Basic properties of the real signals

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***Abstract* – Definitions of the basic parameters of the real signals are given. The disaccord between the engineering world with the real signals and the world of mathematicians with the oversimplified mathematical models for these signals is obvious especially in the following theoretical points: the classical sampling theorem, Fourier series, Dirichlet conditions, Gibbs phenomenon, Dirac function (delta function), Dirac comb, unit step function, staircase functions (based on rectangles) and theirs derivatives. These models are declared misrepresentatives and attempts to reject, modify and replace them were made. The paper is developed for educational and research purposes.**

***Keywords* – real signals, simplest signals approximations**

I. INTRODUCTION

Although during the twentieth century the signal sampling (and reconstruction) theory (SST) and the digital signal processing (DSP) did significant technological progress there are several basic principles and simplified mathematical models (SMM) which could be questioned in order to be rectified or replaced with something more accurate when it is possible. The SMM mention in the paper and widely used in the SST do not represent the real signals (RS) because of the following common errors:

1/ oversimplification and overgeneralization of the RS.

2/ introduction of artificial and misrepresentative *theoretical concepts* which are irrelevant to the engineering practice (they cannot be observed, used or proved practically).

3/ not clear definitions of the RS and their parameters

4/ wrong models of error evaluations of the RS approximations.

Consequently, the SMM in the SST (discussed in this paper) with these drawbacks do not have the same natures as the RS, do not represent it and should be reevaluated. They are oversimplified mathematical models (OSMM).

The disaccord between the RS and the OSMM is obvious especially in the following theoretical points: the classical sampling theorem (CST), Fourier series (FS), Dirichlet conditions (DIC), Gibbs phenomenon (GP), Dirac function or delta function (DEF), Dirac comb (DCM), sin(x)/x function (SXF), unit step function (USF), unit step pulse (USP), rectangular staircase function (RSF) and theirs derivatives.

The present paper is based on the ideas developed in [1, 2]. The notion of the "signal sampling factor" (SSF) $N=F_d/F_s$ was introduced and the maximal amplitude error E_{max} (due to the non sampling the sinusoidal or cosinusoidal signal (SS or CS) into its maximum) during the sampling process was calculated [1,2].

The fundamentals of the classical SST could be found in many sources e.g. [3, 4, 5]. This paper is concentrating on the well-known and widely published fundamentals of the SST because the oversimplification and the overgeneralization have made harm there and are influencing the concepts in the higher levels of the theory.

II. BASIS PROPERTIES OF THE REAL SIGNALS (BPRS)

The RS (the physical signals) are generated, transmitted and processed by material objects. In order to analyze and use the RS simplified but representative mathematical models (MM) are needed. The difference between a representative mathematical model (RMM) and a misrepresentative (non representative) mathematical model (MRM) is that the RMM is tightly related to the RS and has the same "nature". The MRM and the RS have different nature. An MRM cannot be reproduced into material world. More precisely RMM has the same principal parameters as the RS and the differences (the errors) between the RMM and the RS are clearly defined and calculable. Usually, an error < 1% (0.1 dB) is a reason to accept the model as a accurate RMM. An error in one or more of the parameters >10% (1dB) and especially > 30% (3dB) is making the MM misrepresentative.

Unfortunately due to the widely spread oversimplification, overgeneralization and insufficient error definitions a lot of the basic models in SST are oversimplified MM (OSMM) or inappropriate applications of RM. We will mention a few examples: GP, "ideal rectangular pulse (signal)" (IRP), "ideal saw tooth signal" (ISTS), CST, DEF, DCM, SXF, USF and FS. The present paper is calling for a revision, correction and/or abandoning of these basic SMM in the SST.

The RS could be very different and frequently are defined as "fast", "slow", "audio", "video", "wide band", "narrow band", "with one, two or multiple frequency (signal components)", "with one two or many sinusoidal components", "digital", "pulsed" etc. but they have common basic properties, as stated below. During the definition of these properties it is taken into consideration that the inevitable parasitic resistors, capacitors and inductances are involved into generation, transmission and processing of the RS.

Definition 1: The simplest idealized band wide signal (SBLS) is given with one of the following formulas:

(1) $A = A_m \sin(2\pi F+\varphi)+C$

(2) $B = B_m \cos(2\pi F+\varphi)+C$

where Am and Bm are the maximal values, F is the linear frequency, φ is the phase and C is the offset or DC. The noise component of this signal is neglected.

All MM should be tested with SBLS in order to verify their representativeness.

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Definition 2: The postulate for the basic properties of the real signals (PBPRS) is stating that the basic properties of the RS an the RM are as follow:
* Finite spectrum because the RS is produced in the material world
* Finite slew rate (SR) in every parameter due to the reason stated before.
* Finite power and finite amplitude due to the finite among of energy, which could be produced, transmitted and processed by the RS.
* Direct current (DC) component and phase component (FC). (Frequently these two parameters are accepted to be zero or neglected, but they are always present and should be taken into consideration.)
* Noise component (NC). Sometimes it could be predicted or neglected in the simplest RMM.
* Every RS could be represented as a sum of SBLSs with individual DC and FC

Several more notes could be added as comments to the basic properties of the RS and to the SBLS:

1. It is important to repeat that the slew rate (SR) of every RS is always less than infinity and never should be accepted to be infinity. Consequently the SMM with infinite SR as the IRP, RSF, ISTS, DEF and USF are too idealized for the world of engineers.

2. The first derivative of the RS always exists and is less than infinity. Replacing a finite value with infinity is creating an MRM (e.g. GP, DCM, DIC, DEF, etc).

3. The spectrum of the RS is always finite and there is no use to replace it with infinite spectrum (e.g. FS).

4. The spectrum of RS is containing a finite number of spectral lines which could have any value between zero Hertz (or DC) and one maximal frequency F_{smax} determined by the material in which the RS is produced, transmitted or processed. There is no use to represent the RS as an infinity sum of signals (e.g. FS).

5. There is no use to represent obligatory a RS with sum of harmonically related SS and CS. But it is useful to represent RS with an algebraic sum of SS and CS, which could be harmonically or not harmonically related and with individual phase and individual DC component (e.g. FS).

6. If the RS is interrupted in point where the momentum power is equal to zero ($P_{mom} = 0$) this does not change its spectrum. That means that a half of sinusoidal wave, sequence of half waves and a full sinusoidal wave (all of them with no DC component) do have the same FC into theirs spectrums but the amplitude of the FC are different (e.g. FS).

7. The SBLS is containing two spectrum lines. The first is the DC line and the second is any frequency $F_s>0$ approximated with SS or CS. SBLS is used as a basic test signal everywhere when it is possible and as a basic component into the sum for approximation of any RS.

8. The RS does not have ideal “angles”. The “angles” of the RS are always “with curve” or “rounded”.

The RS could be very complex and in order to analyze it a lot of simplified mathematical models (SMM) are used. There are at least several points which are not respected in the SMM used into classical SST:
* The model and the RS have differences in all parameters. These differences should be evaluated and expressed in clear and verifiable form with the corresponding errors. This is not the case with CST, USF, FS, etc.
* There are always limits and if we cross them the SMM and the RS does not have the same nature and the model does not represent the RS. One of the limits is given with the finite value of SR.
* The oversimplification and the overgeneralization applied to the RS leads to wrong conclusions, (e.g. CST).

The points mention before are the main sources of errors between the RS and the SMM in the following cases: the CST, FS, DIC, GP, DEF, DCM, the approximation of the RS with set of DEF or Dirac-like functions and USP. These models have the following common error: “Replacing a RS with too idealized MM with infinite slew rate and ideal angles is leading to the non existing effects and wrong conclusions”. A good example of this error is the replacement of the real “trapezoidal pulse with rounded angles” with “trapezoidal pulse with ideal angles” and even worse with IRP. This replacement is leading to the non-existing phenolmenon called “GP”. Table 1 is giving an idea about which of the basic properties on the RS are not respected into the SMM listed above.

TABLE 1
THE OVERSIMPLIFICATION OF THE REAL SIGNALS INTO SOME THEORETICAL MODELS

#	Basic property of the RS	Frequently oversimplified parameter or signal	Theoretical model in the SST based on this oversimplification
1	Finite amplitude and finite peak to peak amplitude	Infinite amplitude	Delta function, Dirac function
2	Finite power in every moment	Infinite power during the transitions, rectangular signal, saw-tooth signal	Delta function, Dirac function, unit function, rectangular staircase function , unit step function
3	Finite spectrum, finite number of spectral lines	Infinite sum	Fourier series, Infinite polynomial approximation
4	Finite slew rate	Infinite SR, rectangular signal, saw-tooth signal	Gibbs phenomenon, unit step function, unit pulse, RSF
5	Individual DC component	Omission of the DC current component	Classical sampling theorem, Fourier series
6	Individual FC	Omission of the FC	Classical sampling theorem, Fourier series

III. FOURIER SERIES (FS)

From all of the “classical” theoretical concepts mention in the paper perhaps only the FS is still partially useful. Although the FS has several weak points as an attempt to approximate every periodical function (signal) with sum of SS and CS with harmonic frequencies, it is still interesting in the SST because the FS and RS has same nature. The FS is a simple and natural way to approximate some of the RS because it is relatively easy to use set of sinusoidal and cosinusoidal generators working on harmonic frequencies, to forward theirs outputs into analog summing amplifier and to produce many useful RS. But the standard FS has the following disadvantages:

TABLE 2
COMPARISONS BETWEEN THE BASIC PROPERTIES OF THE RS, FS, DIC, GP AND THE PRSS

#	Real signal (RS)	Furrier series (FS)	Dirih-let conditi ons (DIC)	Gibbs phenomen on (GP)	Postu-late (PBRSS)
1	Always finite spectrum	Not necessa rily (NS)	NS	Infinite (*)	Yes
2	Always finite number of spectral lines (of SC)	NS	NS	Infinite (*)	Yes
3	Finite slew rate	NS	NS	Infinite (*)	Yes
4	Finite amplitude (of RS and of all of its SC)	Yes	Yes	Yes	Yes
5	Existing phase	No	No	No	Yes
6	Separate DC for each SC	Comm on DC	Not discu-ssed	Not discu-ssed	Yes
7	Separate FC for each SC	No sepa-rate FC for SC	No separat e FC for SC	No separate FC for each SC	Yes

Notes: SC = Signal component. NS = Not necessarily. The SC could be SS or CS. (*) – Approximation with infinite SR.

* The individual phase of the SS / CS components in the sum is neglected.
* The individual DC component of the SS/CS components in the sum is neglected.
* Sometimes it is simpler to represent a RS as sum of SS/CS signals, which are not harmonically related.
* The sum of SS and CS for approximation of a RS should be finite. The reason is that the RS has always a finite spectrum and finite number of spectral lines in every moment.
* One more problem is that the FS is suffering from wrong applications from the part of the mathematicians. The wrong application of the sum of SS and CS (e.g. to the unreal signals as IRP and ISTS) leads to the DIC and GP, which in fact have no engineering value.

These are the raisons to extend the FS. A more general expression for RS approximation is

1/ with finite sum of non-harmonically related signals
2/ every member is given with formulas (1) or (2).

IV. DUAL-TONE AND MULTI-TONE MULTI FREQUENCY SIGNALS

Dual tone multi frequency signals (DTMF) and multi tone multi frequency signals (MTMF) are used for message transmition, canal testing and testing of the equipment and components. In some cases the frequencies of the tones are not harmonically related and the tones could be not periodical because they are produced and transmitted during short intervals. At least three principal points exist:
1. The phase relations and the DC components could be important. Consequently the FS, which contains harmonically related components without individual phase and DC components, is not always the best or the simplest solution.
2. There is no use to represent a sum of two non-harmonically related SS/CS with an expression, which contains more that two harmonically related SS/CS.
3. There is no use to express a RS with obviously short (finite) spectrum as too long (or infinite) sum of harmonically related SS/CS. Due to the way DTMF and MTMF signals are produced it is known that they have into the spectrum a finite number of SS and CS components with finite parameters.

Any DTMF r(t) could be presented as an algebraic sum of the two signals (SS and/or CS) with the formula below:

(3) $r(t)= s_1(t)+ s_2(t)$,

where

(4) $s_1(t) = (A_{0k} + A_m\sin(2\pi F_k t + \varphi_k))$

and

(5) $s_2(t) = (B_{0k} + B_m\sin(2\pi F_l + \psi_l))$

or the expression for the DTMF is

(6) $r(t) = (A_{0k} + A_m\sin(2\pi F_k t + \varphi_k)) + (B_{0k} + B_m\sin(2\pi F_l + \psi_l))$

DTMF and MTMF have properties not covered by FS:
* Always finite sum of SS and CS
* Individual phase for each signal component (SC)
* Individual DC component for each SC
* Non-harmonic SS and CS into the sum in order to use shorter and simpler mathematical expression.

Keeping SS and CS into the sum is important. Making the difference between the SS and CS is also important and one of them cannot be omitted. Making a difference between the SS and CS is equivalent of making a difference between the four quadrants and the four principal directions into two axe coordinating system (x,y). The classical FS series (which could be found in [3, 5] is stating that every periodical function (in particular a signal) s(t) with period T could be represented as a sum of SS and CS signals with periods n/T (n = 1, 2 , 3…) plus possibly a constant C and the following general expression is applicable [e.g. 5]:

(7) $s(t) = C + \sum (A_{mk}\sin(2\pi F_k t)) + \sum (B_{ml}\cos(2\pi F_l))$

where k and l are integer number which could be infinity.

V. POSTULATE OF RS SPECTRUM PRESENTATION (PRSSP)

The postulate was formulated in order to:
1) Approximate every RS with basic properties in Table 1.
2) Describe in a better way any DTMF and MTMF with non-harmonic frequencies of the SCs
3) Describe RS used during the testing channels, equipment and components.

Definition 3. Postulate PRSSP: Every RS r(t) with basic properties given in Table 1 could be represented as a sum of SS and CS signals components each of them with four basic components ($A_{0,}$ A_m, F and φ) and given with one of the two formulas below:
a/ for every sinusoidal SC $s_{ss}(t)$

(8) $s_{ss}(t)= A_0 + A_m\sin(2\pi F_{ss}t +\varphi))$

b/ for every cosinusoidal SC $s_{cs}(t)$

(9) $s_{cs}(t)= B_0 + B_m\cos(2\pi F_{cs}t + \psi)$

where
A_0, (B_0), A_m $(B_{m)}$, F_{ss} $(F_{cs}$) and φ (ψ) are respectively the DC part, amplitude, frequency and the phase of the SC.

The signal r(t) could de given with the formula below

(10) $r(t) = r_{ss}(t) + r_{cs}(t)$

where
$r_{ss}(t)$ is the sum of all sinusoidal components in the spectrum of RS
$r_{cs}(t)$ is the sum of all cosinusoidal components in the spectrum of RS

The sum of all SS components $r_{ss}(t)$ is given with the formula

(11) $$r_{ss}(t) = \sum_{0}^{p} (A_{0k} + A_{mk} \sin(2\pi F_k t +\varphi_k)) ,$$

k= 0, 1, 2...p

The sum of all CS components $r_{cs}(t)$ is given with the formula

(12) $$r_{cs}(t) = \sum_{0}^{q} (B_{0l} + B_{ml} \cos (2\pi F_l +\psi_l)),$$

l = 0, 1, …q

Consequently the main task of the signal conversion and reconstruction is to convert and reconstruct RS with known error in each signal component. Testing the spectrum of the reconstructed signal should be done separately with analog and digital filters and the results should be compared.

VI. CONCLUSION

Basic properties of the RS are defined. A postulate of the presentation of every RS as a sum of SS and CS components each with four basic parameters is given the postulate is an extension of the standard Fourier series. Example with DTMF and MTMF signals is given. This topic is continued in [6].

The paper is intended to help students and researchers to evaluate the existing MM and to replace them with more representative models when it is necessarily. New ideas and examples in the field are given.

VII. ABBREVIATIONS IN THE PAPER

BRPS – basic properties of the real signal
BLS – band limited signal
SS/CS – sinusoidal/cosinusoidal signal
CST – classical sampling theorem
DC - Direct current
DCM – Dirac comb
DEF – delta function, Dirac function
DIC – Dirichlet conditions
DTMF - Dual tone multi frequency signals
MTMF - Multi tone multi frequency signals
FC - phase component
FS - Fourier series
IRP - ideal rectangular pulse (signal)
ISTS - ideal saw tooth signal
GP – Gibbs Phenomenon
MM – mathematical model
MRM - misrepresentative mathematical model
NS – Not necessarily
NC - Noise component
OSMM – over-simplified mathematical model
PBPRS - postulate for the basic properties of the real signals
PRSSP – postulate for real signal spectrum presentation
RSF - rectangular staircase function
RMM – representative mathematical model
RS – real signal
RSF – rectangular staircase function
TPIA – trapezoidal pulse (signal) with ideal angles
SBLS – simplest band wide signal $A= A_m\sin(2\pi F+\varphi)+C$
SC – signal component
SST - signal sampling theory
SSF – signal sampling factor N=Fd/Fs
SST – signal sampling theory
SR – slew rate
SSM – simplified mathematical model
USF – unit step function

VIII. REFERENCES

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