Some filtering schemes to obtain low aliasing spectrum response

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Abstract: **Some analog and digital filtering schemes are investigated in the paper to be implemented in the systems with nonuniformly sampled data. The proposed filtering schemes are described regards to the hardware requirements, software realization, conversion speed and application field.**

Keywords: **nonuniformly sampled data, filtering schemes**

I. INTRODUCTION

Nonuniformly sampled data occurs in several applications such as geophysics [1], Laser Doppler Anemometry (LDA) [2], oscilloscopes [3] and radar or sonar signal processing [4]-[5]. Such type of data is used by the system designers to avoid aliasing in the signal spectrum or due to the technical problems, it is sometimes impossible to perform regular sampling. But filtering signals, which is nonuniformly sampled, is very difficult task, because the filtering coefficients are time varying [6]. The filtering task is more complicated problem when the signal bandwidth is very wide and the maximum spectrum frequency overcomes the Nyquist limit. In this case several spectrum estimation methods may be used to calculate the signal spectrum [7]-[9], but are distinguished with aliasing effects due to the non – orthogonal basis. The problem solution is concluded in the interpolation spectrum equation utilization [10] with applied filtering schemes to limit the signal bandwidth to the system Nyquist frequency.

The paper considers some possible analog and digital filtering schemes to overcome the aliasing effect problems in the systems, which use nonuniformly distributed grid.

II. FILTERING SCHEMES

The type of the proposed filtering schemes depends basically on the system sampling frequency and signal bandwidth. When the bandwidth is smaller or equal to the equivalent sampling frequency *f*s, then only one analog filter may be used. The filtered signal values are converted to the digital words by analog–to–digital converter (ADC), which sampled the input signal in nonuniformly distributed grid. The ADC output data are analyzed by the microprocessor (μP) by using the digital spectrum shifting according to the equation:

$$u(t_n) = x(t_n)e^{-j\gamma t_n}, \qquad (1)$$

where γ – shifting frequency

t_n – time sample points

The spectral shift analytical expression of the nonuniformly sampled data is implemented using the basic spectral shift equation to prove the equation (1):

$$U\left(e^{j2\pi\omega}\right) = X\left(e^{j2\pi(\omega-\gamma)}\right), \qquad (2)$$

where $U(j\omega)$ – shifted spectral response

γ – frequency shift.

The nonuniformly sampled data $u(t_n)$ are calculated by well known inverse Fourier transform:

$$u(t_n) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} U(j\omega)e^{-j\omega t_n} d\omega \qquad (3)$$

If the equation (2) is substituted at the definition equation (3), the nonuniformly sampled data $u(t_n)$ are estimated according to the equation:

$$u(t_n) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} X(j\Omega)e^{-j(\gamma+\Omega)t_n} d\Omega =$$
$$= \frac{e^{-j\gamma t_n}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} X(j\Omega)e^{-j\Omega t_n} d\Omega = x(t_n)e^{-j\gamma t_n}$$

When the signal bandwidth exceeded the equivalent sampling frequency *f*s, the more complicated filtered schemes have to be used to analyze the input signal. The following sections represent some analog and digital filtering schemes, which may be implemented to estimate the signal spectrum response.

The spectrum estimation procedure consists of the following steps:

1. The input signal is filtered by using the analog or digital filters to select the frequency response of the desired frequency band. The *i*-th filter passband is defined according to the expression $f_p = [(i-1)f_s; if_s], i = 1,...,n$ (Fig.1), so the filter passband is equal to the sampling frequency;
2. If the signal is passed through analog bandpass filter, the input signal is converted to digital words by analog-to-digital converter. The ADC sampling points are nonuniformly spaced and the equivalent sampling frequency *f*s is below Nyquist limit, i.e. it accomplished undersampling process;
3. The spectrum shift procedure is performed by equation (1) to translate the analyzed frequency band to baseband;

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4. The frequency response is calculated by interpolation spectrum analysis of the nonuniformly sampled data to obtain low aliasing spectrum.

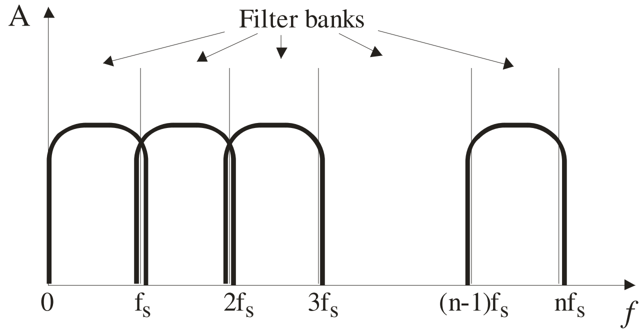

Fig. 1. Filter bank scheme

The next sections discuss the possible filtering schemes to realize the first step of the proposed procedures, while step 2 is analyzed at [11], step 3 is accomplished according to equation (1) and step 4 is discussed in our previous work [10].

2.1. *Analog filtering schemes* – The first proposed analog filtering scheme is defined as a parallel filtering scheme (Fig.2) and it contains multiple bandpass filters (BPFs) and analog-to-digital converters (ADCs). The number of the filter branches is calculated according to the equation:

$$n = \frac{B}{f_s}, \tag{4}$$

where B – signal bandwidth.

The ADC output data are sent to the µP via serial interface (SPI, I^2C, etc.). The microprocessor starts the conversion simultaneously for all ADCs to ensure identical time sample points at the circuit branches, but extracts the converted signal values consecutively.

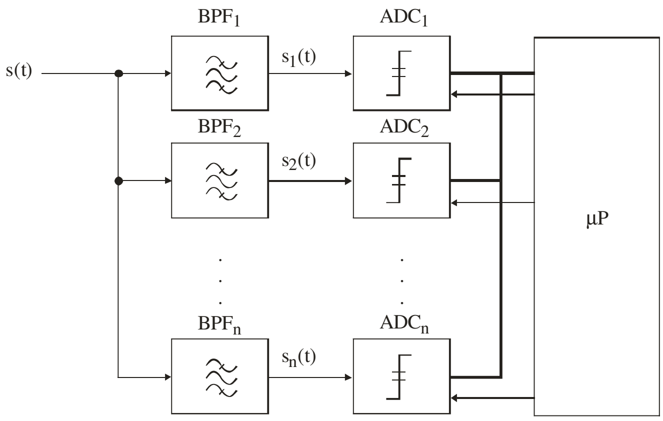

Fig. 2. Parallel filtering scheme

The proposed parallel filtering scheme may produce real time spectrum analysis of the input signal, but requires multiple hardware blocks to accomplish the digital signal processing (n BPFs and n ADCs). The number of the required hardware may be significantly reduced if multiplexed filtering scheme is used (Fig.3), because it requires only one ADC. The additional block is identified as a multiplexer (MUX), which switches the input signals to its output. At the expense of the reduced number of ADCs, this filtering scheme requires faster ADC regard to the parallel filtering scheme.

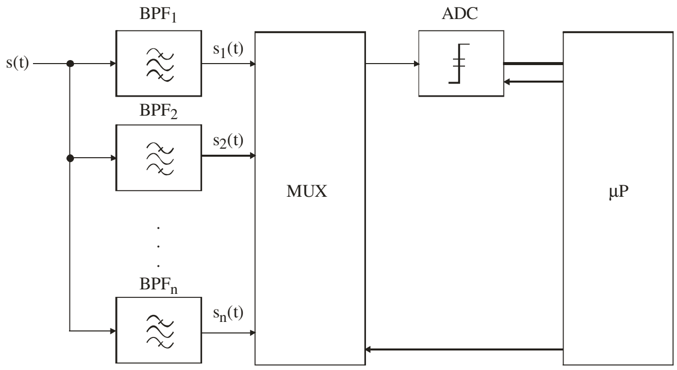

Fig. 3. Multiplexed filtering scheme

When the filter number exceeds the design requirements, then the programmable filtering scheme may be used (Fig.4). It is based on the implementation of the programmable BPF by the included programmable resistors. This scheme is distinguished as the most hardware saving system, but it is the slowest filtering scheme due to the required programming time. Regardless of the selected disadvantage it may be used at the vibration analysis systems (engines, constructions, etc.), where the signal processing time is not a critical parameter.

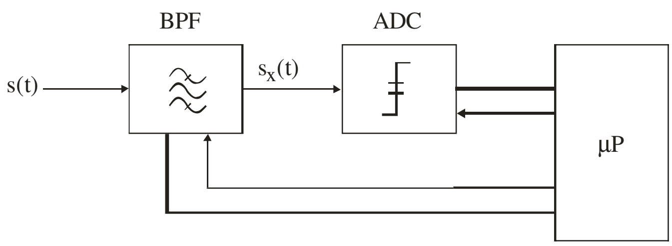

Fig. 4. Programmable filtering scheme

2.2. *Digital filtering schemes* – The digital filtering scheme (Fig.5) is a modern filtering scheme to produce the spectrum analysis of the nonuniform sampled signals. The main problem is concluded in the calculation of the filter coefficients since they have to be time varying [6]:

$$y(\tau) = a_{k\tau}s(t_k) + a_{k-1,\tau}s(t_{k-1}) + \ldots + a_{k-N,\tau}s(t_{k-N}) \tag{5}$$

When the analyzed signals are baseband and bandlimited, Tarczynski *at all* [6] proposed Weighed Least Squares (WLS) approach to solve the problem in the FIR filters. Otherwise, the solution of the selected problem remains unsolved yet. The published filter solution is non-applicable if the signal bandwidth exceeds the sampling frequency, which case is observed in our paper.

Due to its undeniable advantages regards to the analog filtering schemes (higher filter order, bandpass programmable capability, easy software implementation, etc.), the proposed filtering scheme is very suitable for the contemporary digital signal processing solutions.

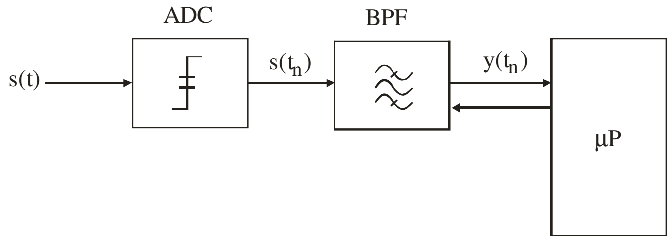

Fig. 4. Digital filtering scheme

Regardless of the selected advantage, the digital filtering scheme theory is not well developed yet, so it remains the unsolved problem at the signal processing field.

III. CONCLUSION

The nonuniformly sampled data is widely used in the modern digital processing systems, so the questions, connected with their spectrum analysis, are actual problems. The spectrum analysis above Nyquist limit is accomplished at four steps as the first one requires implementation of the filtering schemes to obtain low frequency aliasing spectrum response.

The current paper examines some of the possible analog and digital filtering schemes and indicates their main advantages and disadvantages. Also it describes their application fields to define the scheme enclosures. The contemporary requirements to the digital processing systems are connected to the software implementation of the filtering schemes, their power consumption, low price and integration. The most suitable solution of the shown filtering schemes to meet the selected requirements is identified as a digital filtering scheme. Unfortunately, the unsolved problems, connected with filter coefficients calculation, limit their application.

IV. REFERENCES

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