MRC diversity systems in the presence of Log-Normal and Nakagami-m fading

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Abstract – **A performance of dual combined diversity communication systems in the presence of Log-normal and Nakagami-m fading over two uncorrelated branches, is presented in this paper. Performance measures of fading communication systems such as Probability density function of SNR, Amount of fading and Outage probability are calculated and graphically represented for Maximal Ratio Combining.**

Keywords – **Log-Normal fading, Nakagami-m fading, Maximal ratio combining, Amount of fading, Outage probabilty.**

I. INTRODUCTION

In wireless communications, fading causes difficulties in signal recovery. When a received signal experiences fading during transmission, its envelope and phase both fluctuate over time. The overall fading process for land mobile satelite systems is a comlex combination of multipath fading and a log-nomal shadowing. Multipath fading caused by the constructive and destructive combination of randomly delayed, reflected, scattered and diffracted signal components. This type of fading is relatively fast and is responsible for the short-term signal variation. In terrestrial and satellite land-mobile systems, the link quality is also affected by slow variation of the mean signal level due to the shadowing from terrain, buildings and trees.

One of the methods used to mitigate this degradation is diversity. Diversity combining has been considered as an efficient way to combat multipath fading and improve the received signal-to-noise ratio (SNR) because the combined SNR compared with the SNR of each diversity branch, is being increased. In this combining, two or more copies of the same information-bearing signal are combining to increase the overall SNR

Maximal-Ratio Combining (MRC) is one of the most widely used diversity combining schemes whose SNR is the sum of the SNR's of each individual diversity branch. MRC is the optimal combining scheme, but its price is complexity, since MRC requires cognition of all fading parameters of channel.

This paper presents Maximal-Ratio Combining procedure for communication system where the diversity combining is applied over two uncorrelated $(\rho = 0)$ branches, which are given as channels with Log-Normal and Nakagami-m fading.

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In this environment the receiver does not average the envelope fading due to multipath but rather, reacts to the instantaneous composite multipath/shadowed signal. This is often the scenario in congested downtown areas with slow-moving pedestrians and vehicles. This type of composite fading is also observed in land mobile satellite systems that are subjected to vegetative or urban shadowing.

II. SYSTEM AND CHANNEL MODELS

Here, a dual-branch diversity system over two uncorrelated channels in the presence of log-normal and Nakagami-m feding, is being considered. Under these conditions, instantaneous SNR $p(\gamma)$ is obtained by averaging the instantenous Nakagami-m feding average power over the conditional pdf (probability density function) of the log-normal shadowing, which results with combination of gamma distribution (for Nakagami-m feding) and log normal distribution (for log-normal shadowing) [1]

$$p(\gamma/\Omega) = \frac{m^m \gamma^{m-1}}{\Omega^m \Gamma(m)} e^{-\frac{m\gamma}{\Omega}}, \qquad \gamma \geq 0 \tag{1}$$

$$p_\Omega(\Omega) = \frac{\xi}{\sqrt{2\pi}\sigma_i \gamma_i} e^{-\frac{(10\log_{10}\gamma_i - \mu_i)^2}{2\sigma_i^2}} \tag{2}$$

$$p(\gamma) = \int_0^\infty p_\gamma(\gamma/\Omega) p_\Omega(\Omega) d\Omega \tag{3}$$

Substituting Eq. (1) and Eq. (2) in Eq. (3), $p(\gamma)$ can be written as

$$p(\gamma) = \int_0^\infty \frac{m^m \gamma^{m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m\gamma}{\Omega}\right) \times \left\{ \frac{\xi}{\sqrt{2\pi}\sigma\Omega} \exp\left[-\frac{(10\log_{10}\Omega - \mu)^2}{2\sigma^2}\right]\right\} d\Omega$$

$$, \gamma \geq 0 \tag{4}$$

where $\xi = 10/\ln 10 = 4.3429$, μ_i (db) is mean of $10\log_{10}\gamma$, σ_i (db) is standard deviation of $10\log_{10}\gamma$, and m is Nakagami-m factor.

In fig 1., 2. and 3. are shown pdf $p(\gamma)$ for various values of the factors μ, σ and m. For the case where $m = 1$, an overall fading consists of Rayleigh and log-normal feding.

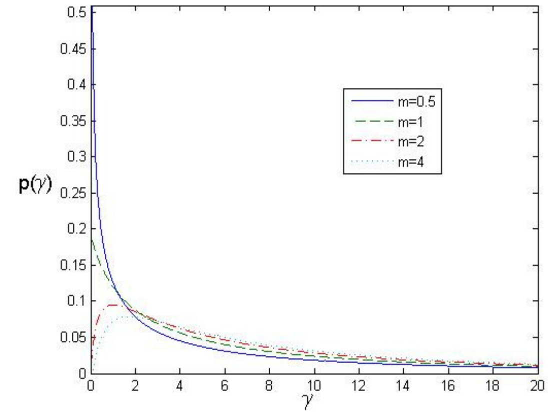

Fig. 1. $p(\gamma)$ for $\mu = 10db$, $\sigma = 5db$, $m = 0.5,1,2,4$

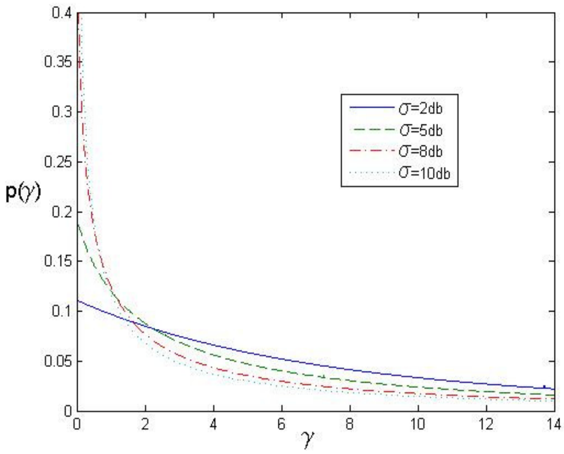

Fig. 2. $p(\gamma)$ for $\mu = 10db$, $\sigma = 2,5,8,10db$, $m = 1$

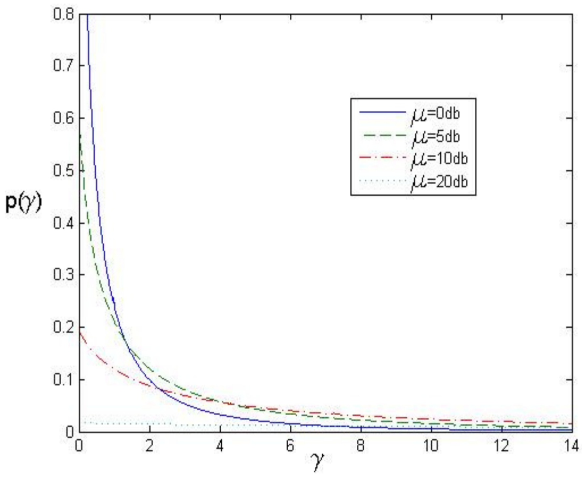

Fig. 3. $p(\gamma)$ for $\mu = 0,5,10,20db$, $\sigma = 5db$, $m = 1$

For further evaluations is important to find moments of combined SNR.

N-th moment of the Output SNR is given by [3]

$$E[\gamma^n] = \int_0^\infty \gamma^n p_\gamma(\gamma) d\gamma \tag{5}$$

N-th moment for distribution in Eq. (4) can be written as [1]

$$E[\gamma^k] = \frac{\Gamma(m+k)}{\Gamma(m)m^k} \exp\left[\frac{k}{\xi}\mu + \frac{1}{2}\left(\frac{k}{\xi}\right)^2 \sigma^2\right] \tag{6}$$

Amount of fading (AF) is a unified measure of the severity of fading for particular channel model and is typically independent of the average fading power, but is dependent of the instantaneous SNR.

Amount of fading is defined by

$$AF = \frac{\mathrm{var}(\alpha^2)}{(E[\alpha^2])^2} = \frac{E[(\alpha^2 - \Omega)^2]}{\Omega^2} \tag{7}$$

$$AF = \frac{E[\gamma^2] - (E[\gamma])^2}{(E[\gamma])^2} = \frac{E[\gamma^2]}{(E[\gamma])^2} - 1 \tag{8}$$

where α is fading amplitude, Ω is average fading power, $E[]$ denotes statistical average and var( ) denotes variance.

Join probability density function in the case of two uncorrelated fading channels is given by [2]

$$p_{\gamma 1,\gamma 2}(\gamma_1, \gamma_2) = p_{\gamma 1}(\gamma_1) p_{\gamma 2}(\gamma_2) \tag{9}$$

III. MAXIMAL RATIO COMBINING

The total SNR at the output of the MRC combiner is given by

$$\gamma_{MRC} = \sum_{l=1}^{L} \gamma_l \tag{10}$$

where L is number of branches.

The average combined SNR at the MRC (maximal ratio combining) output with two branceh is given by:

$$\gamma_{MRC} = \gamma_1 + \gamma_2 \tag{11}$$

Probability density function of the sum of the first and second branch can be written as

$$p_{\gamma_{MRC}}(\gamma_{MRC}) = \int_0^{\gamma_{MRC}} p_{\gamma 2}(\gamma_{MRC} - \gamma_1) p_{\gamma 1}(\gamma_1) d\gamma_1 \tag{12}$$

Substituting Eq. (4) in Eq. (11), $p_{\gamma_{MRC}}(\gamma_{MRC})$ can be obtained as

$$p_{\gamma_{MRC}}(\gamma_{MRC}) = \int_0^{\gamma_{MRC}} \left[\int_0^\infty \frac{m^m (\gamma_{MRC} - \gamma_1)^{m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m(\gamma_{MRC} - \gamma_1)}{\Omega}\right) \times \left\{ \frac{\xi}{\sqrt{2\pi}\sigma\Omega} \exp\left[\frac{10\log_{10}\Omega - \mu)^2}{2\sigma^2}\right]\right\} d\Omega \right] \cdot$$
$$\cdot \left[\int_0^\infty \frac{m^m \gamma_1{}^{m-1}}{\Omega_1{}^m \Gamma(m)} \exp\left(-\frac{m\gamma_1)}{\Omega_1}\right) \times \left\{ \frac{\xi}{\sqrt{2\pi}\sigma\Omega_1} \exp\left[\frac{10\log_{10}\Omega_1 - \mu)^2}{2\sigma^2}\right]\right\} d\Omega_1 \right] d\gamma_1 \tag{13}$$

Relatively simple closed form expressions to represent $p_{\gamma_{MRC}}(\gamma_{MRC})$ can not be derived, because Eq. (13) is too complex for tractable communication system analyses. This pdf can be evaluated numerically using some of software tools (Matlab, Mathematica).

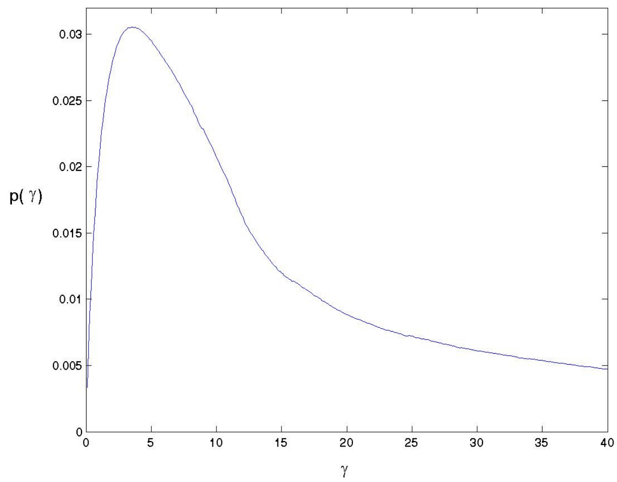

Fig. 4. $p_{\gamma_{MRC}}(\gamma_{MRC})$ for $\mu_i = 10db$, $\sigma_i = 5db$, $m = 1$

Very often it is assumed for performance analysis of communication systems, that channel coefficients are uncorrelated and identically distributed. Fig 4. depicts pdf of γ_{MRC} at the output of MRC combiner for two uncorrelated identically distributed channels with identical parameters $\mu_i = 10db$, $\sigma_i = 5db$, $m = 1$.

N-th moment of γ_{MRC} can be expressed as [1]

$$E\left[\gamma^n{}_{MRC}\right] = E\left[(\gamma_1 + \gamma_2)^n\right] \tag{14}$$

Using binomial expansion, (14) can be written as

$$E\left[\gamma^n{}_{MRC}\right] = E\left[\sum_{k=0}^{n}\binom{n}{k}\gamma_1{}^k\gamma_2{}^{n-k}\right] = \sum_{k=0}^{n}\binom{n}{k}E\left[\gamma_1{}^k\gamma_2{}^{n-k}\right] \tag{15}$$

The average combined SNR $\bar{\gamma}_{MRC}$ at the MRC output can be written as

$$\bar{\gamma}_{MRC} = E\left[\gamma^1{}_{MRC}\right] = 2\bar{\gamma} \tag{16}$$

The second moment of γ_{MRC} is given by

$$E\left[\gamma^2{}_{MRC}\right] = (\gamma_1{}^2 + 2\gamma_1\gamma_2 + \gamma_2{}^2) \tag{17}$$

For the parameters given in Fig 4. ($\mu_i = 10db$, $\sigma_i = 5db$, $m = 1$) it is obtained $\bar{\gamma}_{MRC} = 38.802$.

Amount of fading can be calculated from Eq. (16) and Eq. (17), and for these parametars is $AF_{MRC} = 7.528$.

Outage probabilty is standard performance criterion of diversity systems opereting over fading chanels and it is defined as the probability that the instantaneous error rate exceeds a specified value, or equivalently, that combined SNR of MRC falls below a predetermined threshold γ_{th}

$$P_{out}^{MRC} = P\left[\gamma_{MRC} = \gamma_1 + \gamma_2 \le \gamma_{th}\right] \tag{18}$$

P_{out}^{MRC} is defined in form of integral by

$$P_{out}^{MRC} = \int_0^{\gamma_{th}} p_{\gamma MRC}(\gamma_{MRC})d\gamma_{MRC} \tag{19}$$

Substituting Eq. (10) in Eq. (16) P_{out}^{MRC} can be written as

$$P_{out}^{MRC} = \int_0^{\gamma_{MRC}}\left[\int_0^{\infty}\frac{m^m(\gamma_{MRC}-\gamma_1)^{m-1}}{\Omega^m\Gamma(m)}\exp\left(-\frac{m(\gamma_{MRC}-\gamma_1)}{\Omega}\right)\times\left\{\frac{\xi}{\sqrt{2\pi}\sigma\Omega}\exp\left[\frac{10\log_{10}\Omega-\mu)^2}{2\sigma^2}\right]\right\}d\Omega\right]$$

$$\cdot\left[\int_0^{\infty}\frac{m^m\gamma_1{}^{m-1}}{\Omega_1{}^m\Gamma(m)}\exp\left(-\frac{m\gamma_1)}{\Omega_1}\right)\times\left\{\frac{\xi}{\sqrt{2\pi}\sigma\Omega_1}\exp\left[\frac{10\log_{10}\Omega_1-\mu)^2}{2\sigma^2}\right]\right\}d\Omega_1\right]d\gamma_1 d\gamma_{MRC} \tag{20}$$

Fig. 5 shows Outage probabilty versus instantaneous SNR for the same factors as in fig. 4.

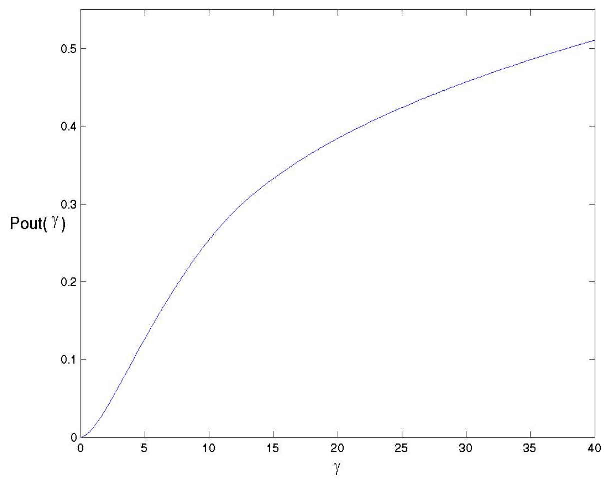

Fig. 5. P_{out}^{MRC} for $\mu_i = 10db$, $\sigma_i = 5db$, $m = 1$

IV. Conclusion

In this letter, a unified performance analysis for the dual diversity MRC over uncorrelated Nakagami-M fading and Log-Normal fading channels is presented. Probability density function of SNR, Amount of fading and Outage probability are derived in the form of multiple integral. It can not be obtained relatively simple closed-form expressions for evaluation of this parametars, because system structure is too complex and it was performed numerical calculation of them. As ilustration of this aproach, caracteristics of receiver are shown for MRC dual diversity case to point out the effect of the overall fading.

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