Denoising of ECG Signal Based on HHT

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Abstract – **This paper introduces a new ECG signal denoising based on Hilbert-Huang transformation (HHT). In order to ensure the estimated level of Gaussian white noise based on threshold denoising is more valid, present the verification of the white noise using probability and statistics. About not white noise intrinsic mode function (IMF), the white noise level can be obtained by the product of the mean frequency and the energy density of each IMF. Point out instantaneous frequency of HHT is an important parameter for removing powerline interference and baseline wander from the ECG.**

Keyword – **HHT, ECG signal, Denoising, Verification of the white noise, Threshold.**

I. INTRODUCTION

Recording of electrocardiograms (ECGs) is an important basis for biomedical diagnosis. ECG signal is a weak low frequency non-stationary signal. When acquiring the data of ECG, because of many random interference such as electromyographical interference, powerline interference and baseline wander, ECG signal is embedded in the noise. The noise causes ECG signal to distort, and influences the validity of medicinal diagnosis. In order to obtain exact signal parameters, identify waveforms and diagnose diseases, we must restrain the noise, and improve the Signal-to-Noise (SNR). ECG signal denoising methods are mostly low pass filtering[1] and wavelet denoising[2]. Low pass filtering can filter high frequency interference, but simultaneously cause ECG signal to lose some information, moreover, it is difficult to restrain additive white noise whose bandwidth is the same as ECG signal. Wavelet transform threshold denoising that proposed by Donoho et al. is an effective method to remove the white noise[3][4]. But, there usually is the selection of wavelet base function when processing material signal. Furthermore, how to choose valid threshold is a problem.

Empirical mode decomposition (EMD) and HHT is a new method pioneered by Huang et al. in 1998 for non-linear and non-stationary time series analysis[5]. Signal is adaptively decomposed into several IMFs, and the IMFs have well-behaved Hilbert transforms. The input data is presented in an energy-frequency-time quantitative distribution by using this method, wavelet transform can't. Threshold denoising method, as described in wavelet analysis, is presented in some signal denoising articles based on EMD[6]. The key of threshold denoising is to choose valid threshold. The threshold must be just larger than the highest level of the noise exactly. So, some effective threshold methods for removing the white noise generate. When the white noise and useful signal component are mixed, with the useful signal component enhancing, existing threshold methods become useless in evidence. So, it is a practical significant subject to study the verification of the white noise series and to choose valid threshold when useful signal component is dominant.

At first, we study the verification of the white noise using probability and statistics. The choosing of the noise level about not white noise IMF is discussed here. Then based on HHT, confirm quantitatively the relation of instantaneous frequency and instantaneous amplitude, present a feasible denoising method by prior information of ECG signal frequency range, and carry this method into ECG signal denoising.

II. HHT

In the HHT, at first, we decompose a complex signal into a series of IMFs based on EMD. Each IMF must satisfy two requirements: (1) the number of extrema and the number of zero crossings are either equal or differ at most by one; (2) at any point, the mean value of the upper envelope defined by the local maxima and the lower envelope that defined by the local minima is zero. The sifting process of the signal involves the following steps: calculate the mean value m (t) of the upper and the lower envelope by using the local maxima and the local minima, then the difference h between x(t) and m(t) is:

$$\mathrm{h} = x(t) - m(t) \qquad (1)$$

Set h as new x(t) and repeat above step until h satisfies the condition of IMF, written as:

$$\mathrm{c}_1 = h \qquad (2)$$

Define c_1 as the 1th IMF, and do:

$$x(t) - c_1 = \mathrm{r} \qquad (3)$$

Set r as new x(t), repeat above steps and then obtain the 2th IMF c_2 ,the 3th IMF c_3 …, until c_n or r satisfies the stopping criterion. Then x(t) can be decomposed into:

$$x(t) = \sum_{i=1}^{N} c_i + r \qquad (4)$$

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For each IMF, the Hilbert transform is defined as:

$$\mathrm{H[c(t)]} = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{c(\tau)}{t-\tau}\mathrm{d}\tau \qquad (5)$$

Construct the analytic signal:

$$z(t) = c(t) + jH[c(t)] = a(t)e^{j\Phi(t)} \qquad (6)$$

The amplitude function:

$$a(t) = \sqrt{c^2(t) + H^2[c(t)]} \qquad (7)$$

And the phase function:

$$\Phi(t) = \arctan\frac{H[c(t)]}{c(t)} \qquad (8)$$

Instantaneous frequency of the IMF component is defined as:

$$\mathrm{f}(t) = \frac{1}{2\pi}\frac{\mathrm{d}\theta(\mathrm{t})}{\mathrm{dt}} \qquad (9)$$

After applying Hilbert transform to each IMF:

$$s(t) = \mathrm{Re}\sum_{i=1}^{N} a_i(t)e^{j\Phi_i(t)} = \mathrm{Re}\sum_{i=1}^{N} a_i(t)e^{j\int\omega_i(t)dt} \qquad (10)$$

The Hilbert spectrum is defined as:

$$H(\omega,t) = \mathrm{Re}\sum_{i=1}^{N} a_i(t)e^{j\int\omega(t)dt_i} \qquad (11)$$

The marginal spectrum is defined as:

$$h(\omega) = \int_0^T H(\omega,t)dt \qquad (12)$$

III. ECG SIGNAL DENOISING USING HHT

The frequency range of ECG signal is 0.05~100Hz, but 90% of ECG spectrum energy concentrates on 0.25~45Hz[7]. P, Q, R, S, and T waveforms are mainly characters of ECG signal analysis. The interference signals are: powerline interference and opposite harmonic interference; baseline wander whose frequency range is 0.15~0.3Hz; electro-myographical interference whose frequency range is 5~2KHz and spectrum characteristics obeys approximate zero-mean Gaussian white noise.

ECG signal is decomposed into high-low frequency IMFs based on HHT. We can obtain instantaneous frequency and amplitude of each IMF, then, the quantitative energy-frequency-time distribution of the signal is obvious. Formally, signal Hilbert-Huang marginal spectrum is very similar as Fourier transform, so, ECG signal dynamic denoising becomes possible. Fig. 1 is the contrast of ECG signal marginal spectrum and Fourier transform. The marginal spectrum hasn't the problem of leakage of spectrum as Fourier transform, and its principal component spectrum is more prominent.

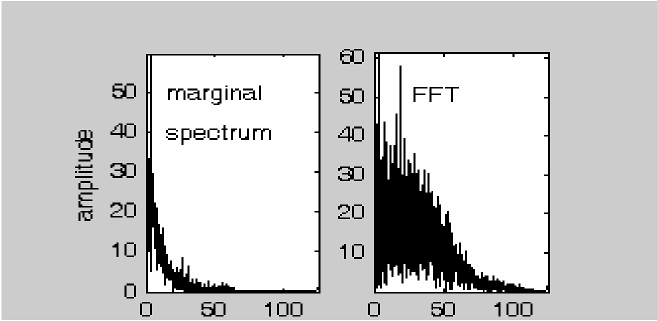

Fig. 1. Marginal spectrum and Fourier spectrum

A. *Problems of EMD threshold Denoising*

EMD-based Threshold denoising[6] is proposed. Its basic idea is decomposing a given signal x(t) into N IMFs based on EMD, Choose a appropriate threshold for each IMF, and cut c_i to $\hat{c}_i$ by this threshold, then reconstruct the signal by using EMD.

$$\hat{x}(t) = \sum_{i=1}^{N}\hat{c}_i + r \qquad (13)$$

The threshold, aiming to removing Gaussian white noise in wavelet denoising, proposed by Donoho et al. is defined as:

$$\tau_i = \hat{\sigma}_i\sqrt{2\log(N)} \qquad (14)$$

$$\hat{\sigma}_i = MAD_i / 0.6745 \qquad (15)$$

Where $\hat{\sigma}_i$ is the noise level of the ith IMF. MAD_i represents the absolute median deviation of the ith IMF and is defined as:

$$MAD_i = Median\{|c_i(t) - Median\{c_i(t)\}|\} \qquad (16)$$

the estimated c_i of the ith IMF is:

$$\hat{c}_i(t) = \begin{cases} sign(c_i(t))\ (|c_i(t) - \tau_i|) & \text{if } |c_i(t)| \geq \tau_i \\ 0 & \text{if } |c_i(t)| < \tau_i \end{cases} \qquad (17)$$

Because the noise and useful signal component are mixed, it becomes difficult to separate the noise from each IMF component. Actually, for high frequency IMFs, including less signal component, consequently, Eq. (15) can be applied to them for calculating noise variance. Because the threshold selection method above is more suitable for Gaussian white noise, in order to ensure the result of Eq. (16) is valid, it is necessary to verify whether the series is the white noise.

Decomposing Gaussian white noise using EMD, each IMF also is Gaussian distribution, and the energy will reduce following N's increase. In theory, EMD decomposition is orthogonal, and the white noise should still be the white noise after orthogonal decomposition. In fact, due to the problem of calculating the mean value of the upper and lower envelopes, EMD is only orthogonal decomposition approximately. The trend of the white noise after approximate orthogonal decomposition is the white noise. All of these offer basis for the white noise verification of IMF.

B. Verification of the White Noise [8]

Verifying to ensure whether the series satisfies the null hypothesis, namely:

The null hypothesis $H_0:\{x(i)\}$ is independent white noise, otherwise hypothesis H_1 is correlative series.

Assume the estimated value of self-correlation coefficient as $\{\hat{\rho}_k$,k=1,2,...,$m\}$,when N is large enough, the distribution of $\sqrt{N}(\hat{\rho}_1,\hat{\rho}_2,\cdots,\hat{\rho}_m)$ obeys approximately m-dimensional standard normal distribution .

$N(\hat{\rho}_1^{\,2}+\hat{\rho}_2^{\,2}+\cdots+\hat{\rho}_m^{\,2})$ obeys approximately χ^2 distribution .Since under the null hypothesis H_0

$$\hat{\rho}_1^{\,2}+\hat{\rho}_2^{\,2}+\cdots+\hat{\rho}_m^{\,2}=0 \tag{18}$$

Under the important level α =0.01, judging whether

$$N(\rho_1^2+\rho_2^2+\cdots\rho_m^2)<\lambda_a \tag{19}$$

is tenable. Where λ_a is critical value of χ^2 distribution with the prominent level α and m degrees of freedom.

C. Estimation of the White Noise Energy in EMD

While IMF component is dominated by useful signal, Eq. (15) can not be applied to estimating the noise energy directly. Gaussian white noise energy is defined as[9]:

$$E_i\overline{T}_i=const \tag{20}$$

Where E_i is the noise energy density of the ith IMF, $\overline{T}_i$ is the mean period of the ith IMF. The mean period can be obtained by averaging the instantaneous frequency of Hilbert transform of current IMF. Based on the noise energy and the mean period that estimated by high frequency IMF, we can estimate the white noise energy of current jth IMF:

$$E_j=const/\overline{T}_j \tag{21}$$

D. Removal of Powerline Interference and Baseline Wander

HHT can obtain instantaneous frequency of each IMF, then, before reconstructing, set signal amplitude whose frequency is close to 50Hz to zero. So, powerline interference can be removed easily by HHT.

About baseline wander, HHT is also an effective method, the low frequency IMF, whose instantaneous frequency is less than 0.3Hz, isn't involved in signal reconstruction process.

IV. EXPERIMENT RESULTS AND ANALYSIS

In order to test the validity of the denoising method above. In experiments, we add Gaussian noise (electromyographical interference), powerline interference, and baseline wander to free noise ECG signal. The correlation coefficients of noisy signal and original signal, denoised signal and original signal are given respectively in Table I.

TABLE I
DENOISING RESULTS

SNR (original signal power and the noise)	5dB	10dB
Correlation Coefficient (noisy signal and original signal)	0.278	0.279
Correlation coefficient (denoised signal and original signal)	0.949	0.958

In Table I, SNR is the ratio of original signal power and the noise (Gaussian noise and powerline interference). Moreover, we experiment many times using real signals from standard MIT/BIH ECG signal database. Experiment results prove that the noise influence of ECG signal can be well removed based on HHT above. The waveform contrast of original signal, noisy signal, and denoised signal using HHT above is given in Fig. 2 (SNR is 5dB).

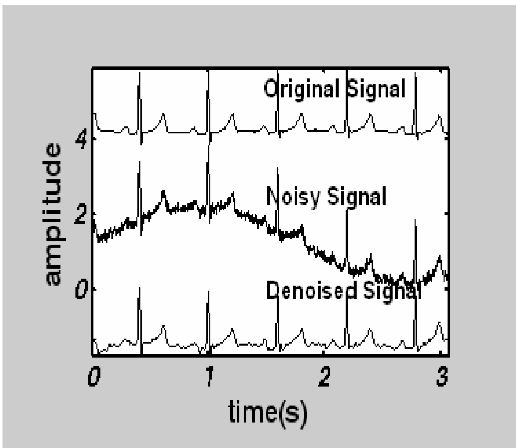

Fig. 2. Contrast of original signal, noisy signal and denoised signal

V. CONCLUSIONS

Through theoretic analysis and large numbers of experiments, conclusions are as follows:

A. Verification of the white noise is necessary for IMFs. It is the base of estimating the white noise variance correctly.
B. About not white noise IMFs, the white noise level can be obtained by the product, which obtained by the energy and mean period of high frequency IMF, and the mean period of current IMF.
C. Instantaneous frequency of each IMF is dependable basis for ECG denoising. HHT is an effective method for ECG signal denoising.

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