Second and Third Order Dispersion Influence on Pulse Propagation in the Presence of the Interference

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Abstract – Chromatic dispersion is one of the most important limiting factors in linear optical transmission systems. Pulse propagation under the common influence of second and third order dispersion is considered in this paper. Analytic solution of pulse shape at a receiver in the presence of interference, second and third order dispersion is determined in this paper, too. Interference is time and phase shifted in regard to useful signal. We consider the worst case, i.e. case when phase shift is π .

Keywords – Second and third order chromatic dispersion, Impulsive response, Coherent interference.

I. Introduction

Chromatic dispersion limits transmission length in optical telecommunication systems that use single-mode fiber. Because of that, it plays a critical role in a propagation of optical pulse. In order to significantly increase transmission speed of optical system the influence of chromatic dispersion on signal propagation, especially its second and third order, must be considered [1].

Another limiting factor in optical transmission systems are incoherent and coherent interferences. Coherent interference is more problematic than incoherent interference because it cannot be controlled by optical filtering in the receiver [2, 3]. It is a reason for research such interference influence on pulse deformation when propagating along optical fiber under second and third order dispersion.

Until now, we studied an influence of interference on signal propagation along linear optical fiber solving nonlinear Schrödinger equation by "split-step" Fourier method [4, 5, 6]. In all of our previous papers, only second order dispersion was included in an investigation. In this paper we obtain general expression of Gaussian pulse at the receiver in the presence of interference, second and third order dispersion. Using this result we consider interference influence on pulse propagation along linear single mode fiber under second and third order dispersion.

II. ANALYTIC SOLUTION OF RESULTING PULSE AT THE RECEIVER

Pulses from many lasers can be approximated by a Gaussian shape. Because of that signal with Gaussian envelope is very often found as a useful signal in optical systems [1, 7, 8]. Gaussian optical pulse is given by:

$$s(t) = \sqrt{P_0} e^{-\left(\frac{t}{T_0}\right)^2 + j\omega_0 t} \tag{1}$$

where ω_0 is the angular frequency of useful signal, P_0 is optical pulse peak power, $T_0' = T_0 / \sqrt{2}$ is pulse half width (at 1/e intensity point).

Coherent interference is of the same frequency as a useful signal and it can be time and phase shifted in regard to useful signal. It can be written as:

$$s_{i}(t) = \sqrt{P_{i}} e^{-\left(\frac{t}{T_{0}} - b^{i}\right)^{2} + j(\omega_{o}t + \varphi)}$$
(2)

where P_i is interference peak power, b ($b = b'T_0$) and φ are time and phase shift, respectively. The envelope and phase of resulting signal $s_r(t)$ are [4]:

$$|s_r(t)| =$$

$$\sqrt{\left(\sqrt{P_0}e^{-\left(\frac{t}{T_0}\right)^2}\right)^2 + 2\sqrt{P_0}e^{-\left(\frac{t}{T_0}\right)^2}\sqrt{P_i}e^{-\left(\frac{t}{T_0}-b'\right)^2}\cos\varphi + \left(\sqrt{P_i}e^{-\left(\frac{t}{T_0}-b'\right)^2}\right)^2}$$
(3)

$$\psi(t) = arctg \frac{\sqrt{P_i}e^{-\left(\frac{t}{T_0}-b'\right)^2}\sin\varphi}{\sqrt{P_0}e^{-\left(\frac{t}{T_0}\right)^2} + \sqrt{P_i}e^{-\left(\frac{t}{T_0}-b'\right)^2}\cos\varphi}$$
(4)

A general expression of impulse response r(t, L) for arbitrary input pulse is [7, 9]:

$$r(t,L) = \frac{1}{2\pi} e^{-\alpha L} e^{j(\omega_0 t - \beta_0 L)} \cdot \int_0^\infty F(\omega) e^{j\left(\omega t - \frac{1}{2!}\beta_2 L \omega^2 - \frac{1}{3!}\beta_3 L \omega^3\right)} d\omega$$
 (5)

when optical fiber is under second and third order dispersion. Second order dispersion is very often dominated in linear optical fiber. Then influence of third order dispersion can be neglected. The situation changes for propagation of ultrashort pulses (with widths in the femtosecond range) in the vicinity of λ_D (λ_D – zero-dispersion wavelength) [1]. That case we

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consider in this paper. $F(\omega)$ is Fourier transform of the input pulse. If we assume that coherent interference appears at the beginning of the optical fiber and that phase shift of interference is π (the worst case), then Fourier transform of the resulting pulse is:

$$F(\omega) = \sqrt{P_0} T_0 \sqrt{\pi} e^{-\frac{\omega^2 T_0^2}{4}} \left(1 - \sqrt{\frac{P_i}{P_0}} e^{-j\omega b T_0} \right)$$
 (6)

If we substitute (6) in (5) and use some mathematical operations, the pulse response for our case becomes:

$$r(t,L) = \frac{\sqrt{P_0}T_0}{2\sqrt{\pi}} e^{-\alpha L} e^{j(\omega_{t} - \beta_0 L + \theta(t))} \sqrt{I_1^2(t) + I_2^2(t)}$$
 (7)

$$\theta(t) = arctg \frac{I_2(t)}{I_1(t)} \tag{8}$$

with

$$I_{1}(t) = \int_{-\infty}^{\infty} e^{-\frac{\omega^{2} T_{n}^{2}}{4}} \left[\left(1 - \sqrt{\frac{P_{i}}{P_{0}}} \cos(b T_{0} \omega) \right) \cos(\omega t - b_{2} \omega^{2} - b_{3} \omega^{3}) - \sqrt{\frac{P_{i}}{P_{0}}} \sin(b T_{0} \omega) \sin(\omega t - b_{2} \omega^{2} - b_{3} \omega^{3}) \right] d\omega$$

$$I_{2}(t) = \int_{-\infty}^{\infty} e^{-\frac{\omega^{2} T_{n}^{2}}{4}} \left[\left(1 - \sqrt{\frac{P_{i}}{P_{0}}} \cos(b T_{0} \omega) \right) \sin(\omega t - b_{2} \omega^{2} - b_{3} \omega^{3}) + \sqrt{\frac{P_{i}}{P_{0}}} \sin(b T_{0} \omega) \cos(\omega t - b_{2} \omega^{2} - b_{3} \omega^{3}) \right] d\omega$$

$$(10)$$

where

$$b_2 = \left(\frac{\beta_2 L}{2!}\right)$$

$$b_3 = \left(\frac{\beta_3 L}{3!}\right) \tag{11}$$

Equations (7)-(10) represent a general expression of Gaussian pulse at the end of optical fiber under second and third order dispersion in the presence of coherent interference.

III. NUMERICAL RESULTS

Results of our research are shown on following figures. Fiber length is expresed via dispersion length which is for n-th order of dispersion given by:

$$L_D = \frac{\left(T_0^{\,\cdot}\right)^n}{\left|\beta_n\right|} \tag{12}$$

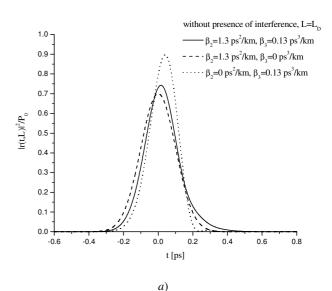
In optical fiber under common influence of second and third order dispersion L_D for n=2 and n=3 is equal. The following values of parameters are used in all cases: $T_0=0.1$ ps, $\alpha=0$ dB/km (optical losses are neglected in the fiber), $\beta_2=1.3$ ps²/km (dispersion coefficient of second order), $\beta_3=0.13$ (dispersion coefficient of third order) ps³/km and $\varphi=\pi$.

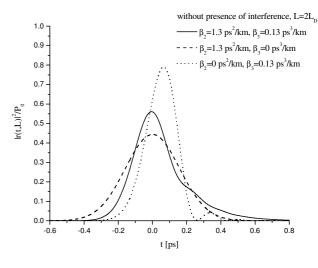
Figure 1 shows propagation Gaussian pulse along the optical fiber. Well known facts can be noticed from Fig. 1:

- second order dispersion influence on pulse broadening.
- third order dispersion influence on pulse deformation,
 i.e. induces strong pulse oscillation in positive region of t.

This figure shows that third ordere disperion decreases pulse broadening induced by second order dispersion. But, it provokes moving of peak power to left.

Figures 2 (SIR = 5 dB) and 3 (SIR = 10 dB) show resulting pulse shape at the receiver for different values of the interference time shift. Interference is the most ruinous when it is not time shifted in regard to useful signal. Negative time shift induced bigger deformation of resulting pulse than positive time shift. Following figures show that in this case we can hardly concluse that Gaussian pulse propagated along the fiber. Presence of interference time shift induces stronger intersymbol interference (ISI), too. Comparasion of Figs. 2 and 3 show that pulse deformation is insofar larger as SIR (Signal – to – Interference Ratio) is less.





b)

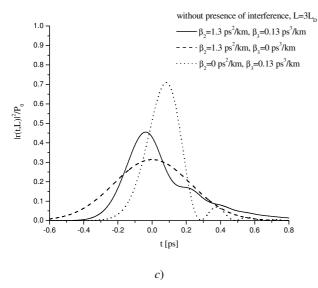
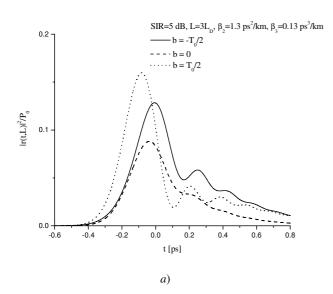


Fig. 1. Propagation Gausian pulse along the linear optical fiber.



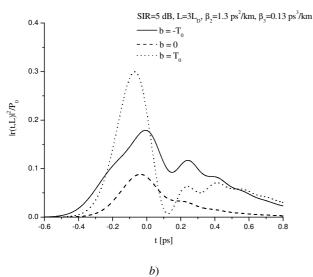
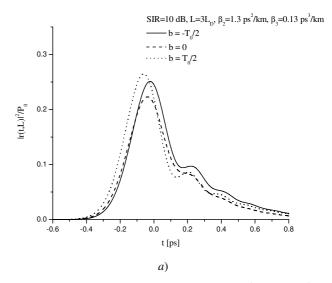


Fig. 2. Resulting pulse shape at the receiver for different values of time shift (SIR = 5 dB).



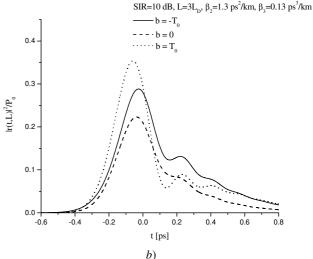


Fig. 3. Resulting pulse shape at the receiver for different values of time shift (SIR = 10 dB).

IV. CONCLUSION

Linear optical fiber is very often used as medium for pulse propagation through telecommunication system. Chromatic dispersion is one of the main factors limiting transmission length in these systems. It is reason for studing common influence of second and third order dispersion on pulse shape along the fiber. Results of this paper show that third order dispersion decreases pulse broadening but moves peak power to left. If there is second order dispersion in the optical fiber, presence of third order dispersion will be advisable because error in detection process is smaler in that case. Coherent interference may be detrimental in optical telecommunication systems because it additionally distorts optical pulse and can cause significant rise to intersymbol interference. We analyzed the worst case of coherent interference, i.e. $\varphi = \pi$. The time shift of interference play significant role. It makes the pulse response take the shape of long trailing skirts which influence on distant pulses. This leads to stronger intersymbol

interference. Until now, these problems we considered using "split-step" Fourier method for solving Schrödinger equation. This method is very sensitive on the number of fiber segments and their width. Obtained results are very important because we used analytical method that has great accuracy. Performances of optical system determined on this way are very objectiv.

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