Digital Matched Filter

Slavcho D. Lishkov[1], Veska M. Georgieva[2]

Abstract – **In the paper are presented the principles of building digital matched filters (DMF), which are used by signal processing with background noise. Their perspective using in signal processing is appropriated by high selectivity, stability, precision of the characteristics and the reliability of the filters.**
On the base of the theory of digital matched filters, the students can made a simulation model for analyze and synthesis of digital matched filter. Some important characteristics can be analyzed: the spectrum, impulse response, correlation function, the influence of the sampling on the level of the sideband of the output signal and on the level of quantizing. The paper can be used in engineering education in studying these filters.

Keywords – **Digital matched filters, ambient noise, correlation functions, detection and identifying of signals, computer simulation.**

I. Introduction

It's described in the paper an idea of exercises for the course on Signals and Systems of students from faculty of communications, from faculty of computer systems and from faculty of electronics.

The digital matched filters are used by signal processing with background noise. Their perspective using in signal processing is appropriated by high selectivity, stability, precision of the characteristics and the reliability of the filters[1].

On the base of the theory of digital matched filters, the students can made a simulation model for analyze and synthesis of digital matched filter. The synthesis of the DMF can be realized on the base of the digital filter by giving the Amplitude/frequency characteristic and Phase/frequency characteristics. The DMF can be realized as FIR or IIR filters[2]. Some important characteristics can be analyzed: the spectrum, impulse response, correlation function, the influence of the sampling on the level of the sideband of the output signal and on the level of quantizing.

II. Problem Formulation

The principles of building digital matched filter are not differed from the usual digital filter. The matched filter is linear filter, which provides maximal signal-to-noise ratio (SNR). Its impulse response can be presented with the following mathematical description, given in Eq.1.

$$g(l\Delta t) = k.U\left[(n_c - l)\Delta t\right] = k.U\left[(n_0 - l)\Delta t\right] \quad (1)$$

where k is a constant; $n_0 = n_c = n_{\max}$ is a max delay of the signal in the output of the filter, which is needed to its physical realize; $n_c = \tau_c / \Delta t$; τ_c is the time of the signal; $l = (0,1,2,.........n_c - 1)$; Δt is the sampling time.

The frequency response of the digital matched filter is given in Eq.2.

$$H(j\omega) = \exp(-j\omega n_0)\sum_{l=0}^{\infty} g(l\Delta t)\exp(-j\omega l) = \\ = U(-j\omega)\exp(-j\omega n_0) \quad (2)$$

According to principles of the matched filtration (Eq.3) [3]:

$$\left|H(j\omega)\right| = k\left|U(-j\omega)\right| = k\left|S^*(j\omega)\right| \quad (3)$$

where $S^*(j\omega)$ is the complex conjugate $\Delta f = -1/2\Delta t \le \Delta f \le 1/2\Delta t$ ed spectrum of the signal.

The noise of the input of the digital matched filter is a background noise can be study as fortuitously stationary sequence an even distribution of the spectrum density $N_0/2$ in the frequency band $\Delta f = -1/2\Delta t \le \Delta f \le 1/2\Delta t$, dependent on sampling frequency. The correlation function of the digital noise in the output of the DMF is coincided with accuracy of constant multiplier on form of the output signal $y(k\Delta t)$, given in Eq.4.

$$\Psi_N(m\Delta t) = N_0/2.y\left[(n_c - m)\Delta t\right] \quad (4)$$

The Energy SNR is $q^2 = \dfrac{2E_0}{\sigma_N^2}$ and $\sigma_N^2 = \Psi(0) = N_0/2\,E_U$ is the mean power noise.

III. Synthesis of the DMF

The synthesis of the DMF can be realized on the base of the digital filter by giving the Amplitude/frequency characteristic and Phase/frequency characteristics. These can be determinate from the amplitude and phase spectrum of the signal.

The simple case is on the base of the FIR filters, which can be defined in Eq.5 [4].

$$y(k\Delta t) = \sum_{l=0}^{n_c-1} g(l\Delta t).U\left[(k-l)\Delta t\right] = \sum_{l=0}^{n_c-1} C_l U(l) \quad (5)$$

where $C_l = g(l\Delta t)$ is the digital impulse response.

[1]Slavcho D. Lishkov is with the Faculty of Communication, TU-Sofia, Kl.Ohridsky str.8, Sofia, Bulgaria.

[2]Veska M. Georgieva is with the Faculty of Communication, TU-Sofia, Kl.Ohridsky str.8, Sofia, Bulgaria, E-mail: vesg@tu-sofia.bg

To preserve the information about the amplitude and phase of the signal and to decrease the sampling frequency can be used the quadrature processing. The signal $u(t)$ can be presented in the complex form, given in Eq.6.

$$\dot{u}(t) = \dot{U}(t)\exp[j\omega t + \varphi_0] = \dot{U}(t)\cos[\omega t + \varphi_0] + j\dot{U}\ (t)\sin[\omega t + \varphi_0] \tag{6}$$

where $\dot{U}(t) = U_1(t)\cos\omega t + jU_2(t)\sin\omega t$;

$U(t) = \sqrt{U_1^2(t) + U_2^2(t)}$; $tg\varphi_0 = U_2(t) / U_1(t)$,

$U_1(t)$ and $U_2(t)$ are respectively the equiphase and the quadrature components of the signal. The signal can be decomposed to the quadrature components using the phase comparators.

The digital quadrature components of the signal can be marked respectively $U_1(k)$ and $U_2(k)$. The complex shape of the input signal and the complex shape of the impulse response can be described in Eq.7.

$$\dot{U}[k] = U_1[k] - U_2[k]\,;\ \dot{g}[k] = g_1[k] + g_2[k] \tag{7}$$

The output signal of the DMF with the accuracy of constant multiplier $\Delta t / 2$ is the following (Eq.8):

$$\dot{Y}[k] = y_1[k] + jy_2[k] = \sum_{l=0}^{n-1_c}\{U_1[k-l] - jU_2[k-l]\}\times\{g_1[n_c - l] + jg_2[n_c - l]\} \tag{8}$$

When $n_c - l = i$, for the quadrature components can be obtained (Eq.9):

$$y_1[k] = \sum_{i=1}^{n_c} g_1(i)U_1[k-(n_c - i)] + \sum_{i=1}^{n_c} g_2(i)U_2[k-(n_c - i)] = y_{11}(k) + y_{22}(k)$$

$$y_2[k] = \sum_{i=1}^{n_c} g_2(i)U_1[k-(n_c - i)] - \sum_{i=1}^{n_c} g_1(i)U_2[k-(n_c - i)] = y_{21}(k) + y_{12}(k) \tag{9}$$

This algorithm can be realized by using the following structural diagram (Fig. 1):

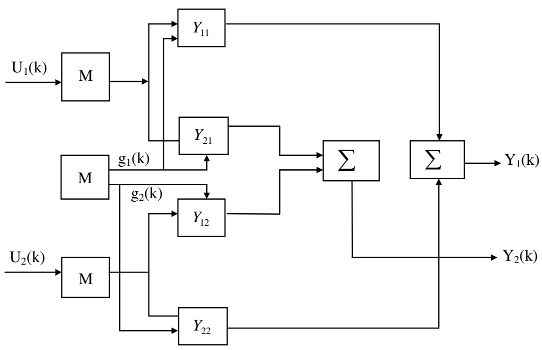

Fig. 1. Structural diagram of the algorithm

The quadrature channels can be realized on the base of the IIR filters. In this case the coefficients and the time of the processing are the same. The advantage of the qudrature processing is the decreasing of the sampling frequency. To realize the DMF on the base of the IIR filters, we can use the following correlation, given in Eq.10.

$$\vec{U} = \vec{U_1} + \vec{U_2}\ ;\ \vec{C} = \vec{C_1} + \vec{C_2} \tag{10}$$

where $\vec{U}$ and $\vec{C}$ are respectively the input signal and the coefficients of the filter. For the output signal can be obtained (Eq.11):

$$\vec{y} = \vec{C}.\vec{U} = \vec{y_1} + j\,\vec{y_2}\ ;$$

$$\vec{y_1} = \vec{U_1}.\vec{C_1} - \vec{U_2}.\vec{C_2}\ ;\ \vec{y_2} = \vec{U_2}.\vec{C_1} + \vec{U_1}.\vec{C_2} \tag{11}$$

By the matched filtration can be realized the operation convolution of the signals in the time domain, what corresponds to the multiplication of their spectrums in the frequency domain. The spectrums can be obtained by Fast Fourier Transform (FFT).

In Fig.2 is given the DMF for processing in frequency domain.

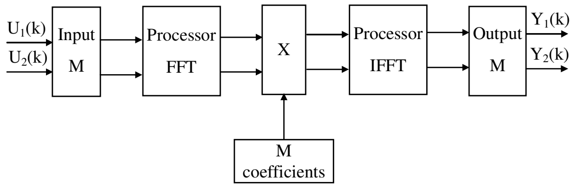

Fig. 2. Schema of DMF for processing in frequency domain

The effectiveness of the filtration is dependent on the sampling frequency and the level of the quantizing. It's known, that the sampling frequency $f_s = 2\Delta f_{SB}$, where Δf_{SB} is the sideband of the signal.

On the base of the theory the students can do the following tasks:

- To made a synthesis of the DMF as FIR or IIR filter
- To analyze the influence of the sampling frequency on the output signal
- To analyze the influence of the level of quantizing on the output signal

IV. Conclusion

In the paper are presented the principles of building digital matched filters (DMF), which are used by signal processing with background noise.

On the base of the theory of digital matched filters, the students can create a simulation model for analyze and synthesis of digital matched filter. As variant for simulation can be used MATLAB environment, respective the SIMULINK TOOLBOX. The synthesis of the DMF can be realized on the base of the digital filter, by giving the Amplitude/frequency characteristic and Phase/frequency characteristics. The DMF can be realized as FIR or IIR filters. The signal can be processed in time as in frequency domain.

To preserve the information about the amplitude and phase of the signal and to decrease the sampling frequency can be used the quadrature processing.

REFERENCES

[1] Тургин Г. Введение в теорию цифровых согласованных филтров, ТИНЕЗр тл64, №7, 1974

[2] Proakis John G., Digital Communications, Cole Publishing Company, 2000

[3] Тихонов В. Н. Статистический анализи синтез радиотехнических устройств и систем, М., Радиосвезь, 1991

[4] Иванов Р. Цифрова обработка на едномерни сигнали, Г., Алмаматер, 1999