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Impulse Response of SI Polymer Optical Fibres

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Abstract – **Using the power flow equation to describe the mode coupling in the fibre and consider the power loss dependent on mode group we have built a mathematical model of the SI polymer fibre, which solution yields all quantities of interest of the fibre: power distribution, attenuation and impulse response as functions of length by UMD launching condition.**

Keywords – **POF, time-dependent power flow equation, impulse Response, Fourier Approach.**

I. INTRODUCTION

Presently, polymer optical fibres are of interest in communication systems in close-up range, automotive, home equipment etc. because of less stringent requirements ofoptical carriers and splices, cheaper connectors and low costs by the production. However, the mode dispersion caused by the relative thick core and high value of power loss reduce the transmission distance and throughput. Moreover the mode coupling caused by the random variations of the core index or the random irregularities of the fibre wall complicates the characteristics of the pulse propagation. It is purpose of this paper to present a satisfactory approach to obtain the impulse response of polymer optical fibres.

II. POWER LOSS

Step index polymer optical fibres are thick core light guides that lead as many as million of modes. The core is larger than the wavelength of the light. The fibre can be analysed with a geometric ray-tracing model. Moreover, an important consequence of this huge number of modes is that light polarization is not preserved along the fibre. There is no correlation between input and output polarization states, except for very short POF (< 0.5m). Further, we base our considerations on this ray model considering lengths in the range of many hundred meters.

One of the most important processes encountered by light as it passes through a fibre is attenuation. When passing through an optical fibre of the length L, the power of the light decreases exponentially according to

$$P(L) = P(0) \cdot exp(-\alpha \cdot L) \tag{1}$$

where $P(0)$ is the input power of the light and α is the attenuation coefficient in 1/km which depends on mode number. The attenuation coefficient can be split into three terms that describe three different phenomena of attenuation, i.e. length difference for each mode, the difference in the number of reflections for each mode and *Goos – Haenchen* effect.

At first, we consider the length difference for each mode. The accumulated path length can be written as

$$L_\theta = \frac{L}{cos(\theta)} \tag{2}$$

where θ is the angle of incidence. With the core attenuation coefficient α_{core} the path-dependent loss can be written as

$$\alpha_l = \alpha_{core} \cdot L_\theta \tag{3}$$

The second phenomenon of power loss is the attenuation caused by ray reflection on the clad-core interface. Rays with a greater propagation angle reflect more often than rays with a smaller angle of incidence θ. The length-dependent attenuation is given by

$$\alpha_r = \frac{L \cdot tan(\theta)}{2\rho} \cdot ln(R) \tag{4}$$

with the reflection coefficient R and the core radius ρ.

Now, the third cause for attenuation, the *Goos – Haenchen* effect, is taken into account. In [4] the extra attenuation coefficient caused by the *Goos - Haenchen* effect α_{GH} is described by

$$\alpha_{GH} = \frac{d(\theta)}{\rho} \cdot \frac{\alpha_{clad}}{cos(\theta)} \tag{5}$$

where α_{clad} is the attenuation coefficient of the cladding and the shift - depth of the reflection plane into the optically thinner cladding is expressed as $d(\theta)$ and the corresponding formula is given in [5] as

$$d(\theta) = \frac{\lambda}{2\pi\sqrt{n_{core}^2 cos^2(\theta) - n_{clad}^2}} \tag{6}$$

Now the attenuation coefficient α can be given as

$$\alpha = \alpha_{core}\frac{1}{\cos\theta} + \frac{\tan(\theta)}{2\rho} \cdot \ln(R) + \alpha_{GH} \tag{7}$$

for the power distribution we yield

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$$P(\theta, z) = P(\theta, 0) \cdot \\ exp\bigg(- \alpha_{core}\frac{z}{cos(\theta)} - \frac{z \cdot tan(\theta)}{2\rho} \cdot ln(R) - \alpha_{GH} z \bigg) \quad (8)$$

III. DIFFUSION EQUATION

By approximation of the modes by a continuum Gloge [1] obtained the fundamental coupled-power equation

$$\frac{\partial P(\theta, z)}{\partial z} = -\alpha(\theta) P(\theta, z) + \frac{D}{\theta}\frac{\partial}{\partial \theta}\bigg(\theta \frac{\partial P(\theta, z)}{\partial \theta}\bigg) \quad (9)$$

which describes the power flow of the coupled modes of stepp – index fibres, with

$P(\theta, z)$	angular power distribution
θ	propagation angle with respect to the core axis
D	coupling coefficient
α	modal attenuation

However, this equation does not consider the time differences of the different modes by the process of power diffusion and blur the time axis so that the delay of the different power parts cannot be determined. On the contrary, the determination of the impulse response requires the consideration of the time differences of the different modes. Gloge develops his idea further building in additionally the time into the equation. With the total variation in P

$$dP = \frac{\partial P}{\partial z} dz + \frac{\partial P}{\partial t} dt \quad (10)$$

he write in [2] the time-dependent power flow equation

$$\frac{\partial P}{\partial z} + \frac{dt}{dz}\frac{\partial P}{\partial t} = -\alpha(\theta) P(\theta, z) + \frac{D}{\theta}\frac{\partial}{\partial \theta}\bigg(\theta \frac{\partial P(\theta, z)}{\partial \theta}\bigg) \quad (11)$$

The derivate dt/dz is the delay of the power part $P(\theta)$ for a unit length and can be written for the ray model of the SI POF as

$$\begin{aligned} \frac{dt}{dz} &= \frac{d}{dz}\frac{z \cdot n_{core}}{c_0 \cdot cos(\theta)} = \frac{n}{c_0 \cdot cos(\theta)} \\ &= \frac{n}{c_0} + \frac{n}{c_0}\bigg(\frac{1}{cos(\theta)} - 1\bigg) = \tau_0 + \tau_{rel} \end{aligned} \quad (12)$$

The common delay to all modes n/c is ignored further.

IV. FOURIER APPROACH

With the Fourier transformation

$$p(\theta, z, \omega) = \mathcal{F}\{P(\theta, z, t)\} \quad (13)$$

we can write Eq. (11) in the form

$$\frac{\partial p}{\partial z} = -(\alpha(\theta) + j\omega\tau_{rel})p + \frac{D}{\theta}\frac{\partial}{\partial \theta}\bigg(\theta \frac{\partial p}{\partial \theta}\bigg) \quad (14)$$

The first term in Eq. (14) describes attenuation and time shift of the different modes and the second one the diffusion respectively. There are two different physical phenomena, that act simultaneously. We can write formally Eq. (14) in the form

$$\frac{\partial p}{\partial z} = (\hat{L} + \hat{D})p \quad (15)$$

where $\hat{L}$ is a differential operator that accounts for attenuation and time difference of the different modes and $\hat{D}$ is the operator that governs the effect of mode coupling. We can give an approximate solution from (15) by assuming that over a small distance Δz the attenuation and coupling effects act independently. Mathematically,

$$p(\theta, z + \triangle z, \omega) \approx exp(\triangle z \hat{L}) exp(\triangle z \hat{D}) p(\theta, z, \omega) \quad (16)$$

The split approach ignores the no commutating nature of the operators $\hat{L}$ and $\hat{D}$. To improve the accuracy, the split method can be modified by adopting a different procedure to propagate the optical pulse over one segment Δz. In this procedure Eq. (16) is replaced by

$$p(\theta, z + \triangle z, \omega) \approx exp(\frac{\triangle z}{2}\hat{L}) exp(\triangle z \hat{D}) exp(\frac{\triangle z}{2}\hat{L}) p(\theta, z, \omega) \quad (17)$$

For equation (14) the solution for the differential operator $\hat{L}$ can be given as

$$p(\theta, z + \frac{\triangle z}{2}, \omega) = p(\theta, z, \omega) \cdot exp(-\alpha(\theta)\frac{\triangle z}{2}) \cdot exp(-j\omega\tau_{rel}\frac{\triangle z}{2}) \quad (18)$$

An analytical solution for the differential operator $\hat{D}$ can be given with the Bessel functions. On the contrary, we use the finite-difference method to solve it numerically.

$$p(\theta, z + \triangle z, \omega) = \int_z^{z+\triangle z} \frac{D}{\theta}\frac{\partial}{\partial \theta}\bigg(\theta \frac{\partial p}{\partial \theta}\bigg) dz \quad (19)$$

Applying Eq. (18) und Eq. (19) successively with the procedure in Eq. (17) we obtain the solution at the end of the fibre

$$p(\theta, L, \omega) \quad (20)$$

The total output over all angles θ is obtained from the integration

$$H(L, \omega) = \int_0^{\theta_{max}} p(\theta, L, \omega) d\theta \quad (21)$$

With the reverse Fourier transformation we obtain the impulse response of the fibre

$$h(L,t)=\mathcal{F}^{-1}\{H(L,\omega)\} \tag{22}$$

To find the quantity power distribution we do reverse Fourier transformation and integrate over all times

$$P(\theta,L)=\int_0^{t_{max}}\mathcal{F}^{-1}\Big\{p(\theta,L,\omega)\Big\}dt \tag{23}$$

A last integral evaluation

$$P_{out}(L)=\int_0^{\theta_{max}}P(\theta,L)d\theta \tag{24}$$

gives the total output power, which can be used to calculate the loss.

V. Simulation Results

Table I presents all important parameters, that we have adjusted to obtain the power distribution and impulse response along the fibre. Impulse responses by different fibre lengths are shown in Fig. 1. The process of diffusion does not influence heavy the impulse response after the first twenty meters, whose curve falls rapidly for longer transit times, that indicates the mode-dependent attenuation. For long fibre the diffusion effects a transition of the impulse form from an exponential shape to the Gaussian shape.

TABLE I
PARAMETER

Parameter	Value
Velocity of light c_0	$3\cdot10^8$ m/s
Radius of fibre ρ	$0.5\cdot10^{-3}$ m
Core refractive index n_{core}	1.492
Wavelength λ	650 nm
Numerical Aperture	0.5
Core attenuation coefficient α_{core}	120 dB/km
Cladding attenuation coefficient α_{clad}	5000 dB/km
Reflection factor R	$1-10^{-4}$
Diffusion coefficient D	$1.5\cdot10^{-4}$ rad^2/m

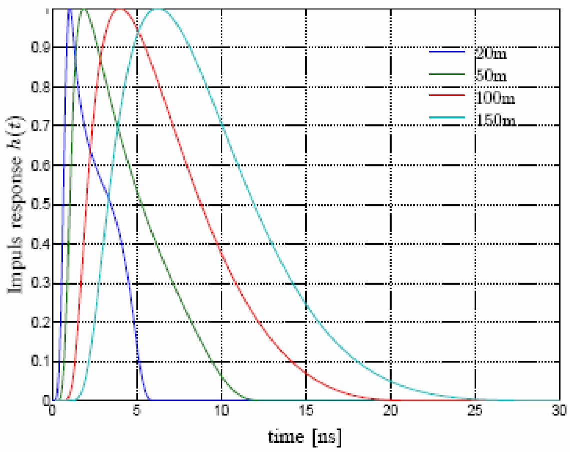

Fig. 1. Normalized impulse response according to Eq. (22) plotted for different fibre length

The power distribution along the fibre is shown in Fig. 2. We feed the fibre from a light emitting diode, that excites equally all modes. The blue curve shows the power distribution on the front of fibre. Modes with a greater number carry more power than lower modes, that propagate with a smaller angle of incidence because of the power contained in their differential solid angle. The diffusion concentrates slowly the power in the middle of the fibre and affects at the same time an averaging of the transit times of the different modes. The coupling length is achieved after 150m. The brown-ish curve results from the overlap of lilac and dark yellow. That indicates that the impulse shape does not change anymore after the coupling length.

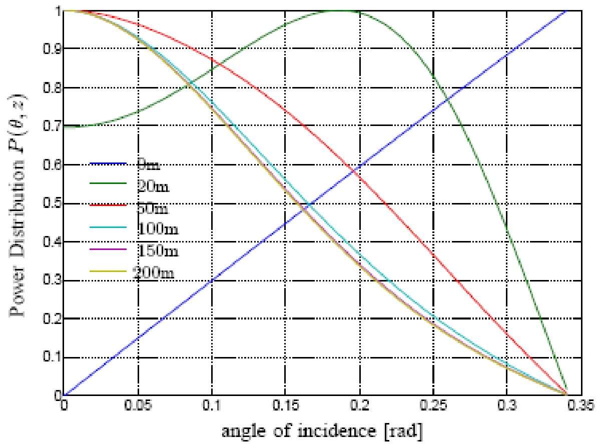

Fig. 2. Normalized power distribution at different fibre lengths calculated with (23) by UMD launching condition

VI. Conclusion

We have introduced shortly the attenuation of a SI polymer fibre based on optical rays model. Under the assumptions of mode coupling occurring between nearest neighbours using the time-dependent power diffusion equation we have built the base description of a step-index polymer optical fibre. We have developed a method to solve this equation transforming it into the frequency domain and giving an analytical and numerical solution for attenuation and diffusion respectively so that power distribution, attenuation and impulse response can be obtained.

Two further extensions of this work seem desirable: the building-in of the effect of mode conversion and to take the stochastical nature of diffusion into account.

As an example of applications, the impulse response can be useful to develop channel model of SI POF's, which can be used by several electrical methods of equalization to improve their transmission characteristics.

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