„Mathematical models“ versus „real signals“

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***Abstract* – The disaccord between the engineering world with the real signals and the world of mathematicians with the mathematical models for these signals is obvious especially in the following theoretical points: the classical sampling theorem, Fourier series, Dirichlet conditions, Gibbs phenomenon, Dirac function (delta function), Dirac comb, sin(x)/x function, unit step function, staircase functions (based on rectangles) and theirs derivatives. These models are declared misrepresentatives and attempts to reject, modify and replace them were made. The paper is developed for educational and research purposes.**

***Keywords* - real signals definition, simplest signals approximations**

I. INTRODUCTION

New ideas in the signal sampling and reconstruction theory (SST) are given in [1,2,3]. But the classical SST is still using a lot of mathematical models (MM) which do not represent real signals (RS) [4, 5, 6]. The students and the researchers in the engineering and mathematical fields are having difficulties to understand and apply these models. The problem is discussed in this paper and solutions are proposed.

A clear difference between the term “mathematical model” (MM) and the term “real signal” (RS) should be made. These two terms have different nature. For example it is acceptable from mathematical point of view to imagine a MM with the following properties:
1/ infinite number of minimums and/or maximums
2/ ideal angles (45 deg, 90 deg. etc)
3/ infinite slew rate (SR)
but obviously it is not possible to produce RS with these parameters. Consequently these MM are not applicable to the SST and to the engineering (physical) world. They are misrepresentatives MM (MRM). The primary objective into the engineering world is to develop and utilize representative MM (RMM).

When a MM of the RS is developed, an evaluation of the errors in each parameter (amplitude, phase, spectrum, frequency, slew rate (SR), power, etc) should be done. This is not an easy process but without it there is a danger to develop a MRM and to draw wrong conclusions. In fact the level or the representativeness of the MM should be evaluated and stated in each particular case.

A lot of differences between the technical process (TP) (e.g. the real signal) and the MM exists. A bidirectional relation should be created and tested as follow:
1. The TP should be replaced with closest and simplest possible RMM. The nature of the TP should be respected and exaggerated simplification and overgeneralization should be avoided.
3. A difference should be made between RS and the MM with theirs mathematical functions (expressions). Not every mathematical function (MF) could be converted into RS but every RS could be represented with mathematical expressions**.**

II. DIRICHLET CONDITIONS (DIC)

The attempt of applying the Fourier series (FS) to “every periodical function” is suffering from overgeneralization, oversimplification and non-respect of the basic rule that “the “original” and “the approximation” (or the RS and its SMM) should have to have the same nature and the same basic properties in order to make parts of a representative couple “signal-model”. The FS could be very useful when applied to a lot of periodical RS with neglected noise components but not to “every periodical mathematical function or RS”.

The DIC are result of attempt to rectify some weak points in the FS but they are suffering from the same weak points. due to the oversimplification, overgeneralization and are misrepresentatives. Perhaps from the mathematical point of view the DIC are acceptable. But if the FS is used to represent a RS from the engineering world the DIC are useless because every RS is satisfying the conditions stronger than DIC For example, a RS with infinite number of minimums and maximums cannot be produced with technical devices Obviously the DIC conditions are not related to the RS and to the SST. A comparison between these conditions and the basic properties of the RS is stated in [3]. It is obvious that the RS are always satisfying the DIC. Consequently there is no use to mention DIC in relation to the FS and RS into SST.

Moreover we could mention the following against the DIC, delta function (DEF), Dirac comb (DCM), GP and unit step function (USF):
* The tension on the capacitor could not be discontinued. The circuit and signal source without parasitic (inherent) capacitor(s) could not exist. Consequently the SR of every signal source is limited and the “angles” of the produced signal are never ideal. How the “angles” will be approximated depends on the application. For example they can be approximated with exponential or sinusoidal functions.
* The same reason as above but applicable to the current traversing the inductance internal to the source and the inductance(s) of the signal connecting and processing elements.
* The electric chargers and waves have finite speed of movement. Consequently the SR of every signal source is limited and depends on the material where the signals is produced, transmitted and processed.
* Replacing a finite RS with infinity is unacceptable source of error in most of the cases and creates non-existing effects and infinity spectrum. The GP is an example of an artificial effect in the engineering world. (Perhaps it has some value in the world of mathematics.)

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III. CLASSICAL SAMPLING THEOREM (CST)

The classical sampling theorem (CST) or so called "sampling theorem of Kotelnikov - Shannon- Whittaker -Nyquist-etc" is based on DEF, DCM, FS and function sin(x)/x (SXF). CST is pretending in general "to represent or reconstruct exactly any band limited signal (BLS) with maximal frequentcy F_{smax} under the condition $Fd>=2F_{smax}$". The CST has many weak points [1, 2] and we will mention:
1/ lack of errors evaluations,
2/ possible exception (sometimes the signal could be reconstructed with signal sampling factor (SSF) N <2),
3/ possible lost of the signal with SSF N=2,
4/ wrong MM and
5/ possible strong and not desirable phase modulation.

The lack of errors evaluation is very important disadvantage of the CST because once converted into digital form the RS is reconstructed with differences (errors) in every parameter and these differences should be evaluated. Or the ADC is leading to the irreversible lost of information and this lost should be evaluated. Obviously CST is a result of the overgeneralization and oversimplification. If we compare the CST with another fundamental theorem (e.g. with the theorem of Pythagoras $c^2 = a^2 + b^2$) we will understand the differences:
* in the definitions (CST is not well defined),
* precision (lack of precision in the CST) and
* usefulness (CST is not useful in practice)
between the two theorems.

In many cases a set of RS with known parameters is used. This is giving the possibility to use the method "a particular solution for a particular case" which means that representative sampling theorems (RST) and RMM could be developed for each case (e.g. for each signal or group of signals) and under appropriate conditions (simplifications).

Several practical examples could be given concerning sampling the basic RS e.g. direct current (DC), sinusoidal signal(SS), co sinusoidal signal (CS), sum of SS and CS, triangular-like signals and trapezoidal-like signals with different form etc. When it is possible and necessary several particular RST and RMM could be put in one common RMM or RST under proper conditions. A good example for this approach is the sampling of SS and CS signals discussed in [1, 2].

Obviously, new RMM should be developed. Definitions of the parameters of RS, evaluations of the existing MMs and proposing new and better (more accurate and closer to the RS) models are necessary.

IV. GIBBS PHENOMENON (GP) VERSUS "RINGING" AND "APPROXIMATION ERROR"

The GP is frequently discussed in the manuals for engineers [e.g. 6] and in software dealing with "signals and systems" such as Matlab. The usual example is an approximation of "ideal rectangular pulse" (IRP) with FS. This example is obviously wrong. In fact the GP is born due to the common error repeated widely in the SST: "Due to the oversimplification (overgeneralization) the SMM (in this case the IRP) does not represent the RS". The nature of the RS is not respected and an artificial (non-existing) and not useful "phenomenon" is "observed". The GP does not exist for the "trapezoidal-like signals" or "square-like signals" (signals with finite SR, with "rounded angles" which resembles to the IRP but have different nature). In fact the "real rectangular pulse" (RRP) is always a "trapezoidal-like signal with rounded angles"! The conclusions are:
1/ the GP and the DCI are not related to the RS and the SST
2/ the error of the approximation of an RS with the FS cannot be considered as a GP.

A clear difference between the process of "ringing" [6] and the GP should be done. The effect of "ringing" or the effects of "overshoots" and "undershoots" are real effects when a RRP is applied to the channel with inappropriate load, narrower frequency band or with not enough equalized frequency band. But the "overshoots" and the "undershoots" have nothing in common with the artificial GP because at least of the following reasons:
1. The IRP used to illustrate the GP is too idealized and does not have a real value. As we said before: "The infinite SR and the ideal angles cannot be generated, transmitted and processed in a material world and there are two mistakes:
a/ replacing a finite number with infinity
b/ replacing a continuous function with discontinuous function."
2. Trying to approximate an ideal signal with ideal "angles" (IRP, ISTS etc.) with set of real functions with different nature (e.g. SS and CS) is wrong because of the different nature of the signal and its approximation. Also, there is a difference between the GP (an inappropriate approximation) and the "approximation error due to appropriate but not exact approximation".
3. There is no sense to approximate a RS with obviously finite spectrum (and with finite number of spectrum lines) with infinity sum of SS and/or CS (e.g. FS).
4. The nature of the process of "ringing" due to the inappropriate loading is different from the process of GP. In fact a RRP passing through a real channel (circuit) with inappropriate load or frequency bandwidth is producing the real effect of "ringing" which is important and informative but this is not a GP.

The conclusion is that "the ringing effect exists but the GP does not exist in the engineering world or more exactly it is not applicable to the SST. Replacing an IRP with FS is like an attempt to replace an imaginary number with infinite sum of real numbers and to discover that there is a "problem" or an "error".

Also, the following main differences between the process of "ringing" and the GP exist:
1. The "ringing" is applicable for the transitions of the signal. It is not applicable for the horizontal parts of the pulse. The GP is applicable for both parts.
2. The "ringing" is not always symmetrical for the positive and the negative transition and frequently is applicable only for the end of the transition. The GP is symmetrical for the both transitions and exists for the beginning and the end of the transitions.
3. The "ringing" is a natural process due to the disaccord between two real objects (the "signal" and the "channel" or

the "load"). The GP is an artificial effect due to the dissacord between a RS (sum of SS and/or CS) and imaginary signal (e.g. the IRS and ISTS).

Also the GP is not equivalent to the normal "approximation error" between a RS and its RM. As it was mention before the RS and its RM has the same nature and the same basic properties and one or more approximation error(s) could be defined. The GP is the result of attempt to approximate an imaginary signal with algebraic sum of real signals.

V. EXAMPLES FOR OVERSIMPLIFICATION

Figure 1 is showing examples of the oversimplification of two pulses.- trapezoidal and triangular.

Figure 1a and Figure 1c are showing idealized (simplified) but still representative models with straight lines and "rounded" angles. They are built by two basic element:

* The first element is a non-ideal "rounded" angle which could be approximated with exponential or sinusoidal function in the simpler cases.

* The second element is a linear function (straight line) with finite SR.

In the connection point these two elements have the same value and same SR.

The models on Figure 1 b and Figure 1d are oversimplified and misrepresentatives. The oversimplified (over generalized) models (OSMM) are containing artificial (non existing) elements as ideal angles and straight lines with infinite SR. They should be abandoned because theirs properties are too far to the properties of the real signals and are leading to the wrong conclusions.

VI. CONCLUSIONS

Several important conclusions could be made:

1. The DIC (discontinuity, derivation, limited number of maximum and minimums, etc.) are not related to the application of FS to approximate RS in the SST. Every RS is satisfying the DIC. Consequently DIC should not be mention in the SST in relation with FS and RS.
2. Although the FS is a good approximation of many periodical RS it is incomplete, e.g. it cannot approximate in a simple way the RS constructed by a sum of the simplest band limited signals (SBLS) and containing SS and/or CS with non-harmonic frequencies and an individual DC component. Consequently, a more general sum containing SS and CS with non-related parameters (amplitude, frequency, phase and DC component) should be used to approximate any RS [3].
3. The Dirac comb (DCM) is based on the rule "take and forget" and does not correspond to the real world application. In fact the RS is always replaced with a step function (SF) based on the rule "take and memorize the present sample until the next sample became available".
4. A difference should be made between the physical (engineering) world and the SMM. In the world of mathematicians a lot of things are possible but in the world of the engineers a few things are realizable and observable. For example an SMM, which have to be used in the engineering world, have to have the same nature as the modeled engineering process. Not respecting the nature of the engineering (physical) process is leading to MRM and to wrong conclusions.
5. The GP is declared, "non-existing in the engineering world" due to the fact that the RRP is in practice "a trapezoidal-like signal with rounded angles". Consequently the GP is not applicable to it and is not observable. The GP and the natural effect of "ringing" due to improper loading or bandwidth are declared with different nature.
6. The CST is considered "non efficient" due to the oversimplification, lack of universality and no errors evaluation. The DEF, DCM and the SXF are considered "irrelevant to the sampling and reconstruction process".
7. With the exception of the FS, which is still utilizable in some cases and after "reparation"[3], the rest of the classical concepts stated above are irrelevant to the SST. In order to be more useful the FS should be extended with new parameters (individual phase and DC component of every SS/CS component) and with non-harmonic frequencies of the SS/CS components.
8. Postulates about the composition of RS are formulated. The examples with DTMS and MTMF are given in [3].
9. The oversimplifications and the overgeneralizations are important sources of errors in the classical SST and should be avoided. Also it is important to test the representativeness of every SMM and to evaluate the errors between SMM and replaced RS. The main sources of the oversimplification into the NRM are the replacement of the finite SR with infinite and the rounded angles with ideal or "broken" angle.
10. During the sampling and reconstruction process the approximation of the RS is with non-ideal trapezoidal stair case function (TRSF with finite slew rate).
11. The SMM but still RMM for RS should be constructed from two segments: "lines" and "rounded angles". This SMM must not be further simplified because an oversimplification will occur and a MRM will be produced with non-existing and not useful artificial effect (e.g. GP and CST)

The paper is making revision of some of the fundamental principals in SST and DSP and replacing them with more realistic models.

The paper is developed for the educational and research purposes. It is giving new ideas and examples for evaluation of the relations between the basic properties of the real signals and the representatives and non representatives simplified mathematical model of these signals. It is helping the students in the engineering and mathematical field to understand the nature of the real signals and the relations between them and the RMM and NRM. New models could be developed with these ideas and the old modes could be replaced.

VII. ABBREVIATIONS IN THE PAPER

BLS – band limited signal
BRPS – basic property of the real signal
CS – co sinusoidal signal
CST – classical sampling theorem (Kotelnikov-Whittaker-Shannon – etc.)
DSP - digital signal processing

DC - Direct current
DCM – Dirac comb
DEF – delta function
FS - Fourier series
IRP - ideal rectangular pulse
ISTS - ideal saw tooth signal
GP – Gibbs phenomenon
MF – mathematical function
MM – mathematical model
MRM - misrepresentative mathematical model
NC - Noise component
OSMM – over-simplified mathematical model
RRP – real rectangular pulse
RSF - rectangular staircase function
RMM – representative mathematical model
RS – real signal
RSF – rectangular staircase function
TP – technical process
TPIA – trapezoidal pulse (signal) with ideal angles
TRSF – trapezoidal stair case function
SBLS – simplest band wide signal $A= A_m \sin(2\pi F+\varphi)+C$
SC – signal component
SST - signal sampling theory
SS - sinusoidal signal
SSF – signal sampling factor N=Fd/Fs.
SST – signal sampling theory
SR – slew rate
SSM – simplified mathematical model
RST – representative sampling theorem
SXF – sin(x)/x function
USF – unit step function

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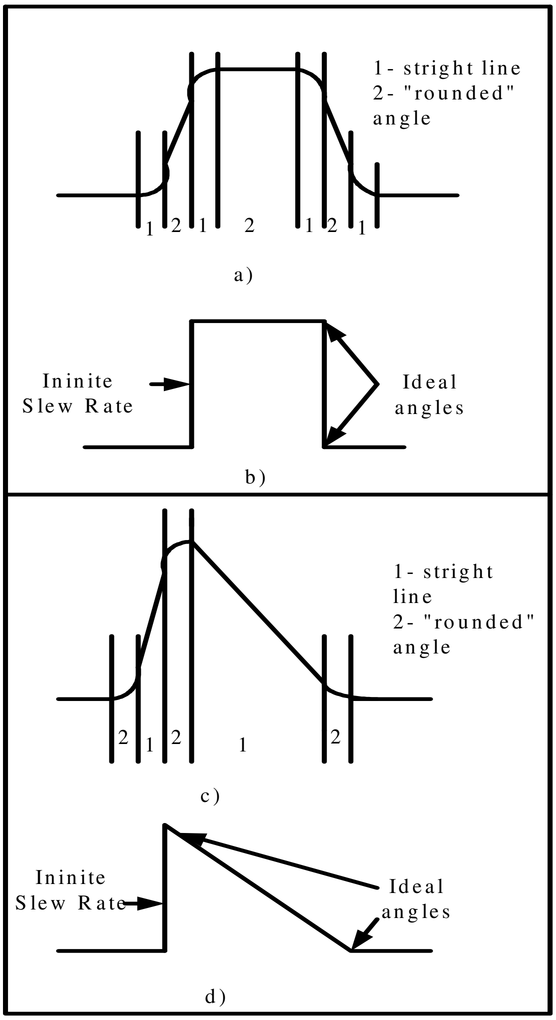

Figure 1