Formation of Signals Harmonized with a Linear Channel of Connection with Limitation of Their Average Rectified Values

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Abstract: **The paper presents a solution of the problem of synthesizing harmonized signals with the limitation of their average values and a report on the efficiency of their transmission along a linear channel of connection.**

Keywords: **signals harmonized with the channel, functional of synthesis**

I. Introduction

The problem of forming signals harmonized with the channel of connection is a variation problem connected with finding the conditional extreme of the quality functional of synthesis, i.e. with certain limitations imposed on the signals. They have determined closed area S*, where the signals searched are in its interior. The kind of the functional has been defined by the criteria of quality and the nature of the noise in the channel while the non-functional limitations are defined by the properties of the transmitter. The signals, which have been imposed certain limitation on, are same in structure and united in one class despite the kind of the criteria of quality.

The paper presents a solution of the problem of synthesizing harmonized signals with the limitation of their average values and a report on the efficiency of their transmission along a linear channel of connection.

II. Formulation of the Problem of Harmonizing with the Limitation of the Average Rectified Values of the Input Signal for the Channel

The most common kind of the limitations on the signals searched is:

$$m_1\left\{\int_{\tau_H}^{\tau_K}\varphi_j\left[x_j(t), s_j(t), \underline{V}(t), t\right]dt\right\} \le S_j \text{ j=1,2,…m,} \qquad (1)$$

where: $s_j(t)$ – input signal;

$x_j(t)$ – output signal;

$\underline{V}$ - vector function of the disturbing signals.

However, the most spread limitation in practice is that of the average value of p-*th* order of the signal being optimized:

$$\left[\frac{1}{\Delta T}\int_{t_H}^{t_K}\left|s_j(t)\right|^p dt\right] \le S_p\text{, } \Delta T=t_K-t_H\text{, j=1,2,…,m} \qquad (2)$$

When p=1 is satisfied in (2), it concerns the limitation of the average rectified value of the signal. Such a limitation can be observed with describing movable radio means where the source used to supply the transmitting device is overcharged earlier than the output series of the transmitter.

Then the problem of signal harmonization can be formulated in the following way: within the class of signals $L_1[t_H, t_K]$ determined by limitations of (2), with (p=1), to find those signals $\{s_i(t);\ i=1,…, m,\ t\in[t_H, t_K]\}$ that maximize the functional for:

$$I=\varphi^{-1}\sum_{i=1}^{m}\sum_{j=1}^{m}a_{ij}\varphi\left(r_{ij}\right) \qquad (3)$$

Here r_{ij} is the distance between the i-th and j- th signals at the output of the channel of connection. The distance for a standardized metric linear area with the limitation of the average rectified value of the signal [1] is determined as:

$$r_{ij}=\frac{1}{\Delta\tau}\int_{\tau_H}^{\tau_K}\left|x_i(t)-x_j(t)\right| dt\text{ ,i,j=1,2,…m,} \qquad (4)$$

$\Delta\tau=\tau_K-\tau_H$ is the interval of the observation on the output signal.

Substituting (4) into (3) and expressing the output signal by the input one, the functional, which has to be maximized, takes the kind of:

$$I=\int_{t_H}^{t_K}...\int_{t_H}^{t_K}s(t_1)...s(t_v)H_v(t_1....t_v)dt_1...dt_v \qquad (5)$$

where H_v is the nucleus of the functional and depends on the channel pulse characteristics.

III. Solution of the Optimization Problem with Limiting the Average Rectified Value

From the mathematical point of view, in the case of limiting the average rectified value of the signal, it is most

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convenient to use the inequality of Haldle [2] to work out the conditions necessary and sufficient for the maximum of functional (5). It can be proved, according to him, that in order to reach a maximum (5), signals s(t) have to satisfy the following condition:

$$s(t_1)=\sum_{i=1}^{N} a_i \delta\left(t_i - t_{i\max_x}\right) signF(t_1) \tag{6}$$

where :

$$F(t_1)=\left|\int_{t_H}^{t_K}\dots\int_{t_H}^{t_K} s(t_2)\dots s(t_v) H_v(t_2\dots t_v)dt_2\dots dt_v\right| \tag{7}$$

– a_i are coefficients that are determined by the condition of the signal norm in $L_1[t_H, t_K]$:

$$\frac{1}{\Delta T}\int_{t_H}^{t_K}\left|s\ (t_1)\right|\ dt = S \tag{8}$$

– $\{t_{i\max}, i=1\dots N\}\equiv\{\Theta\}$ is a set of moments where function $F(t_1)$ reaches a maximum.

Having substituted (6) into (8) and taking into consideration the filtrating property of δ - function, it is obtained that:

$$\sum_{i=1}^{N} a_i = \Delta T S_1 \tag{9}$$

Hence, the harmonized signals of class $L_1[t_H, t_K]$ with limited average rectified values that are determined by expression (6) present a sequence of δ -pulses supplied at the channel input at moment $\{t_{i\max}\}$, when function (7) depending on the channel pulse characteristics has a maximum.

IV. Determination of the Efficiency of Transmitting Signals Harmonized with a Linear Gauss's Channel and with Limited Average Rectified Values.

The linear cable equivalent to a low-pass filter is an appropriate model of some real channels of connection such as cable and hydro-acoustic ones. For the simplest case of one-unit low-pass filter, the pulse characteristic is in the kind of:

$$h(t)=\alpha.e^{-\alpha.t} \tag{10}$$

where α is the damping coefficient of the channel.

The efficiency of transmitting signals harmonized with the channel will be assessed comparing them to signals in the kind of rectangular pulses transmitted along the same channel. That can be done by the coefficient of efficiency expressed by the ratio of the coefficients with transmitting signals of both types:

$$\gamma^2=\frac{K^2_{onm}}{K^2_{np}} \tag{11}$$

and showing what is the relation between the power of the signals at the channel output and the characteristic (10) in the both cases.

To deduce the dependency for (11), the ratio of the norms of the input and output signals has been used as an analog to the coefficient of transmitting harmonized signals of limited average values [2]. If the case of Gauss's noise in the channel is examined when power serves to measure the distance between two signals and the pulse characteristic is substituted with expression (10), it will be obtained that after the transformation:

$$K^2_{onm}=\begin{cases}\alpha\dfrac{\Delta T^2}{2\Delta\tau}e^{2\alpha.t}\left(e^{-2\alpha\tau_H}-e^{-2\alpha\tau_K}\right),t\le\tau_H;\\ \alpha\dfrac{\Delta T^2}{2\Delta\tau}\left(1-e^{-2\alpha(\tau_K-t)}\right),t\succ\tau_H\end{cases} \tag{12}$$

where t is the moment of supplying δ -pulse at the input of the channel of connection.

The dependencies $K^2_{опт}=f(t)$ in the both time intervals for a linear channel in the kind of a low-pass filter are given in Fig. 1 and Fig. 2.

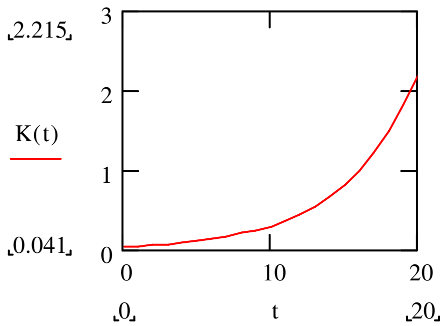

Fig. 1. Dependency $K_{опт}=f(t)$ for $t\le\tau_H$

To find out the best moment $t=t_{\max}$, the dependency $K^2_{опт}=f(t)$ has been examined for a maximum using "Mathcad". It has been obtained that this moment is $t=\tau_H$.

Substituting (12) in (11), the following expression of the efficiency coefficient has been obtained:

$$\gamma^2=\frac{K^2_{onm}}{K^2_{np}}=\frac{\alpha\Delta T\left(1-e^{-2\alpha\Delta T}\right)}{2\left(1-\dfrac{3}{2\alpha\Delta T}+\dfrac{2e^{-\alpha\Delta T}}{\alpha\Delta T}-\dfrac{e^{-2\alpha\Delta T}}{2\alpha\Delta T}\right)}$$

The graphic dependency $\gamma^2=f(\alpha\Delta T)$ is given in Fig. 3.

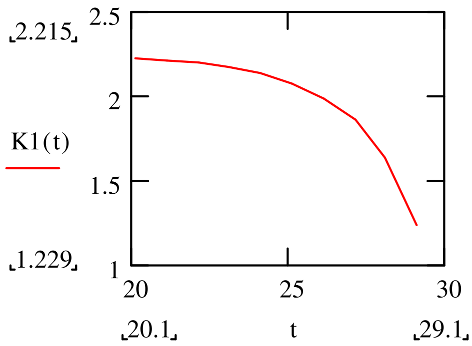

Fig. 2. Dependency $K_{опт}=f(t)$ for $t > \tau_H$

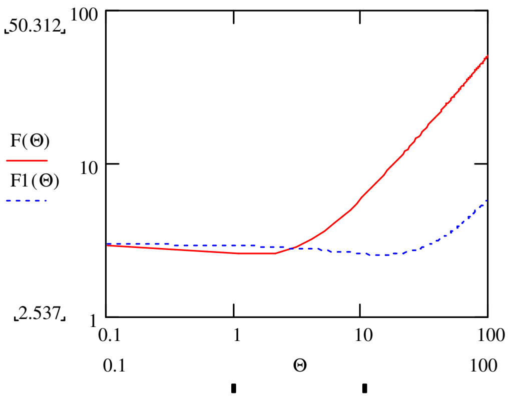

Fig. 3. Dependency $\gamma^2 = f(\alpha\Delta T)$

It is seen that the power of a signal in the kind of a rectangular pulse (dashed line), of duration of $\tau_и$=0,1ΔT, is lower at the channel output than that with transmitting a signal harmonized with the channel and of (dense line). That can be explained by the fact that the limitation of the average rectified value does not result in limiting the power. That is why it is expedient to use those signals in the systems of movable radio connection.

V. Conclusions

A considerable efficiency has been obtained with transmitting signals harmonized with the channel and of a limited average rectified values. It is so because the harmonized signals of the class under examination, $L_1[t_H, t_K]$, are of wide-frequency band (short duration) and therefore their base is B>>1. As it can be seen in Fig.3, the biggest efficiency of transmitting harmonized signals is with the values of $\alpha\Delta$T commensiorable with their base. Hence the transmission of signals in the kind of δ -pulses, such as the harmonized ones with limited average rectified values, results in the increase of their base and in higher power at the channel output.

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