Diversity System with *L* Brunches for the Demodulation of n-FSK Signals

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Abstract – **The diversity system for the demodulation of n-ary digital frequently modulated signal (n-FSK) with *L* brunches is considered in this paper. Such receiver consists of *n* brunches. The signals from the corresponding brunches of all receivers are added. The combiner determines the maximal signal from EGC output. The joint probability density of the signal and the signal derivative at the combiner output is determined in this paper. The average number of the signal crossing rate at the system output can be calculating by this joint probability density. In the paper the probability density of the signal at the combiner output is also calculated.**

Key words – **diversity system, n-FSK signals, probability density, EGC combiner**

I. INTRODUCTION

In this paper the statistical characteristics of the signal at the output of the diversity system with L diversity brunches will be calculated. This system uses for the n-FSK signal demodulation. The signals from the same brunches of each receiver are added. The combiner determines the maximal signal at the output. These diversity systems apply in wireless telephony. The Gaussian noise appears at the receiver input. This noise becomes the narrowband Gaussian noise at the narrowband filter output. The post detection filtering is used. Some models of the diversity systems which are used for the binary FSK signal demodulation are considered in paper [1]. One of models considered is: the combiner determines maximal value of the signal from all brunch outputs of the all receiver. The combiner we consider in this paper is more efficient because of the addition of signals from corresponding brunches of each receiver. In this paper the n-FSK signal will be analyzed whereas in paper [1] the BFSK signal is analyzed. The demodulation in the receivers is coherent. Each receiver consists of narrowband filter and of the correlator. The correlator consists of the multiplier and the low pass filter. We consider the extraction of the referent carrier as ideal. In this paper the error probability will be calculated, the probability density of the signal at the system output and the joint probability density of the signal and its derivative at the system output will be determined also.

We can calculate the number of the signal crossing rate at the system output by the joint probability density of the signal and its derivative.

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II. THE ERROR PROBABILITY CALCULATION

The model of the diversity system which is used for the n-FSK signal demodulation is given at Fig. 1. The diversity system has L diversity brunches. Such receiver has n brunches. The Gaussian noise appears at all receiver inputs. The narrow band Gaussian noise is obtained at the narrow band filter outputs. The signals at the first receiver brunches outputs are $z_{111}, z_{121}, ..., z_{1n1}$ for the hypothesis H_l. The signals at the second receiver brunches outputs are $z_{112}, z_{122}, ..., z_{1n2}$ etc. The signals derivatives are $\dot{z}_{111}, \dot{z}_{121}, ..., \dot{z}_{1n1}$, $\dot{z}_{112}, \dot{z}_{122}, ..., \dot{z}_{1n2}$, etc. The signals at the inputs of the receiver for the hypothesis H_0 are:

$$\begin{aligned} r_1(t) &= A\cos\omega_1 t + n_1(t) \\ r_2(t) &= A\cos\omega_2 t + n_2(t) \\ &\cdots \\ r_n(t) &= A\cos\omega_n t + n_n(t) \end{aligned} \tag{1}$$

A is the signal amplitude, ω_l is the signal frequency for the hypothesis H_l. The Gaussian noises appearing at the diversity brunches inputs are $n_1(t), n_2(t), ..., n_n(t)$. These noises have the mean values zero and variances $\sigma_1^2, \sigma_2^2, ..., \sigma_n^2$. The narrow band Gaussian noise at the filter output in l-th diversity brunch and k-th receiver brunch is

$$n_{lk} = x_{lk}\cos\omega_k t + y_{lk}\sin\omega_k t,$$

$$l = 1,2,...,L; \quad k = 1,2,...,n \tag{2}$$

where x_{lk} and y_{lk} are the Gaussian components in quadrature, independent at the same time instant.

The signals at the corelator outputs are:

$$\begin{array}{llll} z_{111} = A + x_{11} & z_{112} = A + x_{21} & \cdots & z_{11L} = A + x_{L1} \\ z_{121} = x_{12} & z_{122} = x_{22} & \cdots & z_{12L} = x_{L2} \\ \cdots & & & \\ z_{1n1} = x_{1n} & z_{1n2} = x_{2n} \cdots & & z_{1nL} = x_{Ln} \end{array} \tag{3}$$

The signals at the EGC combiner outputs are:

$$\begin{aligned} z_{11} &= z_{111} + z_{112} + \cdots + z_{11L} = L\cdot A + x_{11} + x_{21} + \cdots + x_{L1} \\ z_{12} &= x_{12} + x_{22} + \cdots + x_{L2} \\ &\cdots \\ z_{1L} &= x_{1n} + x_{2n} + \cdots + x_{Ln} \end{aligned} \tag{4}$$

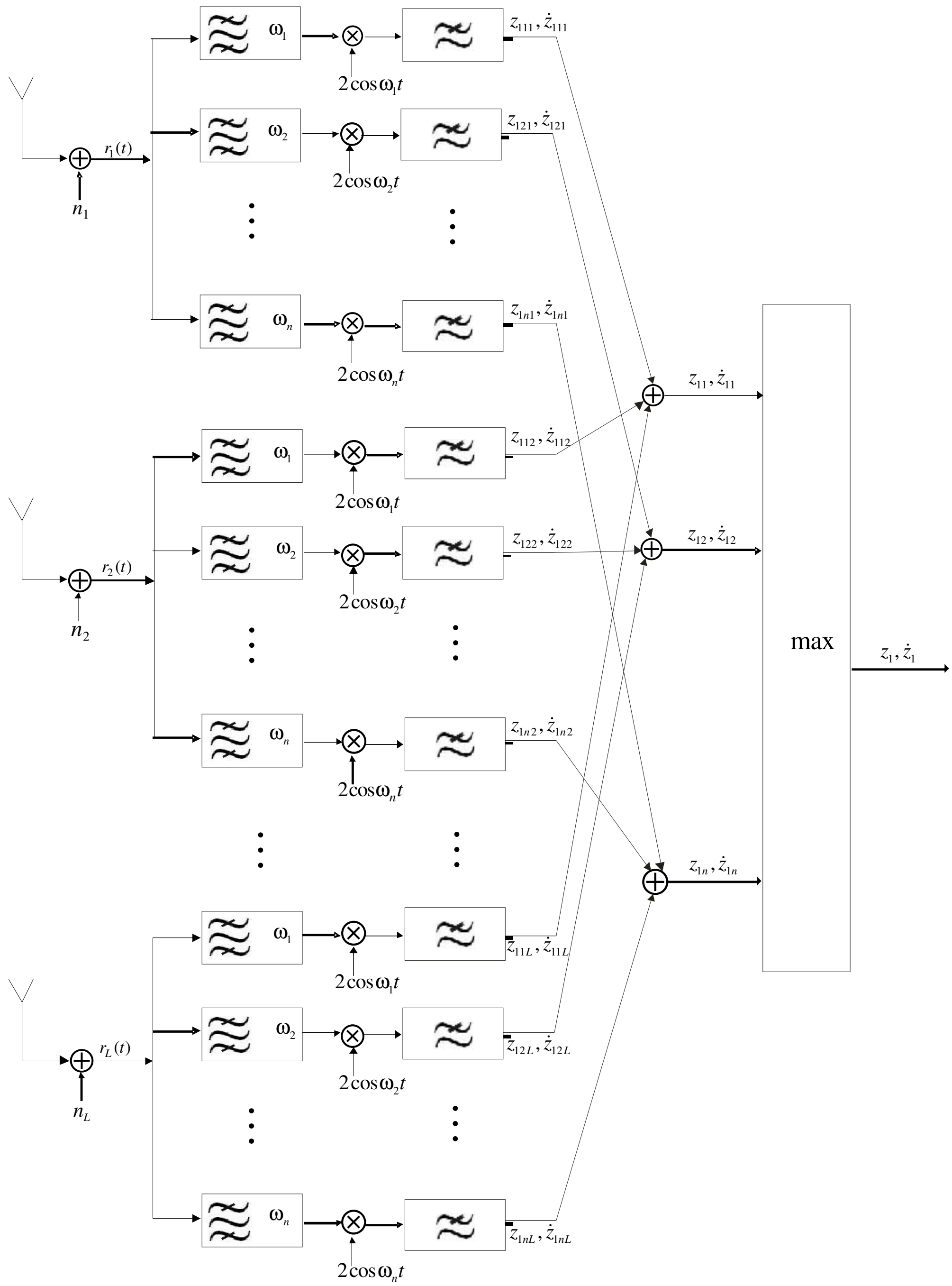

Fig. 1. The model of the diversity system

The signals z_{11l}, $l = 1,2,...,L$ have Gaussian probability density function with mean values A and variances $\sigma_1^2, \sigma_2^2, ..., \sigma_L^2$.

$$p_{z_{11l}}(z_{11l}) = \frac{1}{\sqrt{2\pi}\sigma_l} e^{-\frac{(z_{11l}-A)^2}{2\sigma_l^2}} \tag{5}$$

The signals z_{1kl}, $k = 1,2,3,...,n$; $l = 1,2,...,L$ have mean values zero and variances $\sigma_1^2, \sigma_2^2, ..., \sigma_L^2$.

$$p_{z_{1kl}}(z_{1kl}) = \frac{1}{\sqrt{2\pi}\sigma_l} e^{-\frac{(z_{1kl})^2}{2\sigma_l^2}} \tag{6}$$

The random variable z_{11} has mean value $\overline{z_{11}} = L \cdot A$ and variance

$$\overline{\left(z_{11} - \overline{z_{11}}\right)^2} = \sigma^2 = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_L^2 \tag{7}$$

The random variables $z_{12}, z_{13}, ..., z_{1L}$ have mean values zero and variances:

$$\overline{\left(z_{1l} - \overline{z_{1l}}\right)^2} = \sigma^2 = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_L^2, \qquad l = 2,3,...,L \tag{8}$$

Because of that we have:

$$p_{z_{11}}(z_{11}) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_{11}-LA)^2}{2\sigma^2}} \tag{9}$$

$$p_{z_{1l}}(z_{1l}) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z_{1l}^2}{2\sigma^2}}, \quad l = 2,3,\dots,L \tag{10}$$

The probability for correct decision for hypothesis H_1 is

$$P_C = P\{z_{11} > z_{12}, z_{11} > z_{13}, \dots, z_{11} > z_{1L}\} \tag{11}$$

This probability is:

$$P_C = \int_{-\infty}^{\infty} p_{z_{11}}(z_{11}) \cdot dz_{11} \int_{-\infty}^{z_{11}} p_{z_{12}}(z_{12}) \cdot dz_{12} \cdot$$

$$\cdot \int_{-\infty}^{z_{11}} p_{z_{13}}(z_{13}) \cdot dz_{13} \cdots \int_{-\infty}^{z_{11}} p_{z_{1L}}(z_{1L}) \cdot dz_{1L} =$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_{11}-LA)^2}{2\sigma^2}} \cdot \left(\int_{-\infty}^{z_{11}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z_{12}^2}{2\sigma^2}} \cdot dz_{12} \right)^{L-1} \cdot dz_{11} \tag{12}$$

The conditional error probability for hypothesis H_1 is:

$$P_e = 1 - P_C = 1 - \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_{01}-LA)^2}{2\sigma^2}} \cdot$$

$$\cdot \left(\int_{-\infty}^{z_{01}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z_{02}^2}{2\sigma^2}} \cdot dz_{02} \right)^{L-1} \cdot dz_{01} \tag{13}$$

We can calculate now in the similar way the other conditional error probabilities for hypotheses H_l and then the error probability.

III. STATISTICAL CARACTERISTICS OF THE SIGNAL AT THE DIVERSITY SYSTEM OUTPUT

For the hypothesis H_1, the signal at the system output is

$$z_1 = \max\{z_{11}, z_{12}, \dots, z_{1n}\} \tag{14}$$

The probability density of the signal z_1 is

$$p_{z_1}(z_1) = \sum_{i=1}^{n} p_{z_{1i}}(z_1) \cdot \prod_{\substack{j=1 \\ j \neq i}}^{n} F_{z_{1j}}(z_1) \tag{15}$$

where $F_{z_{1j}}(z_1)$ is cumulative probability of the random variable z_{1j}. The cumulative probability of the random variable z_{11} is

$$F_{z_{11}}(z_{11}) = \int_{-\infty}^{z_{11}} p_{z_{1i}}(x) \cdot dx = \int_{-\infty}^{z_{11}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-LA)^2}{2\sigma^2}} dx \tag{16}$$

If: $$t = \frac{x - LA}{\sqrt{2}\sigma}, \qquad dt = \frac{1}{\sqrt{2}\sigma} dx \tag{17}$$

we obtain

$$F_{z_{11}}(z_{11}) = \int_{-\infty}^{\frac{z_{11}-LA}{\sqrt{2}\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = Q\left(\frac{z_{11} - LA}{\sqrt{2}\sigma}\right) \tag{18}$$

where

$$Q(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{x} e^{-t^2} dt \tag{19}$$

The cumulative probability of the random variables $z_{1k}, k = 2,3,\dots,n$ is

$$F_{z_{1k}}(z_{1k}) = Q\left(\frac{z_{1k}}{\sqrt{2}\sigma}\right) \tag{20}$$

The probability density of the random variable z_1 can be determined at the manner below. The joint cumulative probability of the random variables $z_{11}, z_{12}, \dots, z_{1n}$ is

$$F_{z_{11}, z_{12}, \dots, z_{1n}}(z_{11}, z_{12}, \dots, z_{1n}) = \prod_{k=1}^{n} F_{z_{1k}}(z_{1k}) \tag{21}$$

The cumulative probability of the random variable z_1 is

$$F_{z_1}(z_1) = F_{z_{11}, z_{12}, \dots, z_{1n}}(z_{11}, z_{12}, \dots, z_{1n}) = \prod_{k=1}^{n} F_{z_{1k}}(z_{1k}) \tag{22}$$

The probability density of the random variable z_1 is

$$p_{z_1}(z_1) = \frac{dF_{z_1}(z_1)}{dz_1} = \frac{d}{dz_1}\left(\prod_{k=1}^{n} F_{z_{1k}}(z_1) \right) \tag{23}$$

By using Leibniz formula we can obtain the same expression for the probability density of the signal at the combiner output as previously is derived. The joint probability density of the envelope and the signal derivative at the output is

$$p_{z_1, \dot{z}_1}(z_1, \dot{z}_1) = \sum_{i=1}^{n} p_{z_{1i}\dot{z}_{1i}}(z_1, \dot{z}_1) \cdot \prod_{\substack{j=1 \\ j \neq i}}^{n} F_{z_{1j}}(z_1) \tag{24}$$

where

$$p_{z_{11}, \dot{z}_{11}}(z_{11}, \dot{z}_{11}) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z_{11}-A)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi}\beta} e^{-\frac{\dot{z}_{11}^2}{2\beta^2}}$$

$$p_{z_{12}, \dot{z}_{12}}(z_{12}, \dot{z}_{12}) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z_{12}^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi}\beta} e^{-\frac{\dot{z}_{12}^2}{2\beta^2}}$$

$$\dots$$

$$p_{z_{1n}, \dot{z}_{1n}}(z_{1n}, \dot{z}_{1n}) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{z_{1n}^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi}\beta} e^{-\frac{\dot{z}_{1n}^2}{2\beta^2}} \tag{25}$$

β^2 is the variance of the signal derivative. The signal and his derivative are independent and have Gaussian probability density.

The signals at the system output in two time instants t_1 and t_2 are z_1 and z_2. These signals are:

$$z_1 = \max(z_{111}, z_{121}, ..., z_{1n1})$$
$$z_2 = \max(z_{112}, z_{122}, ..., z_{1n2}) \quad (26)$$

$z_{111}, z_{121}, ..., z_{1n1}$ are the signals at the combiner outputs at time instant t_1 and $z_{112}, z_{122}, ..., z_{1n2}$ at time instant t_2.

We will determine now the joint probability density of the signals z_1 and z_2 at the system output. This probability density is:

$$p_{z_1,z_2}(z_1, z_2) = \sum_{i=1}^{n}\sum_{j=1}^{n} p_{z_{1i1},z_{1j2}}(z_1, z_2) \cdot$$
$$\cdot \prod_{\substack{k=1 \\ k\neq i}}^{n}\prod_{\substack{s=1 \\ s\neq j}}^{n} F_{z_{1k1}}(z_1) \cdot F_{z_{1s2}}(z_2) \quad (27)$$

where

$$p_{z_{111},z_{112}}(z_{111}, z_{112}) = \frac{1}{\sqrt{2\pi}\sigma^2\sqrt{1-r^2}} \cdot$$
$$\cdot e^{-\frac{(z_{111}-A)^2 - 2r(z_{111}-A)(z_{112}-A) + (z_{112}-A)^2}{2\sigma^2(1-r^2)}}$$

$$p_{z_{121},z_{122}}(z_{121}, z_{122}) = \frac{1}{\sqrt{2\pi}\sigma^2\sqrt{1-r^2}} e^{-\frac{z_{121}^2 - 2rz_{121}z_{122} + z_{122}^2}{2\sigma^2(1-r^2)}}$$

...

$$p_{z_{1n1},z_{1n2}}(z_{1n1}, z_{1n2}) = \frac{1}{\sqrt{2\pi}\sigma^2\sqrt{1-r^2}} e^{-\frac{z_{1n1}^2 - 2rz_{1n1}z_{1n2} + z_{1n2}^2}{2\sigma^2(1-r^2)}} \quad (28)$$

r is the coefficient of correlation. $\dot{z}_1$ and $\dot{z}_2$ are signals derivatives at the system output at two time instants. The joint probability density of the signals z_1 and z_2 and theirs derivatives is

$$p_{z_1z_2\dot{z}_1\dot{z}_2}(z_1, z_2, \dot{z}_1, \dot{z}_2) = \sum_{i=1}^{n}\sum_{j=1}^{n} p_{z_{1i}z_{1j}\dot{z}_{1i}\dot{z}_{1j}}(z_1, z_2, \dot{z}_1, \dot{z}_2) \cdot$$
$$\cdot \prod_{\substack{k=1 \\ i\neq k}}^{n}\prod_{\substack{s=1 \\ s\neq j}}^{n} F_{z_{1k1}}(z_1) F_{z_{1j2}}(z_2) \quad (29)$$

where

$$p_{z_{111}z_{112}\dot{z}_{111}\dot{z}_{112}}(z_{111}, z_{112}, \dot{z}_{111}, \dot{z}_{112}) =$$
$$= \frac{1}{\sqrt{2\pi}\sigma^2\sqrt{1-r^2}} e^{-\frac{(z_{111}-A)^2 - 2r(z_{111}-A)(z_{112}-A) + (z_{112}-A)^2}{2\sigma^2(1-r^2)}} \cdot$$
$$\cdot \frac{1}{2\pi\beta^2} e^{-\frac{\dot{z}_{111}^2 + \dot{z}_{112}^2}{2\beta^2}}$$

$$p_{z_{121}z_{122}\dot{z}_{121}\dot{z}_{122}}(z_{121}, z_{122}, \dot{z}_{121}, \dot{z}_{122}) =$$
$$= \frac{1}{\sqrt{2\pi}\sigma^2\sqrt{1-r^2}} e^{-\frac{(z_{121})^2 - 2r(z_{111})(z_{122}-) + (z_{122})^2}{2\sigma^2(1-r^2)}} \cdot$$
$$\cdot \frac{1}{2\pi\beta^2} e^{-\frac{\dot{z}_{121}^2 + \dot{z}_{122}^2}{2\beta^2}}$$

...

$$p_{z_{1n1}z_{1n2}\dot{z}_{1n1}\dot{z}_{1n2}}(z_{1n1}, z_{1n2}, \dot{z}_{1n1}, \dot{z}_{1n2}) =$$
$$= \frac{1}{\sqrt{2\pi}\sigma^2\sqrt{1-r^2}} e^{-\frac{(z_{1n1})^2 - 2r(z_{1n1})(z_{1n2}-) + (z_{1n2})^2}{2\sigma^2(1-r^2)}} \cdot$$
$$\cdot \frac{1}{2\pi\beta^2} e^{-\frac{\dot{z}_{1n1}^2 + \dot{z}_{1n2}^2}{2\beta^2}} \quad (30)$$

IV. CONCLUSION

The diversity system for coherent demodulation of n-ary digital frequently modulated signal in the presence of Gaussian noise at the system input is considered in this paper. The diversity system has *L* diversity brunches and we considered general case. The signals from the corresponding brunches of each receiver are added. The combiner determines (chooses) the brunch of the receiver for whom the signal from the output of the EGC combiner is the biggest. The decision is based on this signal. The results for the error probability are better than in some earlier suggested systems for the FSK signal demodulation [1] because of adding of signals from the same brunches of each receiver previously. The statistical characteristics of the signal and the error probability for the system are calculated in this paper. The probability density function of the signal at the system output, joint probability density function of the signal and signal derivatives at the system output, the joint probability density of the signal at the system output at two time instants and the joint probability density of the signal and signal derivatives at two time instants are derived.

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