiple interferers employing diversity receivers. They are depicted on Figs. 2. -4. for three analyzed types of LMS channels. We consider three techniques of diversity reception involving Selection Combining (SC), Maximal Ratio Combining (MRC) and Optimum Combining (OC).



Fig. 1. Percentage of time received desired signal level is greater than apscisa



Fig. 2. BER dependency of average SNR/bit per branch for M = 2 and SIR<sub>per branch</sub> = 0 and 15 [dB] in channel 1



Fig. 3. BER dependency of average SNR/bit per branch for M = 2 and SIR<sub>per branch</sub> =0 and 15 [dB] in channel 2



Fig. 4. BER dependency of average SNR/bit per branch for M = 2 and SIR<sub>per branch</sub> =0 and 15 [dB] in channel 3

SC selects, among the L diversity branches, the branch providing the largest SNR ratio (or largest fading amplitude). In MRC the signals from all the branches are co-phased and individually weighed by fading amplitude of desired signal and than added. Weighting MRC vector is

$$\mathbf{w} = \mathbf{u}_{s} \tag{13}$$

so the signal at the output of combiner is

$$y = \mathbf{w}^{H}\mathbf{r} = \left|\mathbf{u}_{s}\right|^{2} \cdot x_{s} + \sum_{j=1}^{M} \mathbf{u}_{s}^{H}\mathbf{u}_{j} \cdot x_{j} + \mathbf{u}_{s}^{H}\mathbf{\eta}$$
(14)

This technique is optimal in environments without interference in sense of maximizing SNR and combating fading effect at the output of receiver, but it ignores the effect of cochannel interference [7]. OC addresses both effects of fading of desired signal and the presence of cochannel interference and optimize the signal-to-interference-plus-noise ratio (SINR) at the output of receiver [8]. Weighting OC vector is [7]

$$\mathbf{w} = \mathbf{R}^{-1}\mathbf{u}_{s} \tag{15}$$

where  $\mathbf{R}$  is the interference-plus-noise covariance matrix across the array's elements

$$\mathbf{R} = E\left[\left(\sum_{j=1}^{M} \mathbf{u}_{j} \cdot x_{j} + \boldsymbol{\eta}\right)\left(\sum_{j=1}^{M} \mathbf{u}_{j} \cdot x_{j} + \boldsymbol{\eta}\right)^{H}\right].$$
 (16)

Note that OC weight vector is conditioned on desired signal fading amplitudes vector  $\mathbf{u}_s$ , interference signals fading amplitudes vectors  $\mathbf{u}_j$ , j = 1,...,M and AWGN vector  $\mathbf{\eta}$ , while MRC weight vector is conditioned only on desired signal fading amplitudes vector  $\mathbf{u}_s$ .

As parameter we use different number of diversity branches (L = 1, 2, 4), M = 2 intefering signals and values of SIR<sub>per branch</sub>, 0 and 15 [dB]. This quantity is defined as

$$SIR_{perbranch} = \frac{E[\alpha_s^2]}{M \cdot E[\alpha_j^2]} = \frac{\Omega_s + 2b_s}{2b_i} = 10\log\frac{b_s(K_s + 1)}{b_i}[dB](17)$$

It can be seen that in case that responds to bigger value of  $SIR_{per branch} = 15$  [dB] MRC and OC performs equal, while in case with  $SIR_{per branch} = 0$  [dB], which responds to greater

power of interfering signals, OC outperforms MRC. Also, differences between SC and optimal techniques become more significant in light shadow environment (channel 1) than in heavy shadow influenced channel 3.

### IV. CONCLUSION

We present the analysis of LMS channel performances in the presence of multiple interferers employing various diversity receivers, including SC, MRC, and OC. Simulation results is given for different values of diversity branches (L =1, 2, 4) and SIR<sub>per branch</sub> (0 and 15 [dB]) for three types of communication channel which parameters is taken from literature. It can be seen that in case that responds to bigger value of SIR<sub>per branch</sub> = 15 [dB] MRC and OC performs equal, while in case with SIR<sub>per branch</sub> = 0 [dB], which responds to greater power of interfering signals, OC outperforms MRC. Also, differences between SC and optimal techniques become more significant in light shadow environment (channel 1) than in heavy shadow influenced channel 3.

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