Dual Channel Quadrature Mirror Filter Bank Containing Sigma-Delta Modulators

Jaroslav Vrána[1] and Tomáš Lukl[2]

Abstract – **Quadrature mirror filter bank is used in digital signal processing. Most often it is realized by FIR filters. The signals are decomposed into some subbands by quadrature mirror filter banks. These subbands are further processed. For example it can be stored with various bit accuracy in subband coding. In this paper a structure are described which can be resistant to distortion of subband signals.**

Keywords – **filter bank, sigma-delta modulator, quantization, noise shaping**

I. INTRODUCTION

In subband coding can be used quadrature mirror filter banks. In this case first the input signal is decomposed into some subbands. In individual subbands the signals are processed. The result of their processing is mostly their distortion. In reconstruction part the output signal is reconstructed from individual subbands. Because the individual subbands can be distorted the output signal also can be distorted. But the distortions of subband signal can be affected in the case of sigma-delta modulator are used in decomposition part. Sigma-delta modulator has different transfer functions for input signal and error signal. Therefore the error signal can be adjusted that it should be more suppressed by passing through reconstruction part.

II. QUADRATURE MIRROR FILTER BANK

Quadrature mirror filter bank decomposes input signal into some subbands. In this paper it is used only dual channel quadrature mirror filter bank. The block scheme of this filter bank can be seen in Fig. 1.

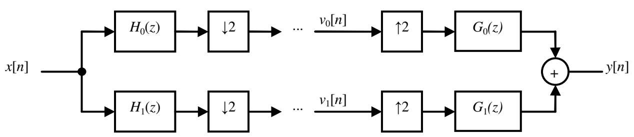

Fig. 1. Block scheme of dual channel filter bank

Input signal is processed in two branches. First branch is low part of half band mirror filter and second branch is high part of half band mirror filter. Input signal enters into low pass filter $H_0(z)$ and high pass filter $H_1(z)$ and thereby it is split in two subbands. Because these parts are frequency limited, the sampling frequency can be decreased. This part is decomposition part. For reconstruction of original input signal at first the sampling frequency must be increased and then the signals are filtered by low pass filter $G_0(z)$ and high pass filter $G_1(z)$ again. This part is reconstruction part. So as the signal is not modified by this structure the Eq. (1) a (2) must be fulfilled.

$$H_0(z)\cdot G_0(z)+H_1(z)\cdot G_1(z)=2z^{-k} \tag{1}$$

$$H_0(-z)\cdot G_0(z)+H_1(-z)\cdot G_1(z)=0 \tag{2}$$

First Eq. (1) describes the passage of input signal through this structure. The signal can be delayed. Second Eq. (2) describes passage of mirror spectrums through this structure, which originates by change of sampling frequency. The most simple filter bank has filters with transfer functions Eqs. (3), (4), (5) a (6).

$$H_0(z)=\frac{1}{2}+\frac{1}{2}z^{-1} \tag{3}$$

$$H_1(z)=-\frac{1}{2}+\frac{1}{2}z^{-1} \tag{4}$$

$$G_0(z)=\frac{1}{2}+\frac{1}{2}z^{-1} \tag{5}$$

$$G_1(z)=\frac{1}{2}-\frac{1}{2}z^{-1}, \tag{6}$$

The processing of signals in individual branches can be expressed as adding error signals to individual subband signals. This situation can be seen in Fig. 2. These error signals pass through filters in reconstruction part of filter bank and they are summed on output. If error signals are white noises with constant power, the power spectrums are summed on output.

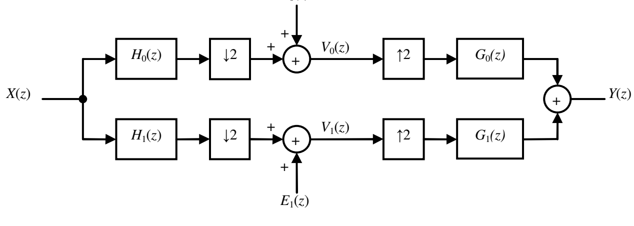

Fig. 2. Block scheme of dual channel filter bank

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III. SIGMA-DELTA MODULATION

Sigma-delta modulators are most often used in AD and DA converters. One of their most advantages is quantization noise shaping. It means that quantization noise is not uniform over whole frequency spectrum, but it is more suppressed in lower frequencies and less suppressed in higher frequencies. In detail it is described in [1]. Block scheme of sigma-delta modulator can be seen in Fig. 3.

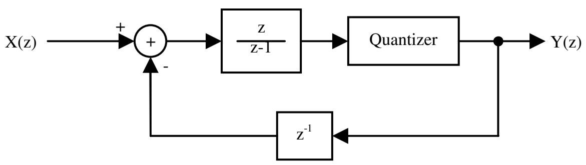

Fig. 3. Sigma-delta modulator

A generalized sigma-delta modulator can be used in filter bank. Digital integrator is replaced by block with transfer function $K(z)$ and block with transfer function $J(z)$ is inserted into feed back. Block scheme of linear model can be seen in Fig. 4.

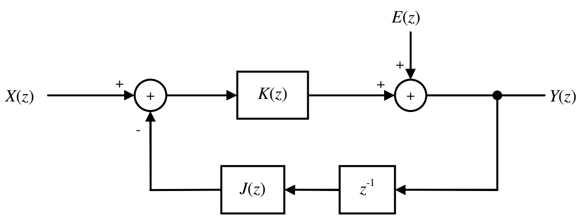

Fig. 4. Linear model of sigma-delta modulator

The input signal enters into summer together with feed back signal. The differential signal pass through block with transfer function $K(z)$. Behind this block a error signal $E(z)$ is added to signal. In AD converter the error signal is quantization noise. Because the quantizer is nonlinear part and the analysis of circuits with this nonlinear part is difficult, the quantizer is replaced by sum with error signal. Thereby the analysis is simpler. The sum of signal and error signal is output signal of generalized sigma-delta modulator. In sigma-delta modulator is also feed back. The output signal is delayed by one sample and then it pass through block with transfer function $J(z)$. This signal is subtracted by input signal. The output signal of this block scheme can be computed by equation:

$$Y(z) = X(z)\cdot\frac{K(z)}{1+z^{-1}\cdot J(z)\cdot K(z)} + E(z)\cdot\frac{1}{1+z^{-1}\cdot J(z)\cdot K(z)}. \quad (7)$$

The first term describes passage of useful signal and the second term describes passage of error signal (quantization noise). So the frequency characteristic corresponds to characteristic of filter in filter bank, the characteristics must be same. Therefore Eq. (8) must be fulfilled, which describes equality between transfer function of sigma-delta modulator and required transfer function $H(z)$.

$$H(z) = \frac{K(z)}{1+z^{-1}\cdot J(z)\cdot K(z)} \quad (8)$$

For realization of sigma-delta modulator it is necessary also to compute transfer function $J(z)$. The transfer function $J(z)$ can be computed by Eq. (8).

$$J(z) = \frac{K(z)-H(z)}{z^{-1}\cdot K(z)\cdot H(z)} \quad (9)$$

By substituting Eqs. (8) and (9) into Eq. (7) the transfer function of sigma-delta modulator is:

$$Y(z) = X(z)\cdot H(z) + E(z)\cdot\frac{H(z)}{K(z)}. \quad (10)$$

From this equation can be seen that the transfer function of input signal is exactly $H(z)$ and sigma-delta modulator has the required transfer function. Transfer function of error signal depends also on transfer function of input signal, but it is divided by transfer function $K(z)$, which can be chosen. By its suitable chose the case can happened that error signal will be more transferred in stop band of reconstruction filter bank and it will be more suppressed in pass band of reconstruction filter bank.

IV. FILTER BANK CONTAINING SIGMA-DELTA MODULATORS

The sigma-delta modulator with general transfer can be used in quadrature mirror filter bank. The advantage of replacement of decomposition filters by sigma-delta modulators is better suppression of subband signals distortion. For example in subband coding the individual coefficients can be stored with lower bit resolution and thereby it produces higher quantization noise into signals which is subsequently suppressed.

Block scheme of dual channel filter bank which use sigma-delta modulators in decomposition part can be seen in Fig. 5. In this structure subsampling is not used yet.

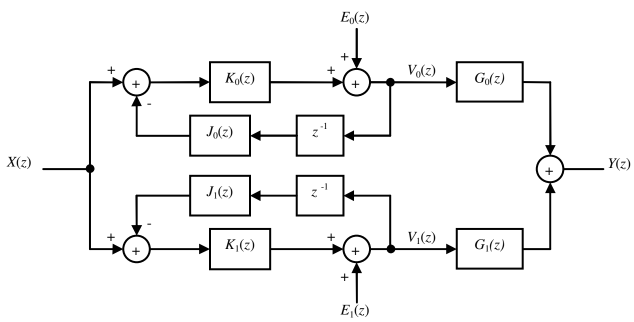

Fig. 5. Dual channel filter bank containing sigma-delta modulators without change of sampling frequency

Both of decomposition filters are realized by sigma-delta modulators and reconstruction part is realized by original filters. The resultant transfer of this structure is:

$$\begin{aligned} Y(z) = X(z)\cdot \frac{K_0(z)}{1+z^{-1}\cdot J_0(z)\cdot K_0(z)}\cdot G_0(z)+ \\ + X(z)\cdot \frac{K_1(z)}{1+z^{-1}\cdot J_1(z)\cdot K_1(z)}\cdot G_1(z)+ \\ + E_0(z)\cdot \frac{1}{1+z^{-1}\cdot J_0(z)\cdot K_0(z)}\cdot G_0(z)+ \\ + E_1(z)\cdot \frac{1}{1+z^{-1}\cdot J_1(z)\cdot K_1(z)}\cdot G_1(z) \end{aligned} \quad (11)$$

If transfer function $J(z)$ is expressed from Eq. (8), which describe required transfer of individual decomposition filter bank parts, and the transfer $J(z)$ is substituted into Eq. (11). The resultant form is:

$$\begin{aligned} Y(z) = X(z)\cdot H_0(z)\cdot G_0(z) + X(z)\cdot H_1(z)\cdot G_1(z) + \\ + E_0(z)\cdot \frac{H_0(z)}{K_0(z)}\cdot G_0(z) + E_1(z)\cdot \frac{H_1(z)}{K_1(z)}\cdot G_1(z) \end{aligned} . \quad (12)$$

From this equation can be seen that first two terms describe passage of input signal and last two terms describe passage of distortion subband signals. If Eq. (1) is fulfilled, the input signal is not changed by passage through this structure, only the subband error signals are added to input signal. By choosing transfer function $K_0(z)$ and $K_1(z)$ the transfer of subband error signals to output signal can be influenced. From this transfer functions it is necessary to compute transfer functions $J_0(z)$ and $J_1(z)$ from Eq. (9). Because filters with transfer functions $J_0(z)$ and $J_1(z)$ can not be causal the next condition has to be fulfilled in choosing transfer functions $K_0(z)$ and $K_1(z)$.

If the filters with transfer functions Eqs. (3), (4), (5) and (6) are used in reconstruction part, which are simple FIR filters with two coefficients, then the transfer function $K_0(z)$ is chosen as a product of transfer functions $H_0(z)$ and $G_0(z)$ and the transfer function $K_1(z)$ is chosen as a product of transfer functions $H_1(z)$ and $G_1(z)$. When the transfer functions are chosen by this way the uniform transfer of subband error signal should be happened, because the transfer functions are changed from polynomials to only constants. In choosing it is necessary to fulfill condition for causality thereby multiplication by constant are added to product. The chosen transfer functions are:

$$K_0(z) = 2\cdot H_0(z)\cdot G_0(z) = \frac{1}{2} + z^{-1} + \frac{1}{2}z^{-2}, \quad (13)$$

$$K_1(z) = 2\cdot H_1(z)\cdot G_1(z) = -\frac{1}{2} + z^{-1} - \frac{1}{2}z^{-2}. \quad (14)$$

The functions $F_0(z)$ and $F_1(z)$ are established for next computing which describe transfer of individual error signals into output signal. The resultant transfer of subband error signals after substitution $K_0(z)$ and $K_1(z)$ are:

$$F_0(z) = \frac{H_0(z)}{K_0(z)}\cdot G_0(z) = \frac{1}{2}, \quad (15)$$

$$F_1(z) = \frac{H_1(z)}{K_1(z)}\cdot G_1(z) = -\frac{1}{2}. \quad (16)$$

For requirement of sigma-delta modulator the functions $J_0(z)$ and $J_1(z)$ are causal and they are in followed form:

$$J_0(z) = \frac{K_0(z) - H_0(z)}{z^{-1}\cdot K_0(z)\cdot H_0(z)} = \frac{2+2z^{-1}}{1+3z^{-1}+3z^{-2}+z^{-3}}, \quad (17)$$

$$J_1(z) = \frac{K_1(z) - H_1(z)}{z^{-1}\cdot K_1(z)\cdot H_1(z)} = \frac{2-2z^{-1}}{1-3z^{-1}+3z^{-2}-z^{-3}}. \quad (18)$$

The resultant output error signal is sum of individual subband signals multiplied by individual transfer functions $F_m(z)$. If the subband error signals are white noises with same power in both of branches then output error signal is square root of sum of square of absolute values individual subband error transfers.

$$|F(z)| = \sqrt{|F_0(z)|^2 + |F_1(z)|^2} = \frac{1}{\sqrt{2}} \quad (19)$$

In this case the maximum of total transfer of error signal into output signal is about 0.707. In the case of classic realization of this filter bank, the maximum transfer of error signal into output signal is unitary. Dual channel filter bank with subsampling is in future research.

V. Results of Filter Bank Simulation

In simulation four simple filters with impulse characteristic by Eqs. (3), (4), (5) and (6) was chosen. The simulation was processed in Matlab. Differential equations (20), (21), (22), (23), (24) and (25) were used in sigma-delta modulator in decomposition part of filter bank. The computing complexity is higher. In classic realization of this filter bank two products and one sum need to be computed for each sample. In the case of sigma-delta modulator is used eleven products and nine sums need to be computer for each sample.

$$v_0[n] = k_0[n] + e_0[n], \quad (20)$$

$$v_1[n] = k_1[n] + e_1[n], \quad (21)$$

$$\begin{aligned} j_0[n] = v_0[n-1] + 2\cdot v_0[n-2] - 3\cdot j_0[n-1] - \\ -3\cdot j_0[n-2] - j_0[n-3] \end{aligned}, \quad (22)$$

$$\begin{aligned} j_1[n] = v_1[n-1] - 2\cdot v_1[n-2] + 3\cdot j_1[n-1] - \\ -3\cdot j_1[n-2] + j_1[n-3] \end{aligned}, \quad (23)$$

$$k_0[n] = \frac{1}{2} \cdot x[n] + x[n-1] + \frac{1}{2} \cdot x[n-2] - \\ - \frac{1}{2} j_0[n] - j_0[n-1] - \frac{1}{2} j_0[n-2], \tag{24}$$

$$k_1[n] = -\frac{1}{2} \cdot x[n] + x[n-1] - \frac{1}{2} \cdot x[n-2] + \\ + \frac{1}{2} j_1[n] - j_1[n-1] + \frac{1}{2} j_1[n-2]. \tag{25}$$

A signal $x[n]$ was chosen as sum of two harmonic signals (first with low frequency and second with high frequency) and signals $e_0[n]$ and $e_1[n]$ were chosen as random signals with uniform distributed functions. These signals were input signals to classic filter bank without subsampling and to filter bank with sigma-delta modulators which can be seen in Fig. 5.

Output signals from decomposition part can be seen in Fig. 6. The input signal is decomposed into two parts and one harmonic component is in each subband. Noise is uniform spread over whole spectrum of subband signals.

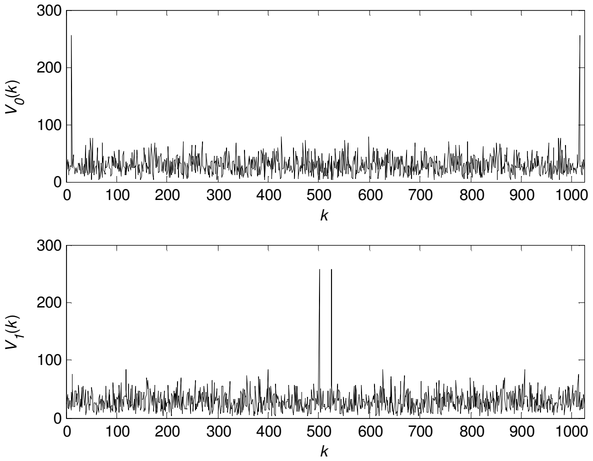

Fig. 6. Spectrum of output signals from original decomposition dual channel filter bank

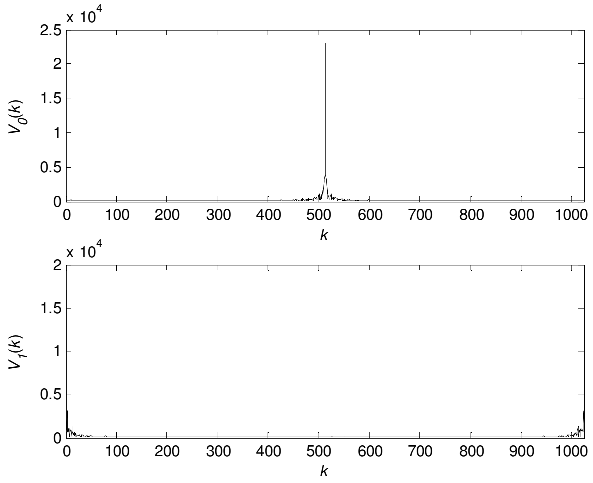

Fig. 7. Spectrum of output signals from decomposition dual channel filter bank with sigma-delta modulators

In the case the sigma-delta modular is used the noise is emphasized in these parts where the signals are suppressed in the reconstruction part. Reconstructed signals can be seen in Fig. 8. In picture can be seen that the noise in output signal is lower if sigma-delta modulator is used in decomposition part of filter bank. This decrement of the noise matches equation (19).

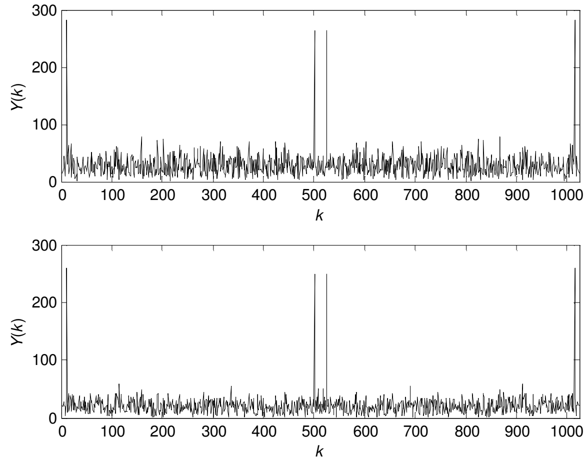

Fig. 8. Spectrum of output signals from original reconstruction dual channel filter bank (upper) and reconstruction dual channel filter bank with sigma-delta modulators (lower)

VI. Conclusion

The filter bank with sigma-delta modulators is next realization possibility of filter bank. In proper design of this structure it is more resistant to distortion of individual subband signals. Its advantage is in individual subband error signal shaping so that it is asserted in output signal at least. In comparison with classic realization of filter bank, this structure is more compute complexity.

VII. Acknowledgement

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