New Complex Orthogonal Narrowband IIR First-Order Filter Sections – an Input Quantization Noise Analysis

Zlatka Nikolova[1], Georgi Stoyanov[2]

Abstract – **In this paper a very low sensitivity first-order orthogonal complex narrow-band band-pass filter section is developed. Then, a noise analysis due to the quantization of the input signal is performed. A comparative study with other well-known complex orthogonal sections has been made on order to show the advantages of the developed filter circuit.**

Keywords – **complex orthogonal digital filters, sensitivity, quantization errors, noise analysis.**

I. Introduction

Quantization of the multiplication products and the input signals in the digital filters is causing parasitic noises usually described as an error signal. Quantization of the multiplier coefficients is deteriorating the filter characteristics. Development of low sensitivity structures is reducing considerably this deterioration and additionally is decreasing the roundoff noises.

Methods of computing the quantization effects in real digital systems are well developed so far. In [1] an evaluating recursive formula for roundoff errors estimation is proposed. The "inners" approach for output noise variance evaluation is suggested in [2]. An alternate method of calculating the noise, suitable both for analytical and numerical computations and useful for higher order filters is presented in [3]. A new algebraic technique proposed by Bomar in [4] provides computationally efficient state-space realizations preserving low roudoff noise, low coefficient sensitivity, and freedom from zero-input cycles.

Although complex coefficients digital filters are gaining popularity in the recent years their quantization noise analysis theory is still not well developed and there are few publications treating the problem. New structures for complex multipliers and their noise analysis are proposed in [5]. In [6] theoretical upper bounds on the amplitude of limit cycles oscillations are determined for direct-form orthogonal second order digital filter. In all these works usually local problems are solved and no general method for roundoff noise estimation is proposed. In this work we present a technique for complex input signal quantization noise analysis. Then we apply it on a newly developed very low sensitivity complex orthogonal first order sections.

The paper is organized as follows. An approach to a complex noise analysis is proposed in section II. The first-order orthogonal complex digital filters are derived in section III. In section IV the resulting error signals at the outputs of these orthogonal complex sections after input signal quantization are examined. In section V, simulation results for quantized input narrow-band complex signals are presented and discussed. Finally, section VI concludes the paper.

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II. Quantization Noise Analysis for Complex Input Signals

The effect of quantization of an input signal is equivalent to a set of noise samples added to the actual input. In case of uniformly distributed noise samples, the variance of the input noise is:

$$\sigma_e^2 = \frac{\delta^2}{12} = \frac{2^{-2B}}{12}, \tag{1}$$

where δ^2 is the quantization step and B is the word-length in bits. Then the steady-state (nominal) value of the output noise variance is given by:

$$\sigma_v^2 = \sigma_e^2 \frac{1}{2\pi j} \oint H(z) H\left(z^{-1}\right) z^{-1} dz = \sigma_e^2\, \sigma_{v,n}^2, \tag{2}$$

where $H(z)$ is the transfer function of the digital filter, $\sigma_{v,n}^2$ denotes the noise gain and is called also normalized output noise variance.

Complex coefficient digital filters are capable of processing both real and complex signals. The quantization of a complex input signal presumes a complex error signal as it is shown in Fig. 1. Then the complex output signal $y(n) = y_{\mathrm{Re}}(n) + j y_{\mathrm{Im}}(n)$ will be mixed with complex output noise:

$$v(n) = v_{\mathrm{Re}}(n) + j v_{\mathrm{Im}}(n). \tag{3}$$

Through this model it is possible to examine the complex output noise variance in a similar way as in the real case.

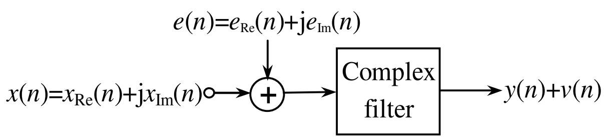


Fig. 1: Noise model for a complex input signal quantization

Evaluating the complex output noise variance presumes all complex quantities to be considered in Eq.(2). Analytic input signals with inphase and quadrature components are processed by special class of complex coefficient digital filters called "orthogonal". The orthogonal complex transfer function can be presented by its real and imaginary parts as follows:

$$H(-jz) = H_{\mathrm{Re}}(z) + jH_{\mathrm{Im}}(z). \tag{4}$$

Realized by real elements, an orthogonal complex structure (Fig. 2) will have two inputs and two outputs (both couples real and imaginary), producing thereby four real coefficient transfer functions two by two equal with ± sign:

$$H_{\mathrm{Re}}(z) = H_{RR}(z) = H_{II}(z);$$
$$H_{\mathrm{Im}}(z) = H_{RI}(z) = -H_{IR}(z). \tag{5}$$

Normally the real and imaginary parts of the complex input signal undergo the same quantization, i.e. $\sigma^2_{e,\mathrm{Re}}=\sigma^2_{e,\mathrm{Im}}=\sigma^2_e$.

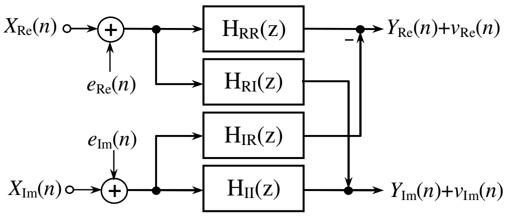


Fig. 2: Block-diagram of complex digital filter structure - noise model for a complex input signal quantization

The structure in Fig.2 shows that the complex output noise variance real and imaginary components should be composed as follow:

$$\sigma^2_{v,\mathrm{Re}} = \sigma^2_e\left(\sigma^2_{v,H_{\mathrm{Re}}} - \sigma^2_{v,H_{\mathrm{Im}}}\right) \tag{6}$$

$$\sigma^2_{v,\mathrm{Im}} = \sigma^2_e\left(\sigma^2_{v,H_{\mathrm{Re}}} + \sigma^2_{v,H_{\mathrm{Im}}}\right) \tag{7}$$

where

$$\sigma^2_{v,H_{\mathrm{Re}}} = \sigma^2_e \frac{1}{2\pi j}\oint H_{\mathrm{Re}}(z)H_{\mathrm{Re}}\left(z^{-1}\right)z^{-1}dz \tag{8}$$

$$\sigma^2_{v,H_{\mathrm{Im}}} = \sigma^2_e \frac{1}{2\pi j}\oint H_{\mathrm{Im}}(z)H_{\mathrm{Im}}\left(z^{-1}\right)z^{-1}\,dz \tag{9}$$

are the output noise variances for the real and imaginary components of the transfer function (4).

III. Complex Orthogonal Digital Filter Circuit Development

One of the best methods of complex digital filter derivation is the method of circuit transformation proposed in [7] and permitting also to obtain orthogonal complex filters with canonical number of elements. We select this method expecting that it will permit the new circuit to inherit good qualities of the real prototype.

According to this expectation and after a careful study of the more often used real-coefficient first-order digital filter structures we selected the best two very low-sensitivity real-prototypes – MHNS and LS1b low-pass sections [8] (Fig. 3).

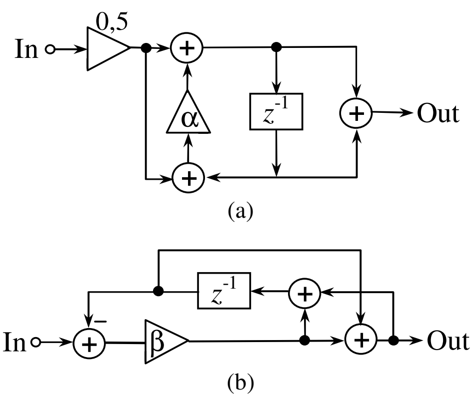


Fig. 3: Real-coefficients first-order low-sensitivity LP filters sections (a) MHNS; (b) LS1b.

The transfer functions these real sections realize are:

$$H^{LP}_{MHNS}(z) = \frac{1-\alpha}{2}\frac{1+z^{-1}}{1-\alpha z^{-1}}; \tag{10}$$

$$H(z)^{LP}_{LS1b} = \beta\frac{1+z^{-1}}{1-(1-2\beta)z^{-1}}. \tag{11}$$

The rotation transformation [7] in its orthogonal case:

$$z^{-1} = jz^{-1} \text{ or } z = -jz, \tag{12}$$

applied on the LP real transfer functions converts them into orthogonal complex coefficients BP transfer functions of doubled order. For the MHNS-based orthogonal complex structure shown in Fig. 4a they are as follow:

$$H^{RR}_{MHNS}(z) = H^{II}_{MHNS}(z) = H_{\mathrm{Re}}(z) = \frac{(1-\alpha)}{2}\frac{1-\alpha z^{-2}}{1+\alpha^2 z^{-2}}, \tag{13}$$

$$H^{RI}_{MHNS}(z) = -H^{IR}_{MHNS}(z) = H_{\mathrm{Im}}(z) = \frac{\left(1-\alpha^2\right)}{2}\frac{z^{-1}}{1+\alpha^2 z^{-2}}, \tag{14}$$

whereas for the LS1b-based orthogonal section (Fig.4b) they are:

$$H^{RR}_{LS1b}(z) = H^{II}_{LS1b}(z) = H_{\mathrm{Re}}(z) = \beta\frac{1+(2\beta-1)z^{-2}}{1+(2\beta-1)^2 z^{-2}}, \tag{15}$$

$$H^{RI}_{LS1b}(z) = -H^{IR}_{LS1b}(z) = H_{\mathrm{Im}}(z) = \beta(1-\beta)\frac{2z^{-1}}{1+(2\beta-1)^2 z^{-2}}. \tag{16}$$

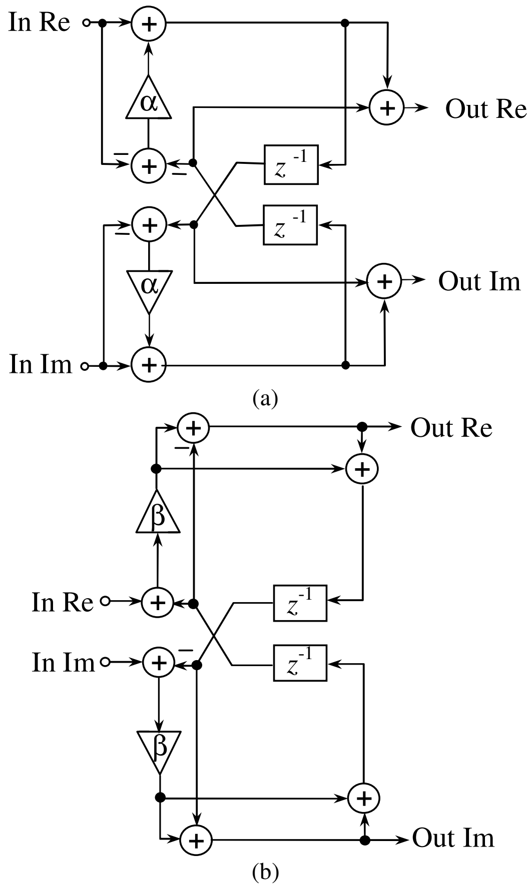


Fig. 4: First-order BP orthogonal structure based on the (a) MHNS; (b) LS1b real sections.

Narrow-band orthogonal BP filters are most often used for practical purposes and they can be derived from narrow-band LP filter-prototypes (pole near $z=1$). Achieving low sensitivity for such pole position is a very difficult task. In [8] the orthogonal structures from Fig.4 have been investigated with respect to their coefficient sensitivity for a given poles-disposition. It was clearly shown that LS1b-based structure preserves its magnitude shape even when the coefficients are quantized to 2 bits, while MHNS-based structure response is considerably changed when the word-length is limited to 3 bits only. The real prototype-sections (Fig. 3) keep the same performance.

IV. Noise Analysis of Complex Input Quantization Errors

In this section both real and orthogonal structures regarding input quantization errors are investigated.

Initially the real input signal for the LS1b and MHNS real sections is quantised to different word-length. First the output noise variance of the real prototype sections, realised with very narrow pass-band (α=0.98 and β=0.01) is calculated. Some experimental results for input signal quantization from 2 to 8 bits are shown in Fig. 5. Apparently, the low sensitivity LS1b section output noise variance is about ten times lower than this of the MHNS-section when the input signal is limited to 2 bits only.

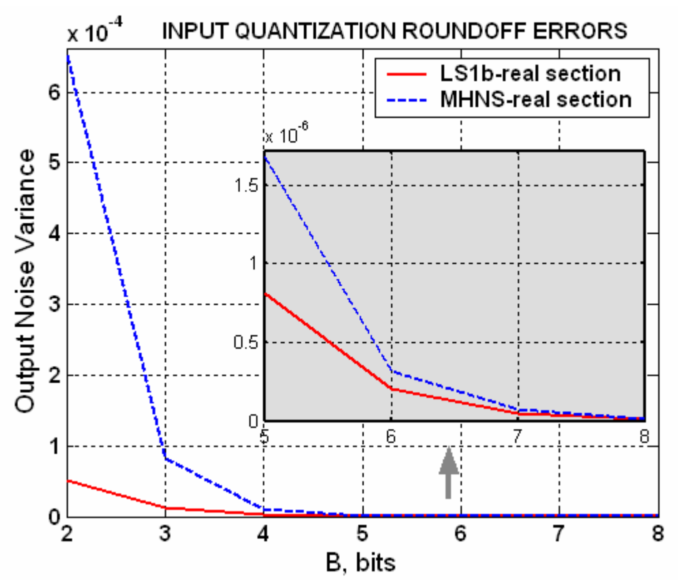


Fig. 5: The output noise variance under input signal roundoff quantization for LS1b and MHNS real sections

Then, the corresponding narrow-band BP orthogonal complex filter sections (Fig. 4) are investigated by using the method from section II.

Following the described in section II method, a complex input signal quantization noise analysis is performed. The calculation results for complex output noise variances for the LS1b and MHNS orthogonal complex sections in different input signal word-length are presented in Tabl. 1.

In order to compare the obtained complex output signal noise variances, their complex modules are shown in Fig. 6.

Tabl. 1

Input signal quantization in bits	Complex output noise variances of the orthogonal complex sections
	MHNS-based (x 10^{-3})
2	0.0120638996891700 + j 0.1817906894536900
3	0.00301597492229 + j 0.04544767236342
4	0.00075399373057 + j 0.01136191809086
5	0.00018849843264 + j 0.0028404795227 1
6	0.00004712460816 + j 0.00071011988068
7	0.00001178115204 + j 0.00017752997017
8	0.0000294528801 + j 0.00004438249254
	LS1b-based (x 10^{-4})
2	0.00514438391361 + j 0.51766175890637
3	0.00128609597840 + j 0.12941543972659
4	0.00032152399460 + j 0.03235385993165
5	0.00008038099865 + j 0.00808846498291
6	0.00002009524966 + j 0.00202211624573
7	0.00000502381242 + j 0.00050552906143
8	0.0000012559531 0 + j 0.00012638226536

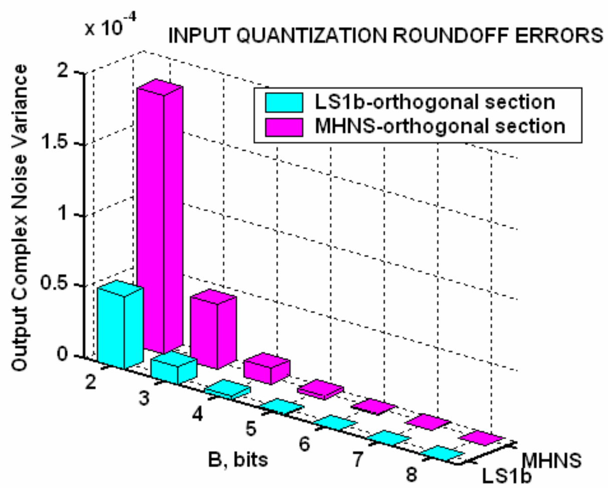


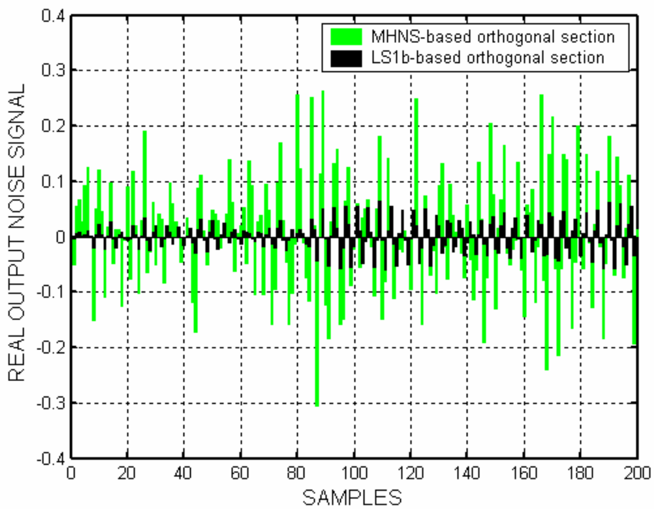
Fig. 6: The output noise variances under the input signal roundoff quantization for LS1b and MHNS -based orthogonal complex first-order sections

It is seen that, the low-sensitivity LS1b-based orthogonal complex section is having more than three times lower output noise after 2 bits input signal quantization. The shorter word-length quantization of the input signal means lower power consumption and faster computation process. For low-sensitivity circuits the resistance against quantization effects provides better signal to noise ratio (SNR), i.e. higher quality digital signal processing.

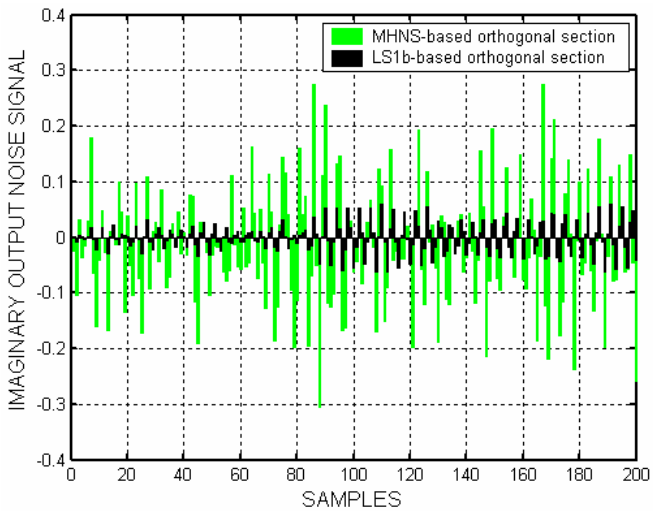
V. Experiments

The narrowband orthogonal first-order filter sections were investigated in a limited word-length complex signals processsing. The complex input signal is a mixture of white noise and analytic sinusoidal signal. The uniformly distributed white noise samples correspond to the word-length of the input complex signal after the quantization.

In case of 2 bits quantization both real and imaginary parts of the input analytic signal and filter coefficients, some experimental results are shown in Fig. 7. In Fig. 7a the real output noise signals for both orthogonal sections are presented, whilst the imaginary output noise signals are shown in Fig. 7b. Obviously the noise reaching to the complex output of the orthogonal circuits is significantly higher for the MHNS-based than for the LS1b-based section.



(a)



(b)

Fig. 7: The output noise signals after input quantization to 2 bits for LS1b and MHNS - based orthogonal complex sections (a) real output; (b) imaginary output.

The output SNR for the LS1b-orthogonal section is about 1,5 times higher in comparison to the MHNS-based circuit. To achieve the same good results as LS1b section demonstrates in 2 bits word-length environment, the MHNS orthogonal filter should be quantized to no less than 6 bits.

It is clear that the famous phrase from the real circuit theory "low sensitivity and low noise go together" is valid also for their complex counterpart.

VI. CONCLUSIONS

In this paper an approach to the complex noise analysis is proposed. The resulting error signals at the outputs of orthogonal complex first-order digital filter sections after input signal quantization are examined. The proposed method is general enough to be applied for complex filter sections of higher order. After relevant alterations it could be effectively applied for all other types of roundoff errors estimation in complex coefficient systems like multiplication product quantization.

The expectation that the real prototype properties will be inherited by its complex filter counterpart was confirmed ones again with respect to the quantization noise analysis. It was shown that both real and orthogonal complex LS1b-based filter sections having very low coefficient sensitivity demonstrate low output noise variance due to the input signal quantization – many times lower than that of the MHNS-based circuit.

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